CHAPTER ONE

GENERAL INTRODUCTION

Loads and the behavior of structure under loads action:

Action (Loads)		Internal Resistance	Reaction (Forces)	Structures
Axial Loads	Tension	Elongation	• **Tension Force**	• Truss • Arch • Beam with inclined load • Inclined beam • Plane frame
	Compression	Contraction	• **Compression Force**	
Transverse Loads	Concentrated	Bending	• **Shear force** • **Bending Moment**	• Beam • Plane frame • Arch
	Distributed (Uniformly)			
	Distributed (Varying linearly)			
	Distributed: (Trapezoidal)			

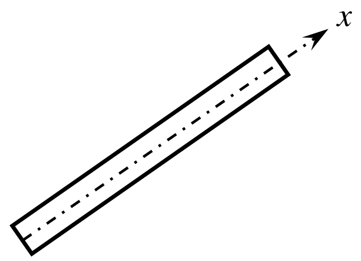
Coordinates System:

There are many types of system coordinates like: Cartesian coordinates system, polar coordinate system, cylindrical coordinate system, natural coordinate system, curvilinear coordinate system. The most general coordinate system used in structural analysis is Cartesian coordinate system, which classify to two axes the one is the ***local axis*** and the other is ***global axis***.

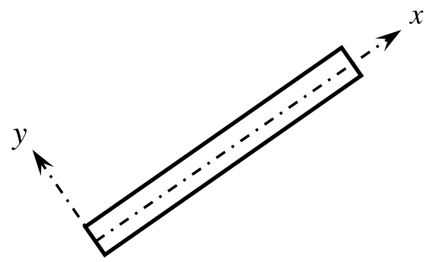
Local axis: is the axis defines the element locally and the terms used for it is *x*, *y* and *z*.

Where:

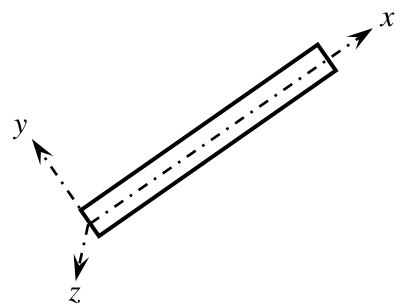
Axis x: is the ***axial axis*** which is the axis pass through the element between ends and its direction follow the element direction.



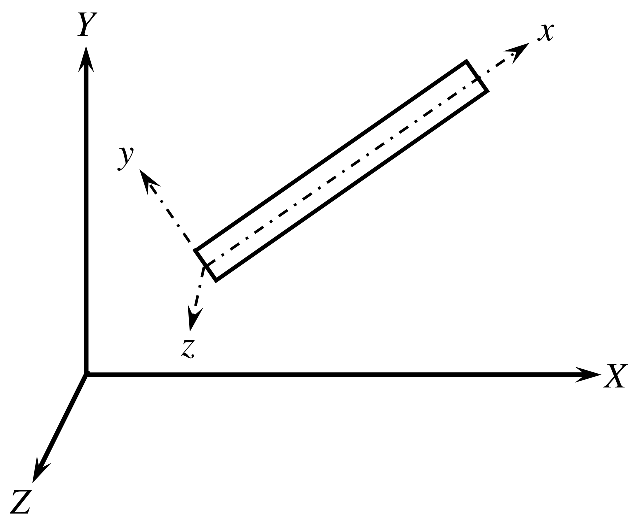
Axis *y*: is ***transverse axis*** and always perpendicular to *x*.



Axis *z*: is perpendicular to the plane *xy*.



Global axis: is the axis defines the structure, in other words all the elements are corporate the same axis termed as X, Y and Z.



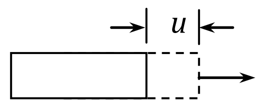
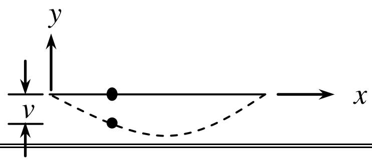
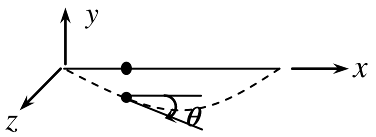
Displacements:

There is two types of displacements:

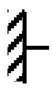
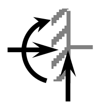
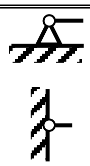
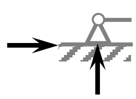
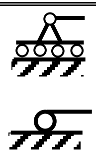
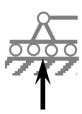
Translation: is the *moving* from point to point.

Rotation: is the sloping of element at appoint from straight position.

According to the direction, the displacements are described as:

Type of Displacement	Description	
Translation	*Axial translation along axial axis x (u)*	
	Transverse translation along transverse axis y (v)	
Rotation	*Rotation by an angle from straight horizontal about z axis*	

Supports:
The supports are special joints where located at the ends of elements and where the applied loads are finally transmitted to it. The resisting force against translation/rotation at the support are called "support reactions"

Type of Support	Symbol	Forces at Support	Displacements at Support
Fixed			*Translation* → $= 0$ *Translation* ↑ $= 0$ *Rotation* $= 0$
Pinned or Hinge			*Translation* → $= 0$ *Translation* ↑ $= 0$ *Rotation* $\neq 0$
Hinge (Roller)			*Translation* → $\neq 0$ *Translation* ↑ $= 0$ *Rotation* $\neq 0$

Equilibrium Equation:

For a two dimensional body occupied a region on the X-Y plane, equilibrium of this body implies that all forces and moments applied to the body must satisfy the following three equations:

$$\sum F_X = 0 \qquad (1)$$
$$\sum F_Y = 0 \qquad (2)$$
$$\sum M_Z = 0 \qquad (3)$$

SHEAR FORCE AND BENDING MOMENT

Introduction:

The internal forces in any point or section can be determined by making cut at specified point or section and there are three forces appear which are:

1. *Axial forces (acting along the axial axis **x**).*
2. *Shear force (acting at transverse axes **y** and/or **z**).*
3. *Bending moment (acting about transverse axes **y** and/or **z**).*

Axial force:

Is the force tending to produce elongation in case of tension force or reduction in case of compression force.

The axial force results from *axial loads* act at elements like *straight beam have inclined loads* or *inclined beam* or in *frames*. It's value in an element equal to the algebraic sum of all axial loads act at this element The unit of axial force is N or kN.

Shear force:

Is the force tending to produce a shear failure at a given point in a beam or frame due to *transverse loads*, the value of it at any point in a beam or frame = the algebraic sum of all upward and downward forces to the left of the point. (The term "*algebraic sum*" means that upward forces are regarded as being positive and downward forces are considered to be negative). The unit of shear force is N or kN.

Bending moment:

Is the magnitude of the bending effect at any point in a beam or frame which is a force multiplied by a perpendicular distance, it is either clockwise or anticlockwise and is measured in kN.m or N.mm. The value of bending moment at any point on a beam or frame = the sum of all bending moments to the left of the point.

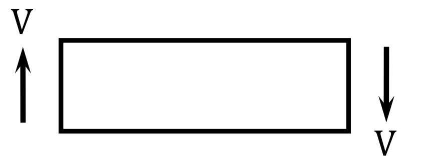
The sign convention:

The sign convention is the determination of the signs of forces (positive or negative) according to the rule set from starting.

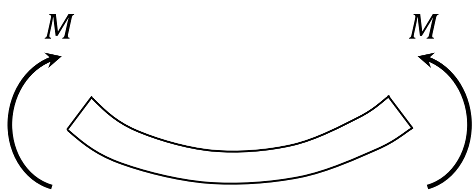
Take the following rule:

- For the axial force N, if the force tends to elongate the segment consider this force is positive.

- For the shear force V, if the force tends to rotate the segment clockwise, consider this force is positive.



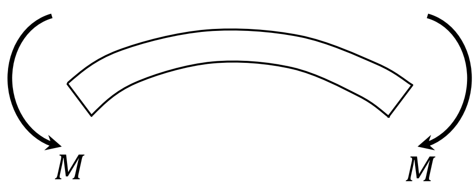
- For the bending moment if the moment tends to make a sag (concave or downwards curvature) in the segment, consider this moment is positive.



Sagging Moment (+ ve), tension bottom

Note that:

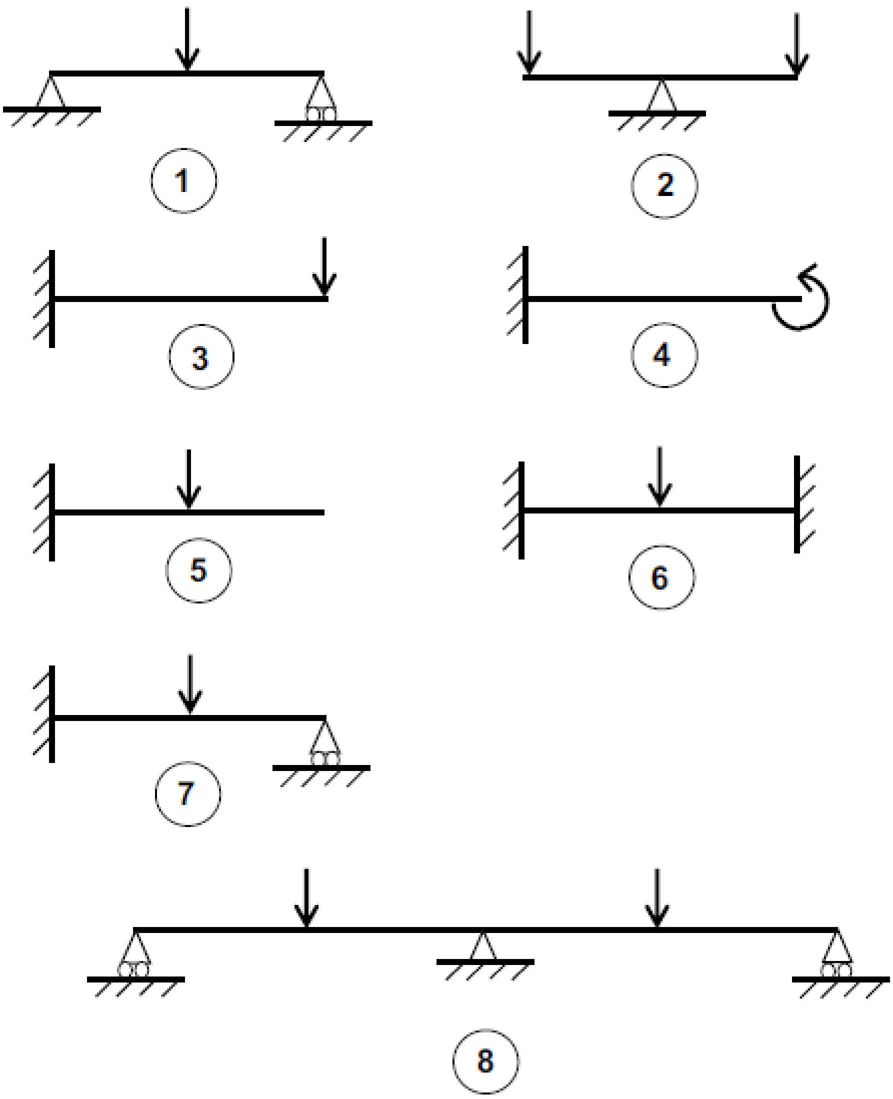
- *the opposite to **sag** is **hog** which is the curvature upwards*



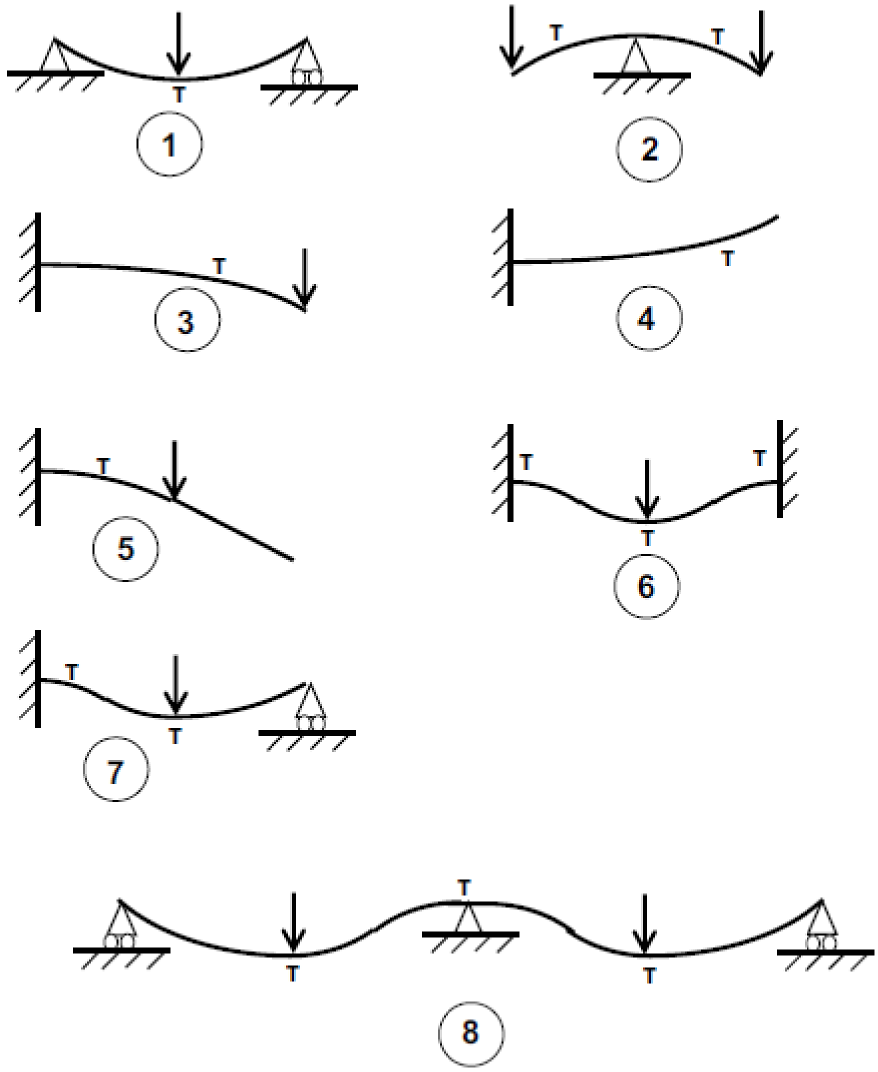
Hogging Moment (– ve), tension top

Example:

Draw the deflected shape under the action of applied loads, by determining ***sag*** *and* ***hog*** *for the following beams:*



Solution:



Figures explains the deflected shape under loads

Notes about shear force:

- *You can observe that shear force is constant between concentrated loads, but it varies linearly along uniformly distributed load.*
- *The maximum values of shear force are at ends and equal in values to the reactions.*

Notes about bending moment:

- *The bending moment is varies linearly between concentrated loads, but it draw curve throw the distributed loads.*
- *Maximum bending moments obtained at points of concentrated loads and points of reactions.*
- *The points of zero moment fall in-between ends called the points of contraflexure.*

Relation between Shear Force and Bending Moment

If the bending moment at any section x-x (M_x), is the function of x, where x is the distance measured from left side to the section considered, then the slope at any point is equal to the shear force.

$$S_x = \frac{dM_x}{dx}$$

When the applied load is distributed load, notices that:

- *the **maximum bending moment** corresponds **zero shear force**.*
- *The first derivative of shear force gives the distributed load*

$$w = \frac{dS_x}{dx}$$

Shear force and bending moment diagram:

The shear force and bending moment diagrams is the drawing represents the variation of shear force and bending moment along the beam or frames by draw the values using graphical representations.

To draw the shear force diagram follow the following steps:

1. Draw the datum (Zero Shear Force Line).
2. Marks any points of:
    a. Concentrated loads.
    b. Start and end of distributed load.
    c. Reactions.
3. Calculate the values of shear force at the marking points, and notice that at any point of concentrated load the shear force is calculated twice one exclude the load and the other include the load.
4. Select suitable scale and put a dot at any marked point. The dot must be upward or downward the datum according to the sign of shear force value if positive put the dot upward if negative put the dot downward the datum.
5. Connect between the dots start from the zero point at the left and continue in series until reach the zero point at right. The final shape is shear force diagram.

To draw the bending moment diagram follow the following steps:

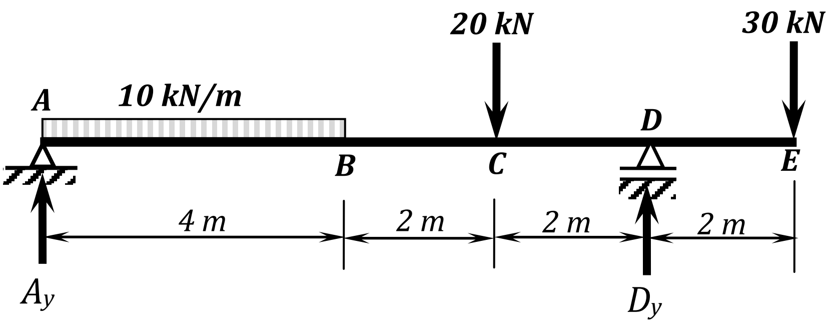
1. Draw the datum (Zero Bending Moment Line).
2. Marks any points of:
    a. Concentrated loads.

b. Start and end of distributed load.
c. Reactions.
d. Zero shear force.

3. Calculate the values of bending moment at the marking points.
4. Calculate the values of maximum moment at the points that are coincide with the points of zero shear force.
5. Select suitable scale and put a dot at any marked point. The dot must be upward or downward the datum according to the sign of shear force value if positive put the dot upward if negative put the dot downward the datum.
6. Connect between the dots start from the zero point at the left and continue in series until reach the zero point at right with notice that the line drawn under distributed load is not drawn as straight line but it drawn as curve. The final shape is shear force diagram.

Example 1:

Draw the shear force and bending moment diagram for the beam shown in Figure below and find the points of contraflexure if exist.



Solution:

Calculate Reactions

By taking moment about D ($\sum M_D = 0$):

$$A_y(8) - 10(4)\left(\frac{4}{2} + 4\right) - 20(2) + 30(2) = 0$$

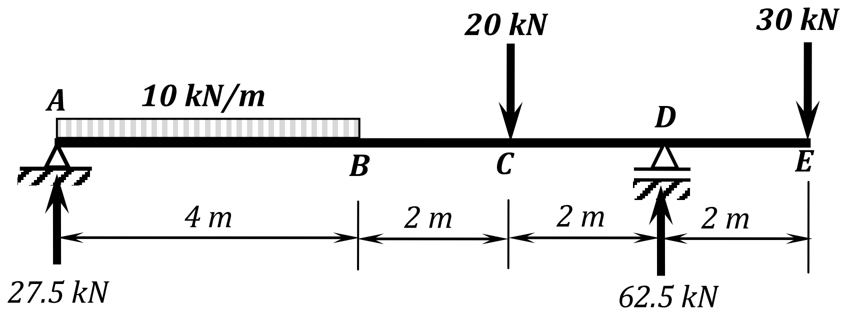
$$A_y(8) = 240 + 40 - 60$$

$$A_y = \frac{220}{8} = 27.5\ kN$$

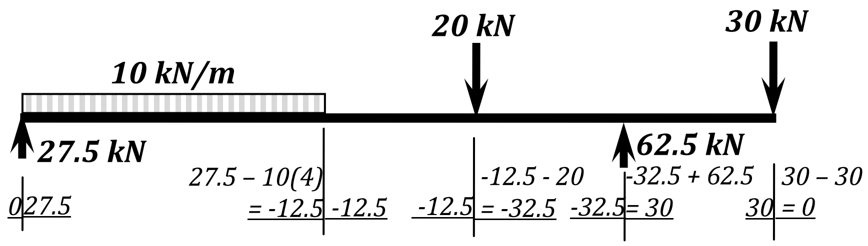
By taking the vertical balance: ($\sum F_y = 0$):

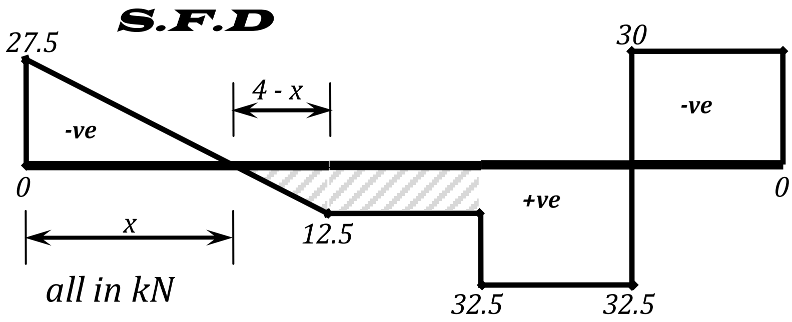
$$A_y + D_y = 10(4) + 20 + 30$$

$$D_y = 90 - 27.5 = 62.5\ kN$$



Shear Force:





Between A & E there is a maximum bending moment coincide with zero shear force at distance x from A
Shear force between A & E can be calculated from the equation:

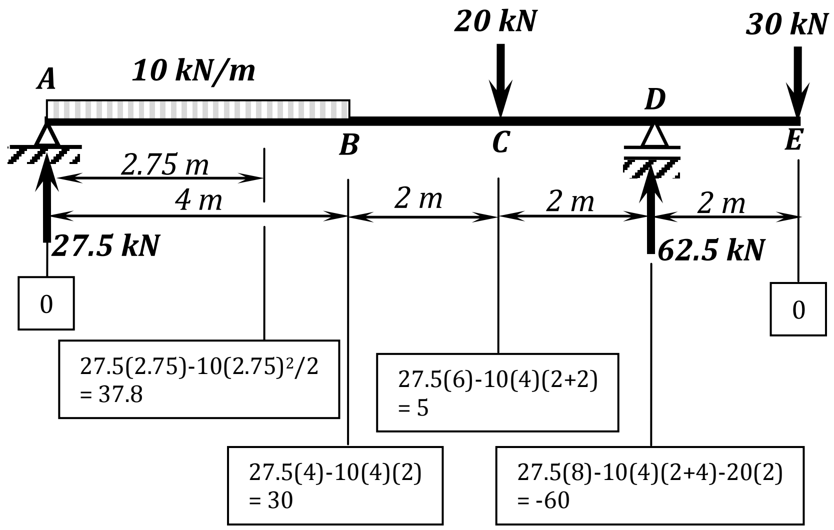
$$S_x = A_y - w(x)$$

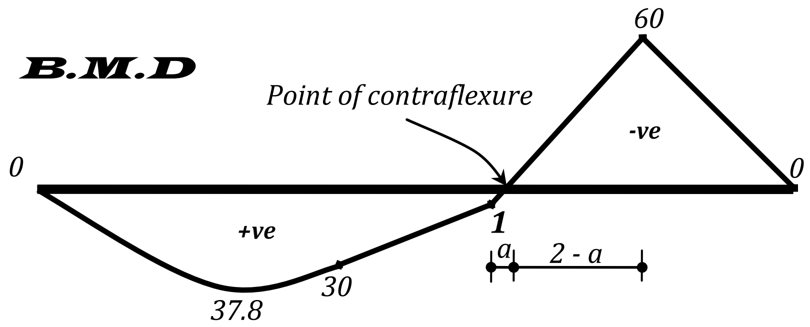
When $S_x = 0 \Rightarrow w(x) = A_y$

From that $x = A_y/w$

$$x = \frac{27.5}{10} = 2.75\ m$$

Bending Moment:





all in kN.m

To find the location of point of contraflexure:
Assuming the point is at distance a from C, then:

$$\frac{a}{5} = \frac{2-a}{60}$$

$$\frac{60(a)}{5} = 2 - a$$

$$12a + a = 2$$

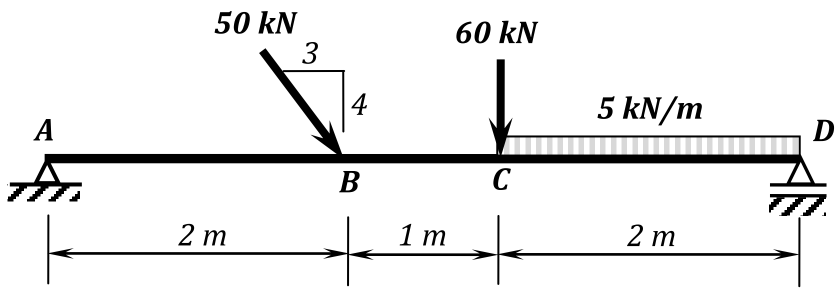
$$a = \frac{2}{13} = 0.154\ m$$

This means the point of contraflexure is at distance of 0.154 m from C.

Example 2:

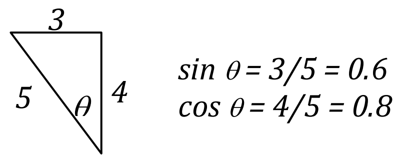
For the beam shown in Figure below, draw:

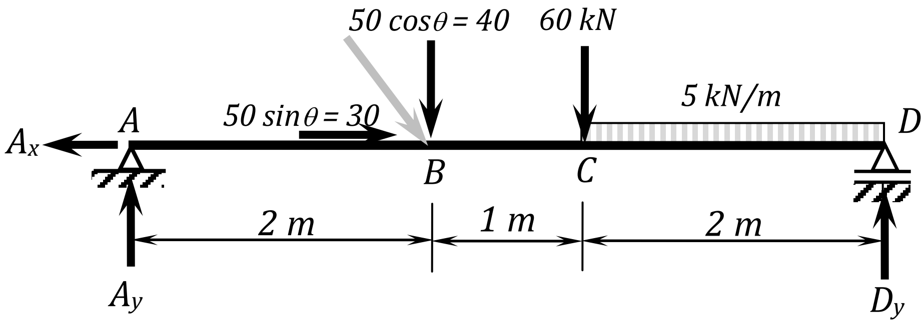
1. *Axial force diagram.*
2. *Shear force diagram*
3. *Bending moment diagram.*



Solution:

Firstly resolve the inclined load:





Calculate Reactions

By taking moment about D $(\sum M_D = 0)$*:*

$$A_y(5) - 40(3) - 60(2) - 5(2)\left(\frac{2}{2}\right) = 0$$

$$A_y(5) = 120 + 120 + 10$$

$$A_y = \frac{250}{5} = 50\ kN$$

By taking the vertical balance: $(\sum F_y = 0)$*:*

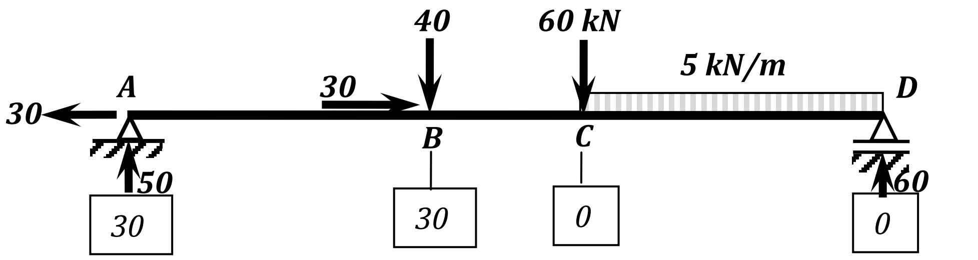
$$A_y + D_y = 40 + 5(2) + 60$$

$$D_y = 110 - 50 = 60\ kN$$

By taking the horizontal balance: $(\sum F_x = 0)$*:*

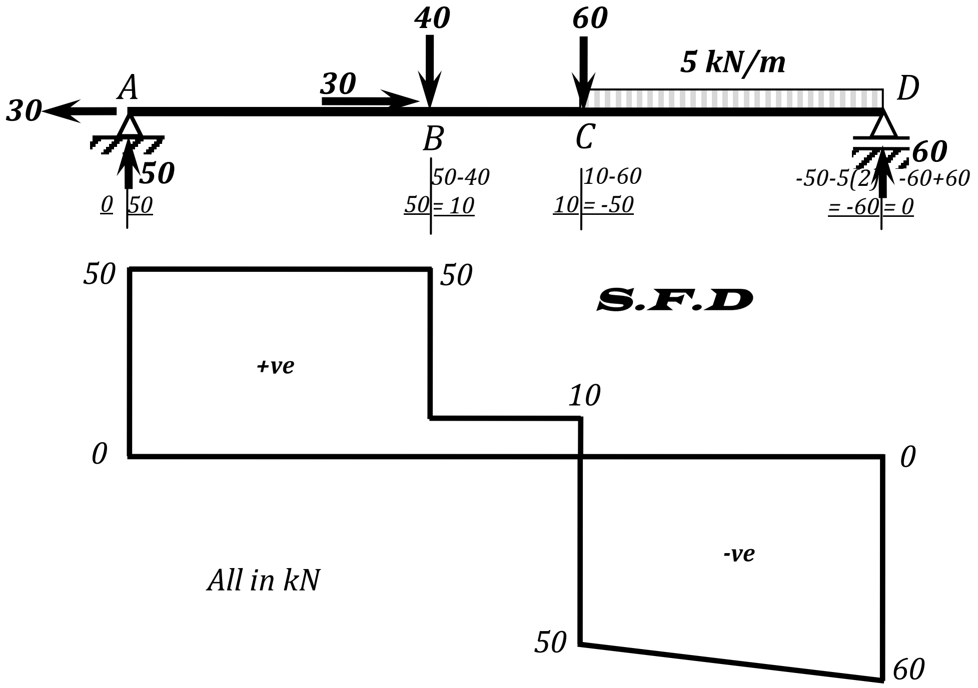
$$A_x = 30\ kN$$

Axial force:

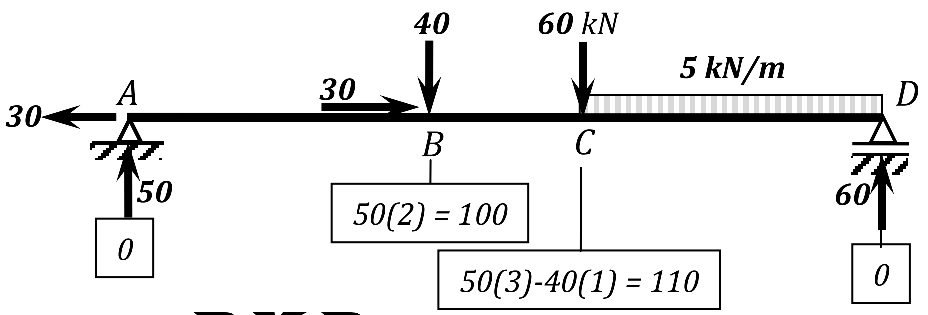


A.F.D

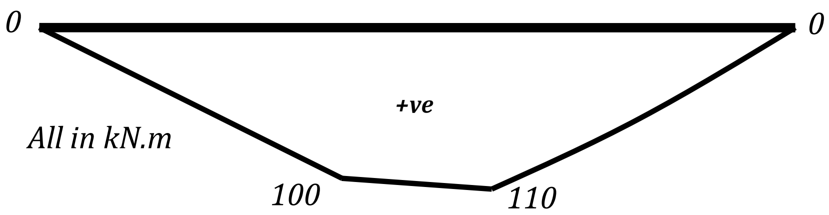
Shear force:



Bending moment:

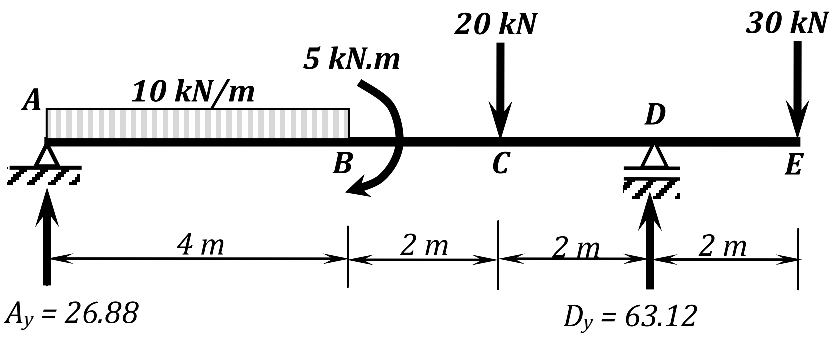


B.M.D



Example 3:

Draw the shear force and bending moment for the beam shown in Figure below.



Take moment about D ($\sum M_D = 0$):

$$A_y(8) - 10(4)(2+4) + 5 - 20(2) + 30(2) = 0$$

$$A_y(8) = 215$$

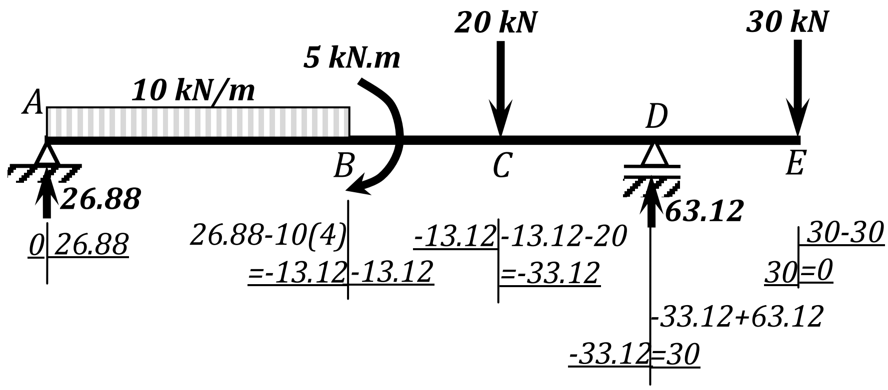
$$A_y = \frac{215}{8} = 26.88\ kN$$

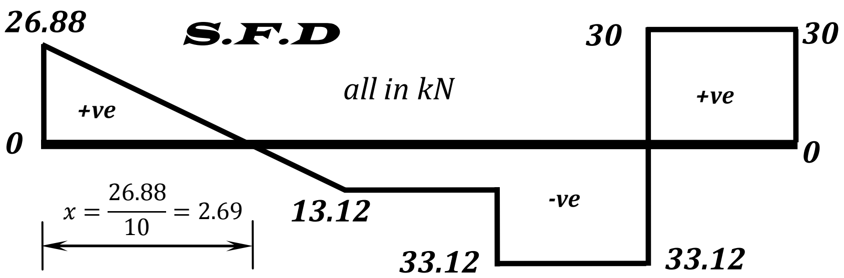
By taking the vertical balance: ($\sum F_y = 0$):

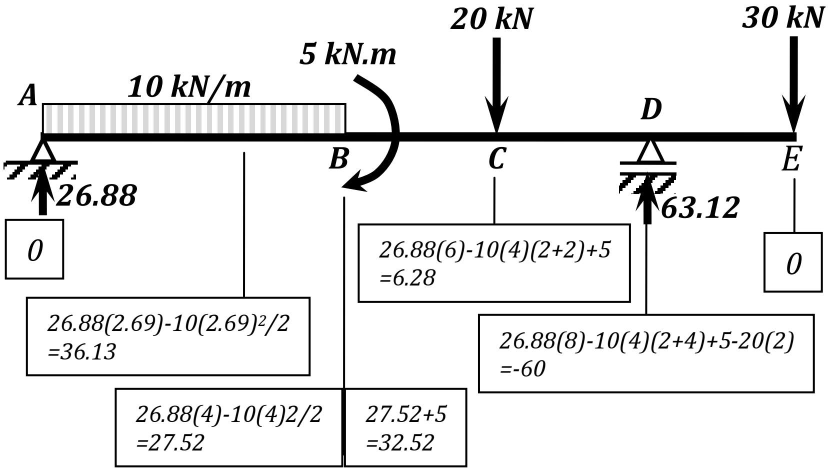
$$A_y + D_y = 10(4) + 20 + 30$$

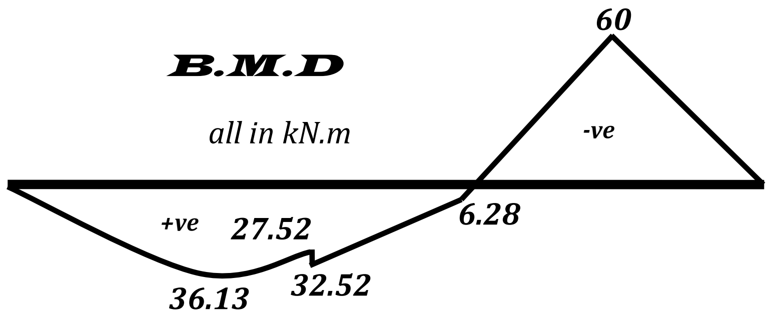
$$D_y = 90 - 26.88 = 63.12\ kN$$

Shear force:



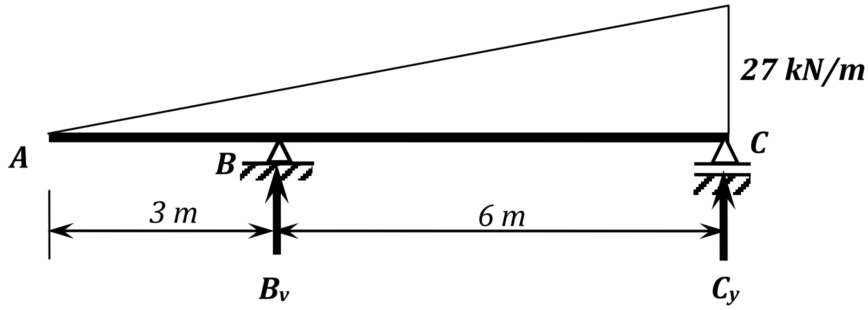






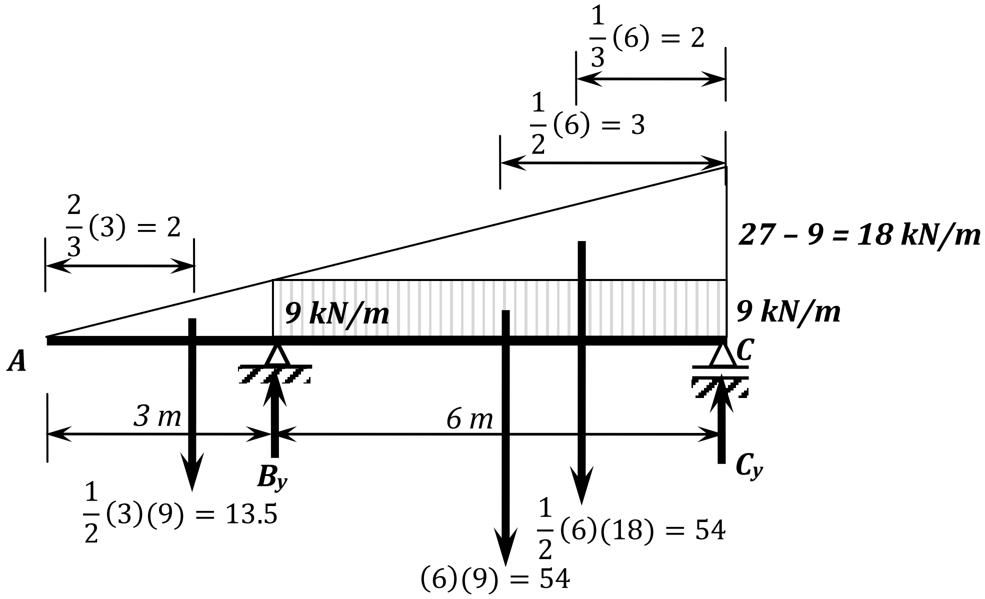
Example 4:

Draw the shear force and bending moment for the beam shown in figure below



Solution:

Reactions:



Take moment about C $(\sum M_C = 0)$*:*

$$-13.5(9-2) + B_y(6) - 54(3) - 54(2) = 0$$

$$B_y(6) = 364.5$$

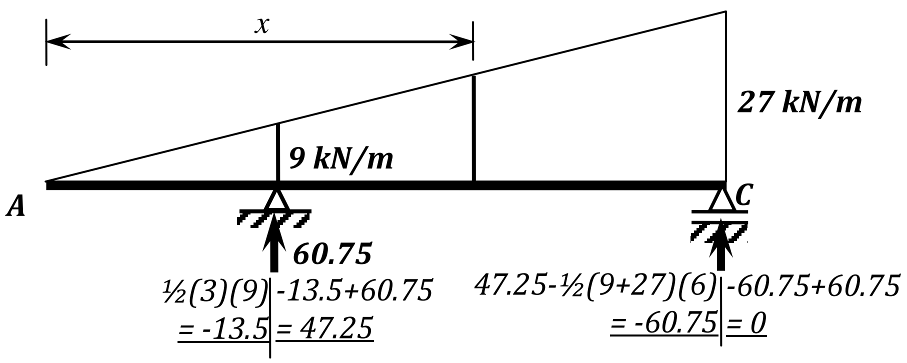
$$B_y = \frac{364.5}{6} = 60.75\ kN$$

By taking the vertical balance: $(\sum F_y = 0)$*:*

$$B_y + C_y = \frac{1}{2}(9)(27)$$

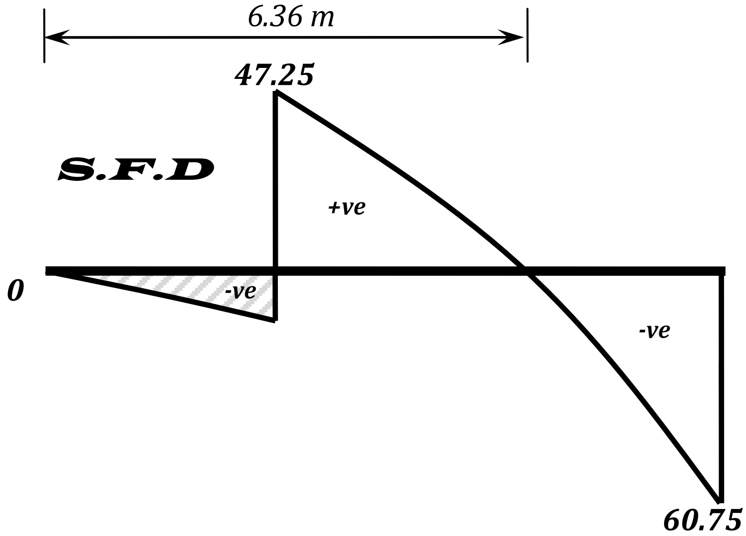
$$C_y = 121.5 - 60.75 = 60.75\ kN$$

Shear force:



$$S_x = -\frac{1}{2}(x)(3x) + 60.75 = 0$$

$$x = \sqrt{\frac{60.75}{1.5}} = 6.36\ m$$



Bending moment: