CHAPTER SEVEN

INFLUENCE LINES

For statically determinate structures

The influence line is the diagram of function of, reaction or shear force or bending moment at a particular section when a unit load move over the span. It is used to study the magnitude of forces in structures due to the moving loads, specially in bridges and cranes.

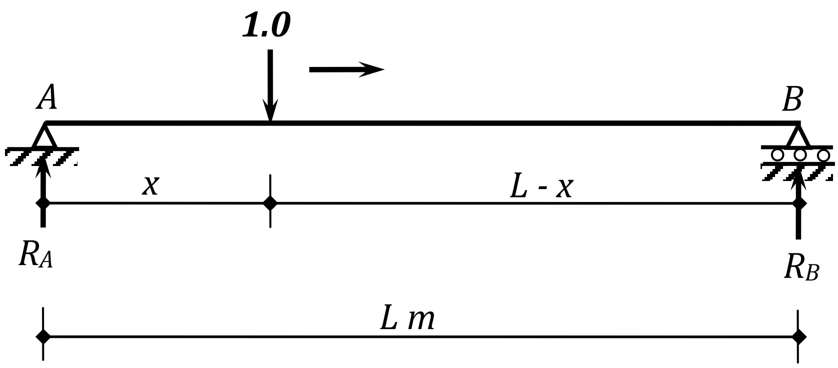
Influence line for beams:

For the forces:

1. Reactions.
2. Shear force.
3. Bending moment.

Example1 (Reactions):

*For the beam shown in figure draw the values of reaction at joints **A** and **B** when the unit load move from **A** to **B**.*



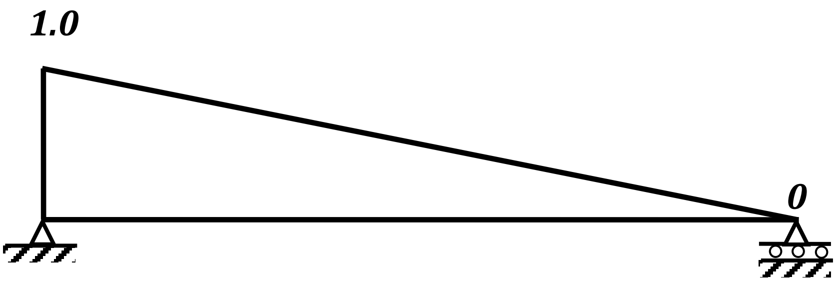
$$R_A = \frac{L-x}{L} = 1 - \frac{x}{L}$$

$$R_B = \frac{x}{L}$$

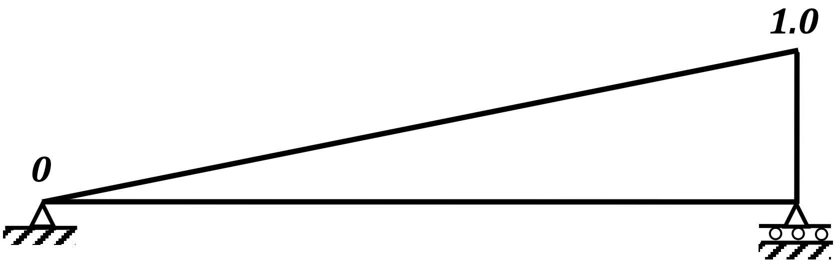
Use the above expression to calculate the value of R_A *and* R_B *when the load move from* **A** *to* **B** *and the values calculated to* **10** *distances start from* **0** *to* **L** *by an increment of* **0.1L**, *the values appears in table below.*

Distance from A	R_A	R_B
0	1	0
0.1L	0.9	0.1
0.2L	0.8	0.2
0.3L	0.7	0.3
0.4L	0.6	0.4
0.5L	0.5	0.5
0.6L	0.4	0.6
0.7L	0.3	0.7
0.8L	0.2	0.8
0.9L	0.1	0.9
L	0	1

By drawing these values:



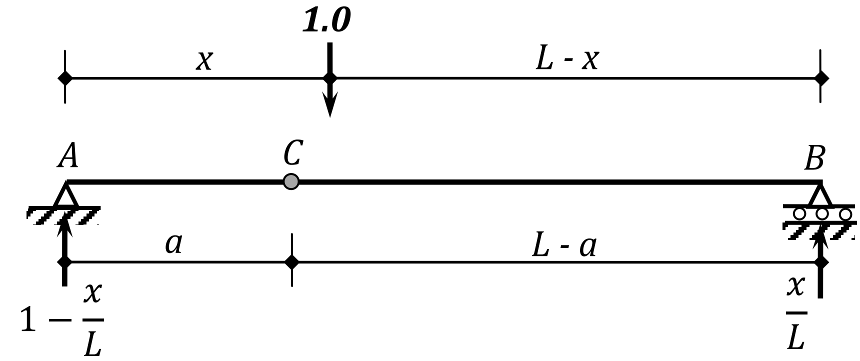
Influence line for R_A:



Influence line for R_B:

Example2 (Shear Force):

*For the beam shown in figure draw the values of shear force at section **a** from **A** when the unit load move from **A** to **B**.*



If x ≤ a the shear force at C is:

$$S_C = 1 - \frac{x}{L} - 1 = -\frac{x}{L} \qquad (1)$$

If x ≥ a the
force at C is:

$$S_C = 1 - \frac{x}{L} \qquad (2)$$

use equation (1) & (2) to calculate the shear force as follows:

at A: x = 0, then the shear force at C is:

$$S_C = -\frac{0}{L} = 0$$

at C: x = a, then the shear force at C is:

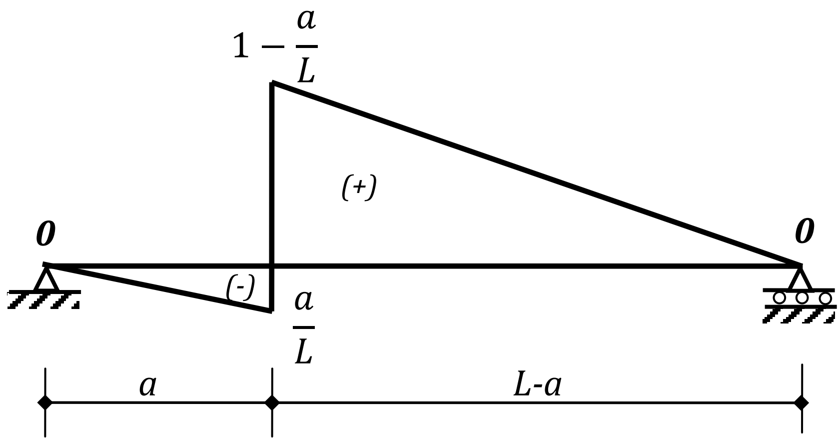
$$S_C = -\frac{a}{L}, from\ equation\ (1)$$

$$S_C = 1 - \frac{a}{L}, from\ equation\ (2)$$

at B: x = L, then the shear force at C is:

$$S_C = 1 - \frac{L}{L} = 0$$

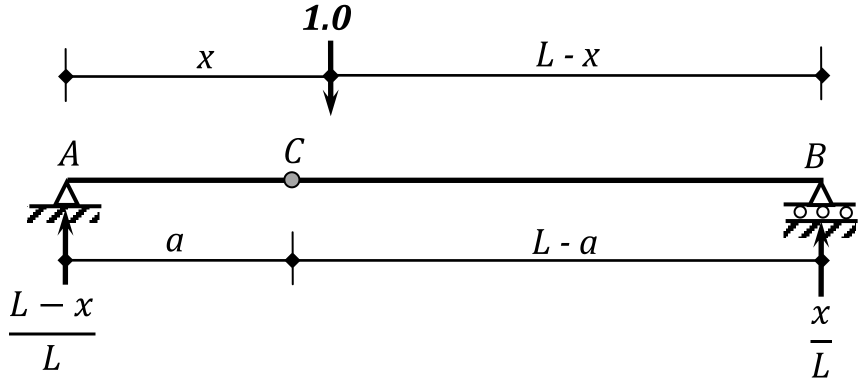
Draw the values in table:



*The diagram is the influence line for **shear force** at **C** of distance **a** from **A**.*

Example3 (Bending Moment):

*For the beam shown in figure draw the values of bending moment at section **a** from **A** when the unit load move from **A** to **B**.*



at A: x = 0, then the moment at C is:

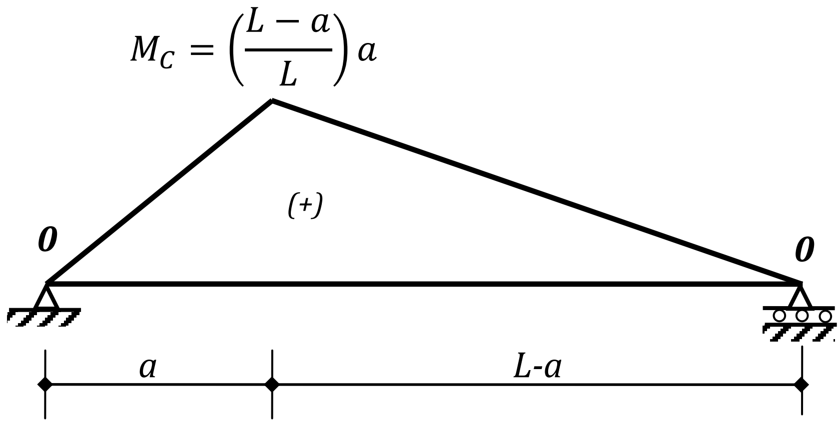
$$M_C = \left(\frac{L-0}{L}\right)a - 1.0(a) = a - a = 0$$

at C: x = a, then the moment at C is:

$$M_C = \left(\frac{L-a}{L}\right)a$$

at B: x = L, then the moment at C is:

$$M_C = \left(\frac{L-L}{L}\right)a = 0$$



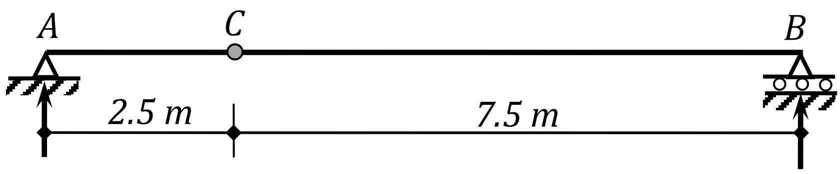
The diagram is the influence line for ***bending moment*** *at* ***C*** *of distance* ***a*** *from* ***A****.*

Example4:

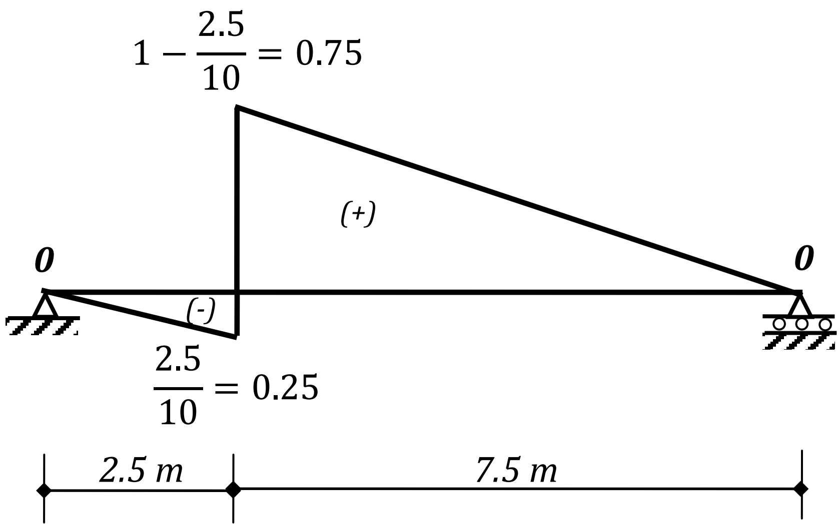
A simply supported beam of length 10 m, draw the influence line for shear force and bending moment at a quarter of the beam.

Solution:

At a quarter distance mean a = 10/4 = 2.5 m

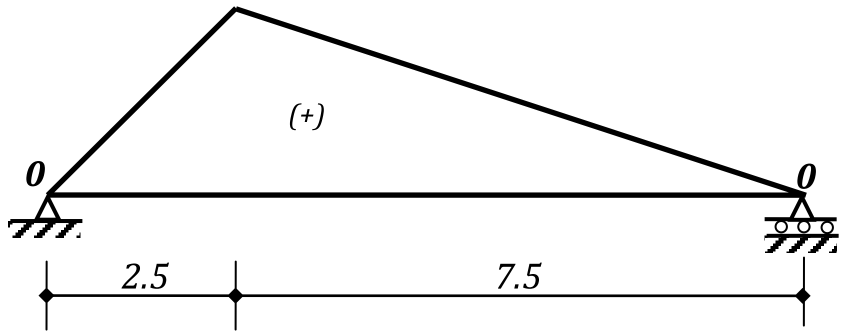


Influence line for shear force:



Influence line for bending moment:

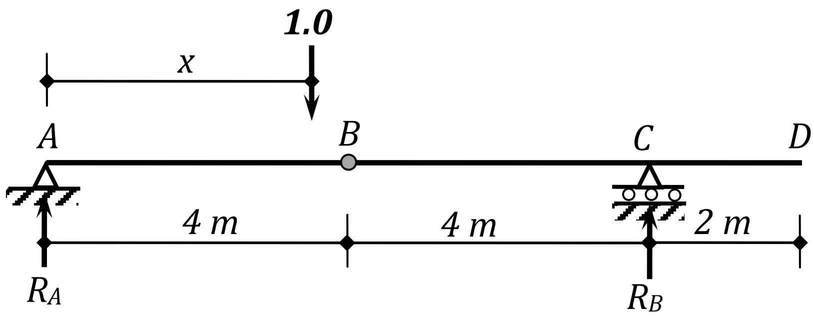
$$M_C = \left(\frac{10 - 2.5}{10}\right) 2.5 = 0.1875$$



Example5:

A simply supported beam of length 8 m with overhang 2 m from B as shown in figure, draw the influence line for:

1. *Reactions R_A and R_B.*
2. *Shear force at B.*
3. *Bending moment at B.*



Solution:

1.

when load 1.0 moves from A to D the reactions are:

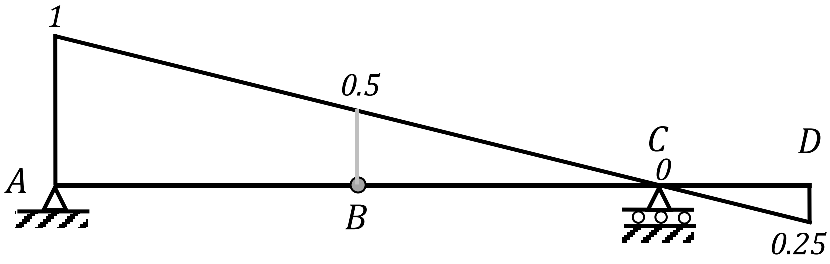
$$R_A = 1 - \frac{x}{8}$$

$$R_B = \frac{x}{8}$$

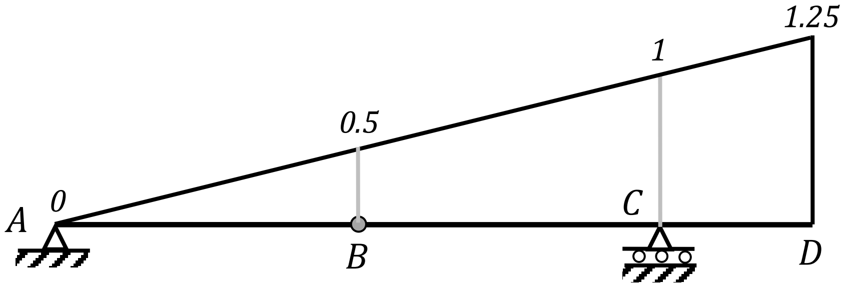
The values of reactions according to the values of x, are shown in table below:

Joint	*x*	*R_A*	*R_B*
A	0	1	0
B	4	0.5	0.5
C	8	0	1
D	10	-0.25	1.25

Influence line for R_A:



Influence line for R_B:



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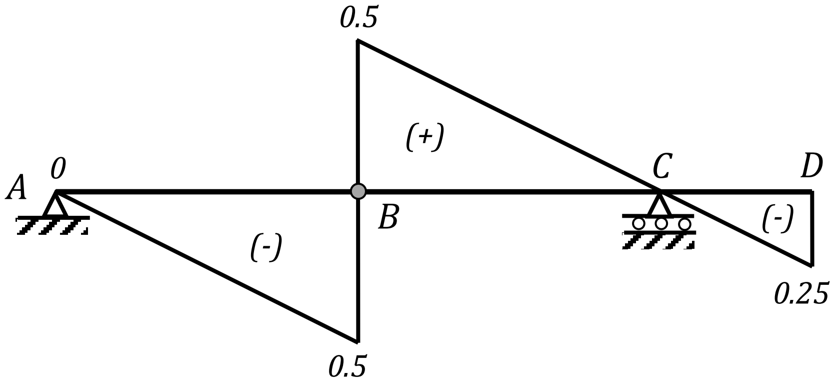
2.

The values of shear force at joint B according to the values of x, are shown in table below:

Joint	x	R_A	S_B
A	0	1	1 – 1 = 0
B (just to left)	4	0.5	0.5 – 1 = -0.5
B (just after)	4	0.5	0.5
C	8	0	0
D	10	-0.25	-0.25

Influence line for Shear force at B:



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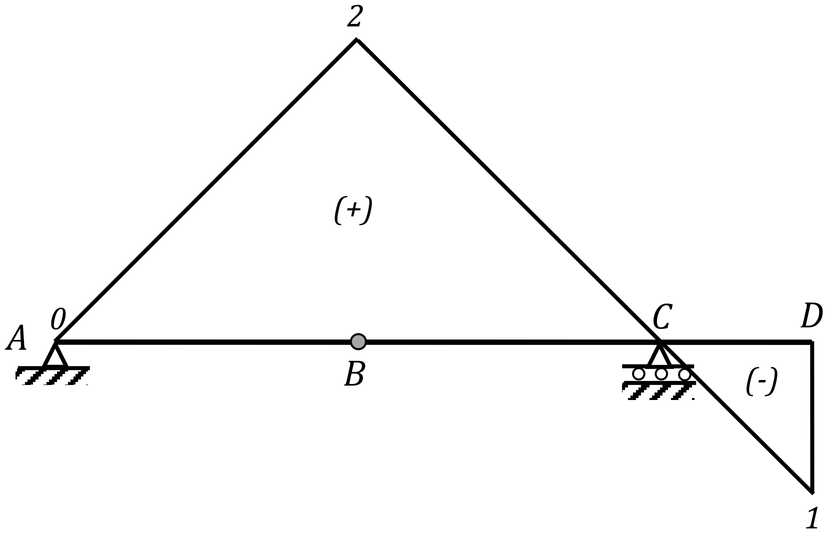
3.

The values of bending moment at joint B according to the values of x, are shown in table below:

Joint	x	R_A	M_B
A	0	1	1(4) – 1(4) = 0
B	4	0.5	0.5(4) = 2
C	8	0	0(4) = 0
D	10	-0.25	-0.25(4) = -1

Influence line for bending moment at B:



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Moving loads:

In order to calculate the maximum values of shear force and bending moment in simple beam when there are moving loads across the beams by using the influence lines, these loads can be classified to many cases, these cases are:

1. Case of single concentrated load.
2. Case of series of concentrated loads.
3. Case of distributed load.

1. <u>Case of single concentrated load:</u>

Shear force:

*The maximum **positive** shear force obtained when the load located just to the right of section and the value is equal to the load multiply by the positive influence line ordinate.*

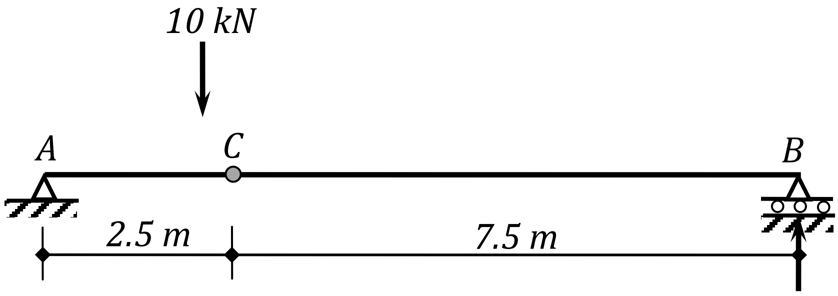
*The maximum **negative** shear force obtained when the load located just to the left of section and the value is equal to the load multiply by the negative influence line ordinate.*

Bending moment:

The maximum bending moment obtained when the load is at the section and the value is equal to the load multiply by the ordinate at section.

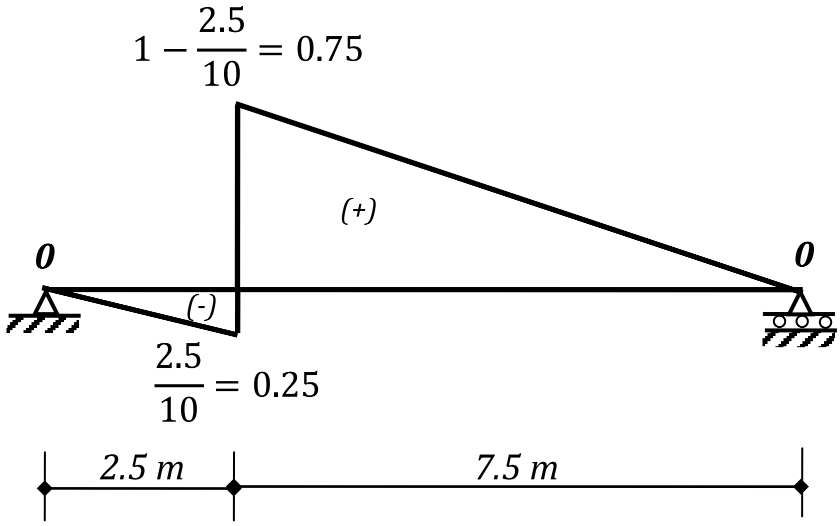
Example:

Find the maximum positive and maximum negative of shear force and maximum bending moment for the beam shown in figure at point C when the load 10 kN move across the beam.



Solution:

Draw the influence line for shear force.



The maximum shear force obtained when the load at section, so multiply the load by positive ordinate for maximum positive and by negative ordinate for maximum negative.

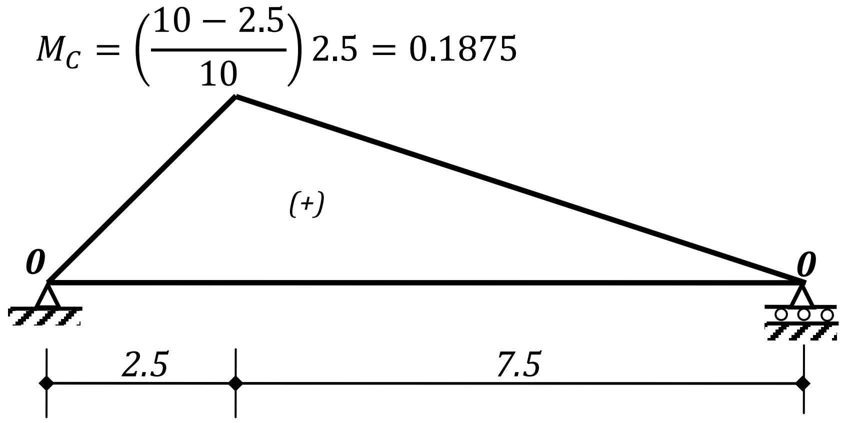
The positive ordinate = 0.75, so:

the maximum positive shear force = 10 x *0.75 = 7.5 kN*

The negative ordinate = 0.25, so:

the maximum negative shear force = 10 x *0.25 = 2.5 kN*

Draw the influence line for bending moment:

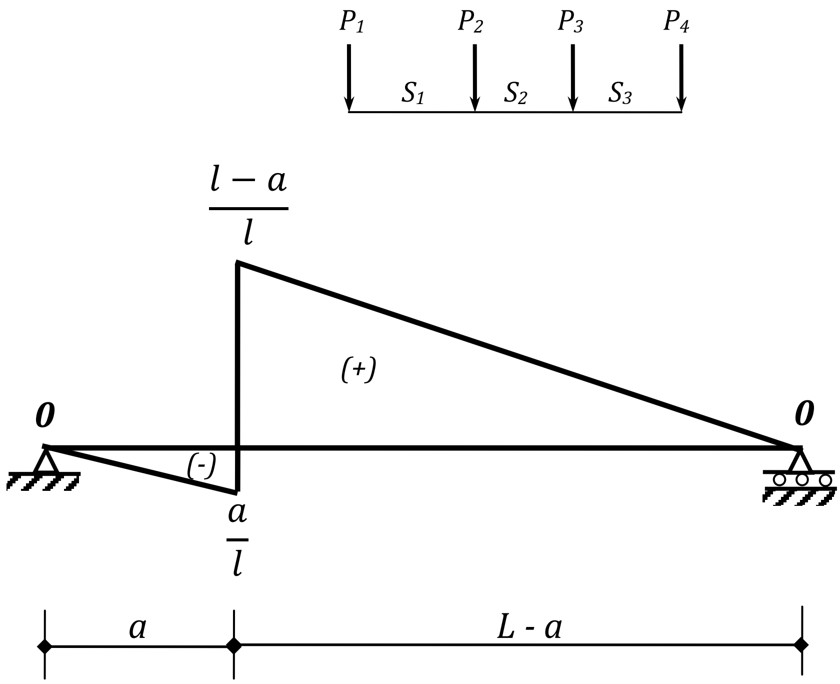


The ordinate = 0.1875, so:

The maximum bending moment = 10 x *0.1875 = 1.875 kN.m*

2. *Case of series of concentrated loads:*

Shear force:



The maximum shear force at any section at distance a from left for a series of concentrated load travel through the beam as shown in figure (from right to left), obtained by applying the following criteria:

- *Calculate the ratio between the total load and the total length:*

$$R_t = \frac{\sum P_i}{L}$$

- *when the first load (P_1) reach the section find the ratio:*

$$R_1 = \frac{P_1}{S_1}$$

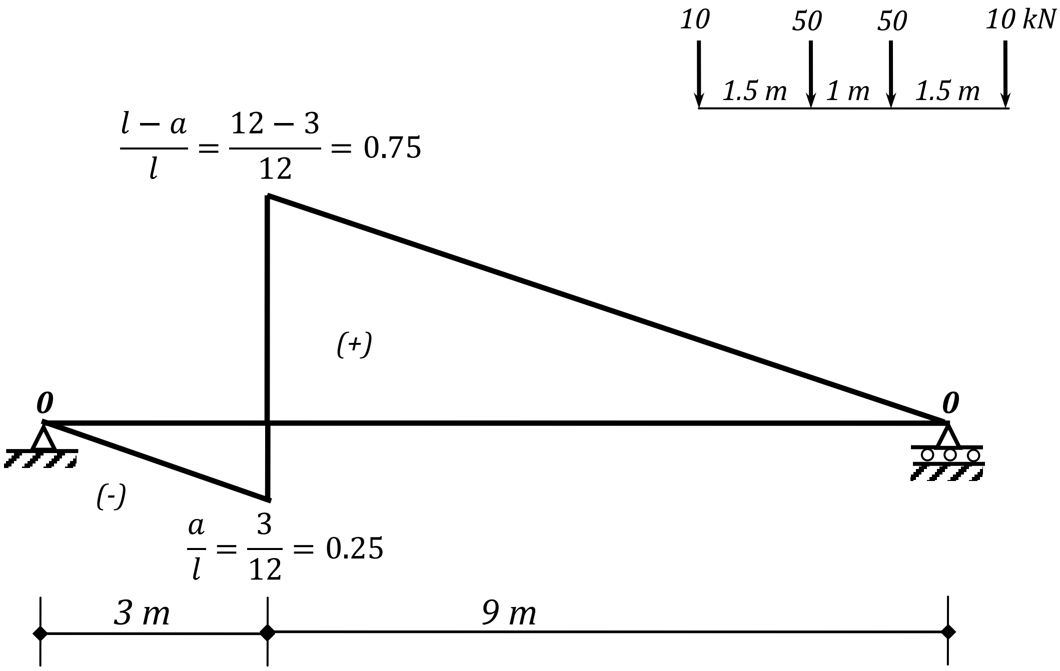
- *If $R_1 < R_t$ let the load pass and put P_2 in section and find the ratio:*

$$R_2 = \frac{P_2}{S_2}$$

- *if $R_2 < R_t$ repeat for P_3 and so on for any number of loads.*
- *Stop when observed that the ratio $R_i > R_t$ and conclude that the load P_i when be at section this situation gives maximum shear force and find its value by multiplying any load by its coincide ordinate and some these values.*

Example:

Find the maximum shear force for a series of concentrated load passes through the beam from right to left and that at section distance



Solution:

$$R_t = \frac{\sum P_i}{L} = \frac{10+50+50+10}{4} = 30$$

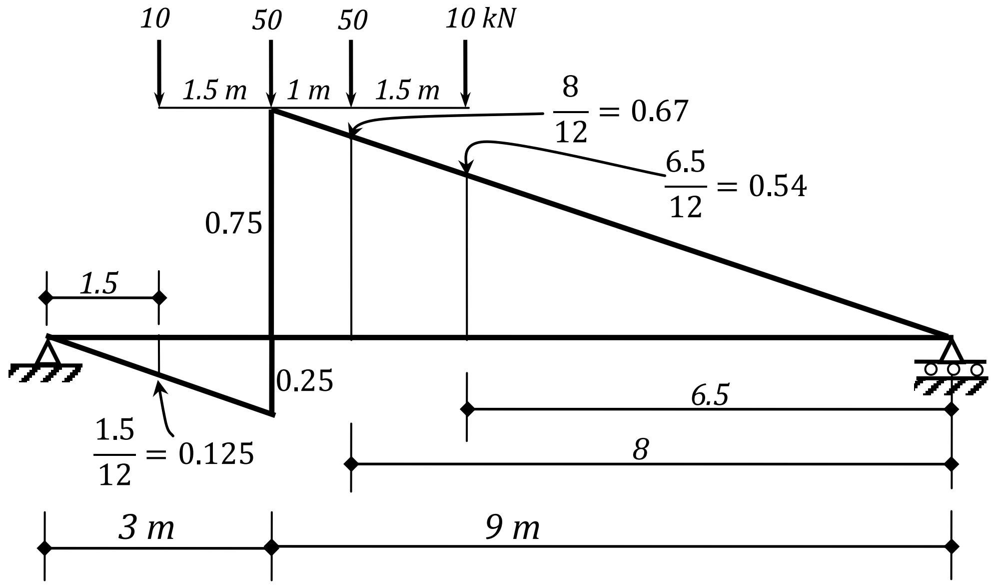
when the first load 10 kN be at section:

$$R_1 = \frac{10}{1.5} = 6.67 < 30$$

when the second load 50 kN be at section:

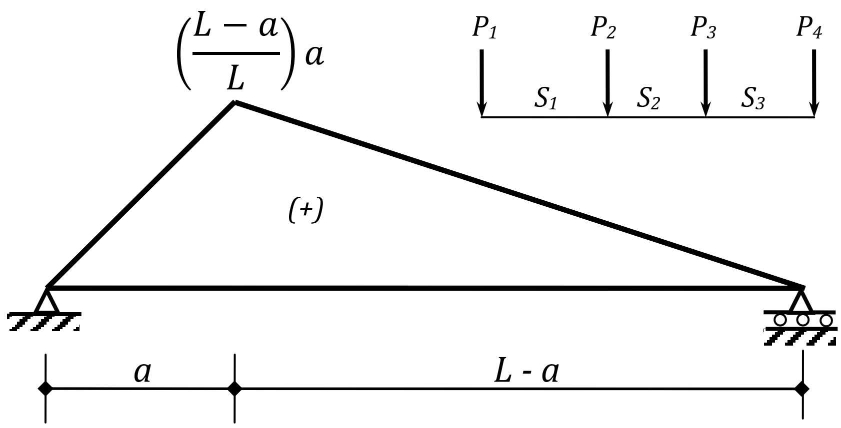
$$R_1 = \frac{50}{1} = 50 > 30$$

so the load 50 kN when be at section satisfy the case of maximum shear.



$$S_{max} = 10(-0.125) + 50(0.75) + 50(0.67) + 10(0.54) = 75.15\ kN$$

Bending moment:



The maximum bending moment at any section at distance a from left for a series of concentrated load travel through the beam as shown in figure (from right to left), obtained by applying the following criteria:

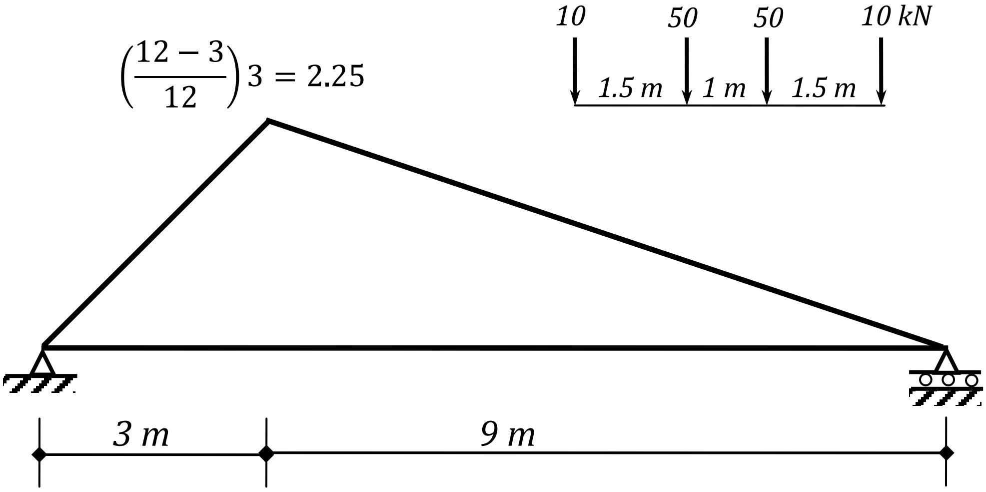
- *Consider the section divide the beam to right side and left side.*
- *When any load be at the section, consider this load and all the loads left to it, at left section and the remainder loads at the right section.*
- *Calculate the ratios between the total load at each and its length as in the following:*

$$R_r = \frac{\sum P_r}{L-a}, R_l = \frac{\sum P_l}{a}$$

- *Star put the first load at section and calculate the two ratio, if the ratio $R_r < R_l$ this is the case of maximum moment, if not put the second loads and so on until you observed that the ratio change from greater than to less than and the load satisfy that this is the case of maximum moment.*

Example:

From the previous example, find the maximum bending moment.



Solution:

When the first load 10 kN at section:

$$R_l = \frac{10}{3} = 3.33$$

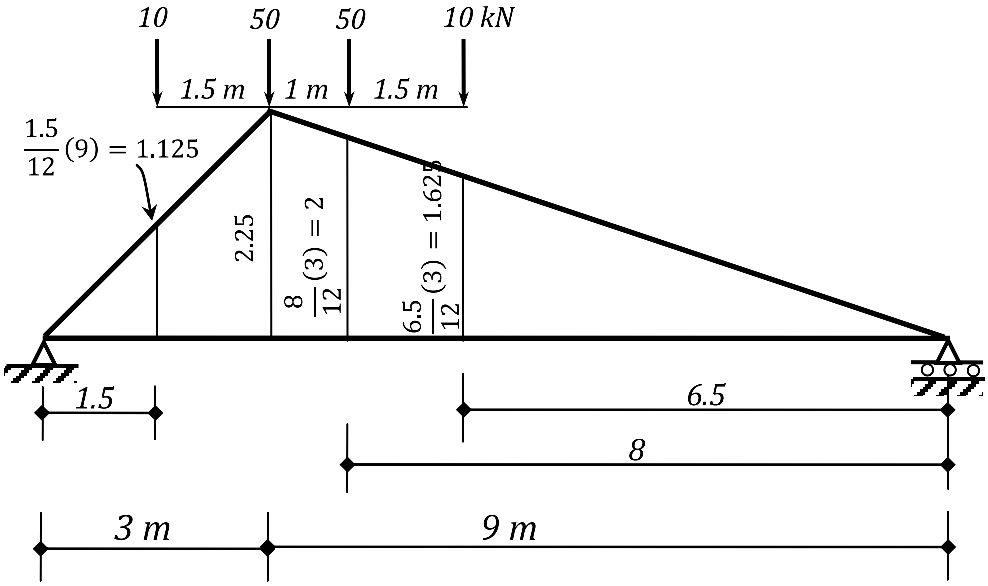
$$R_r = \frac{50+50+10}{9} = 12.22 > 3.33$$

When the second load 50 kN at section:

$$R_l = \frac{10+50}{3} = 20$$

$$R_r = \frac{50+10}{9} = 6.67 < 20$$

So this case of maximum moment.



$$M_{max} = 10(1.125) + 50(2.25) + 50(2) + 10(1.625) = 240\ kN.m$$

Maximum absolute moment:

The maximum absolute moment can be calculated according to the following steps:

1. *Find the resultant of the series of loads and its location (centre gravity of the load).*
2. *The distance between the centre gravity of loads and the nearest load can be divide equidistance and let this location be coinciding with the centre of the beam; this location gives the maximum absolute moment.*

3. Case of distributed load:

Shear force:

*The maximum **positive** shear force obtained when the **front** of the load just reach the section from the left.*

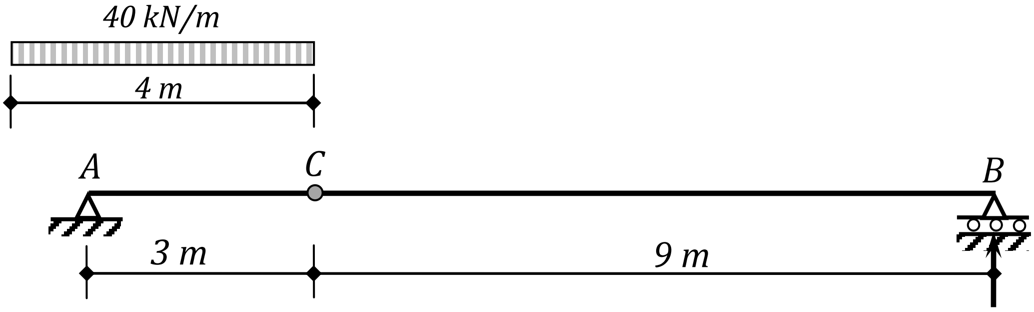
*The maximum **negative** shear force obtained when the **rear** of the load just reach the section from the left.*

Example:

For the beam shown below and if a uniformly distributed load of length 4 m and intensity of 40 kN/m cross the beam from left to right, for a section at distance find:

1. *The maximum negative of shear force.*

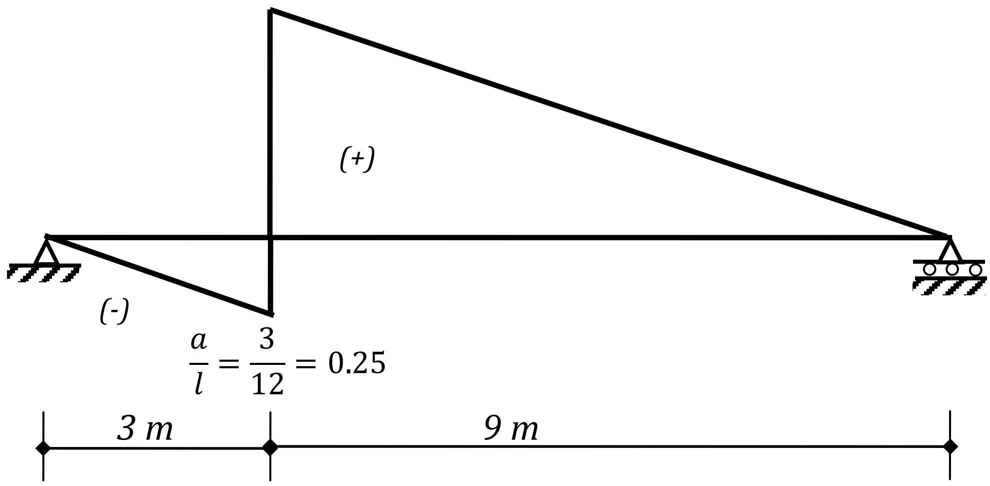
2. *The maximum positive shear force.*



Solution:

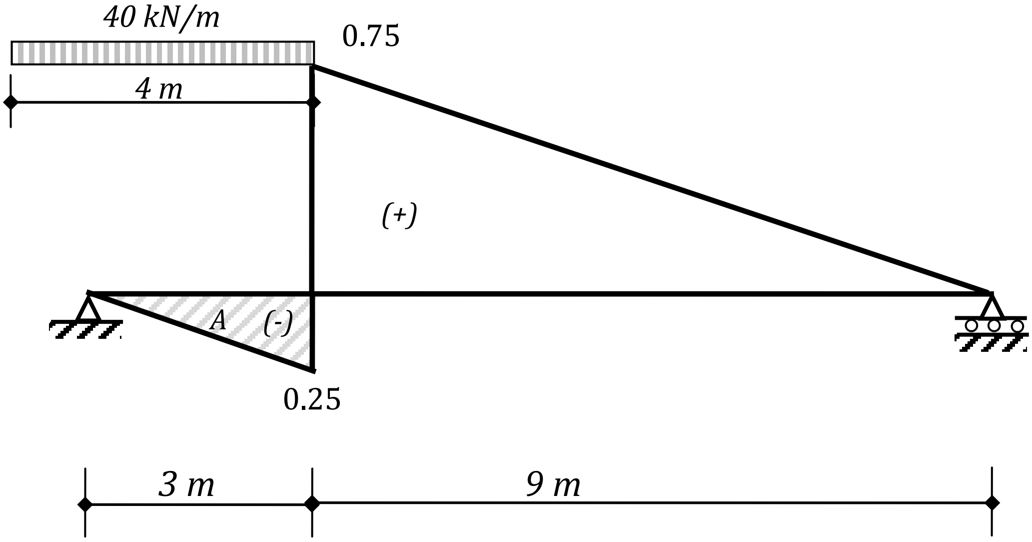
Draw the influence line for shear force.

$$\frac{l-a}{l} = \frac{12-3}{12} = 0.75$$



$$\frac{a}{l} = \frac{3}{12} = 0.25$$

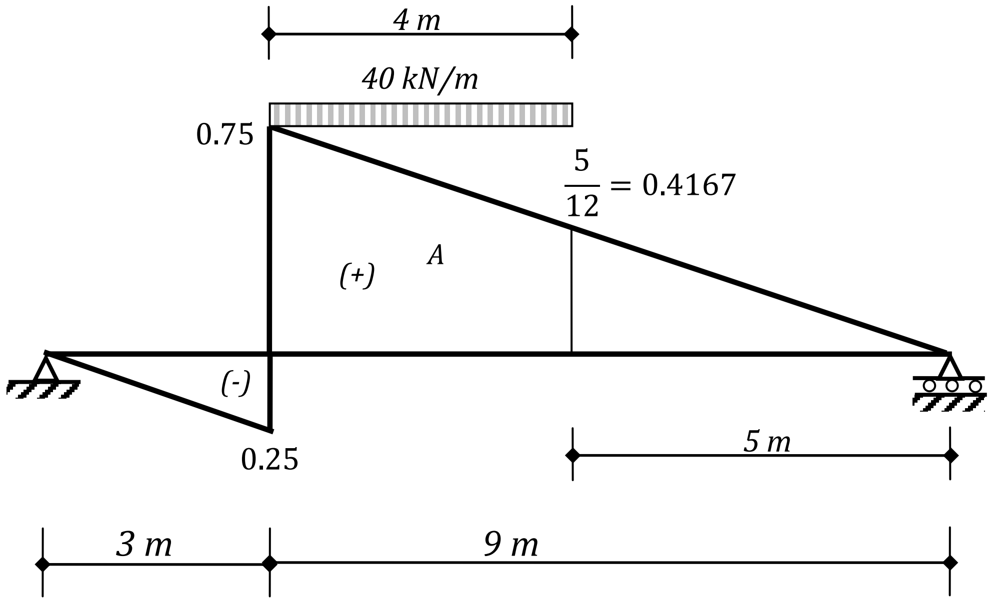
1. *Negative shear force:*
   Put the front of load at C.



$$A = \frac{1}{2}(3)(0.25) = 0.375\ m$$

$$-ve\ S_{max} = 40(0.375) = 15\ kN$$

2. *Positive shear force:*
   let the rear of distributed load at C.



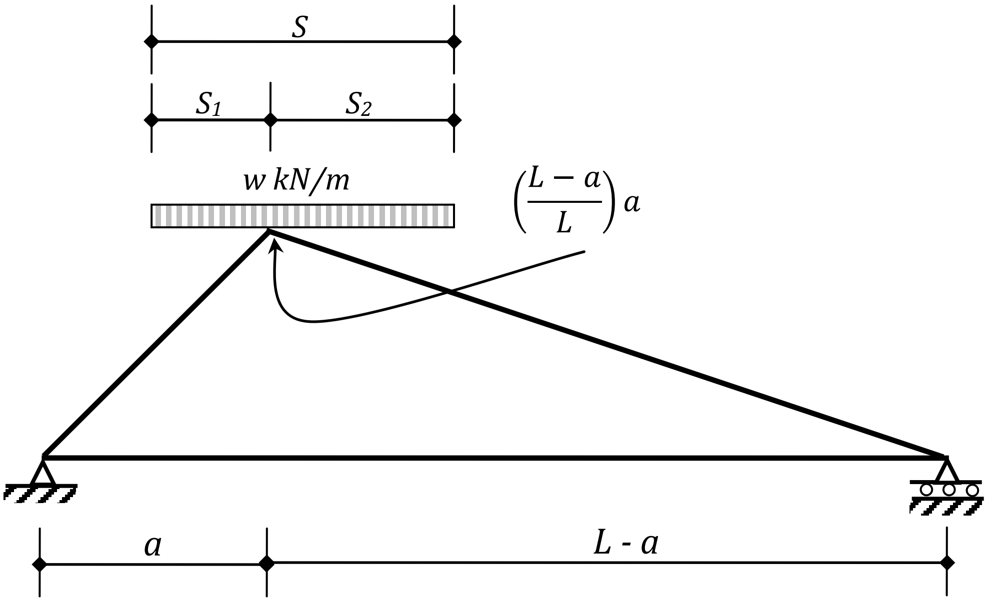
$$A = \frac{1}{2}(0.75 + 0.4167)(4) = 2.33\ m$$

$$+ve\ S_{max} = 40(2.33) = 93.33\ kN$$

Bending moment

If a uniformly distributed load across the beam shown in figure below, then the maximum bending moment obtained when the following condition met:

$$\frac{S_1}{S_2} = \frac{a}{L - a}$$



From the diagram:

$$S = S_1 + S_2,\ S_2 = S - S_1$$

From the expression and let:

$$r = \frac{a}{L-a}$$

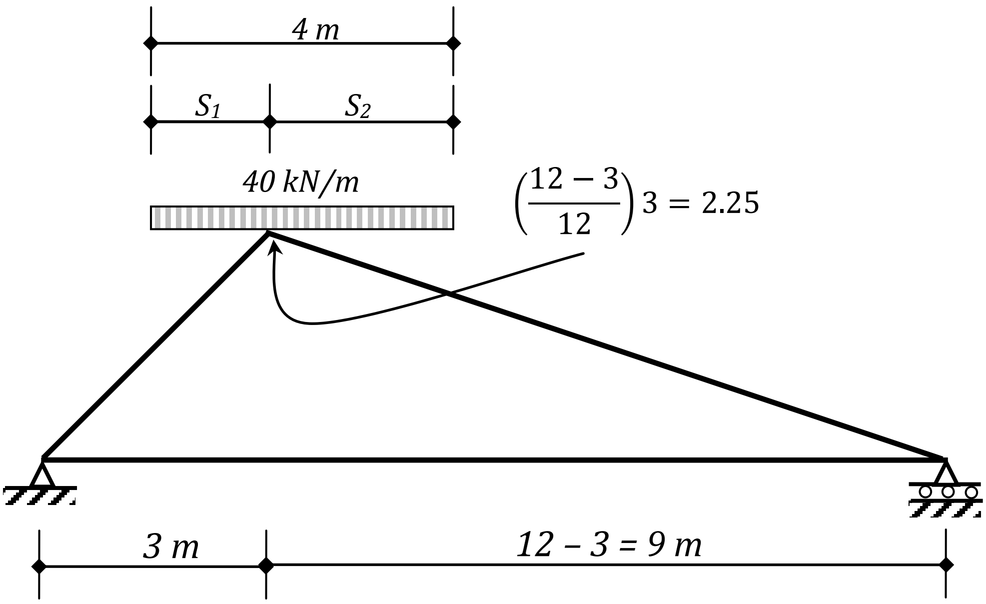
$$S_1 = S_2 r = (S - S_1)r$$

So:

$$S_1 = \frac{r\,S}{1+r}$$

Example:

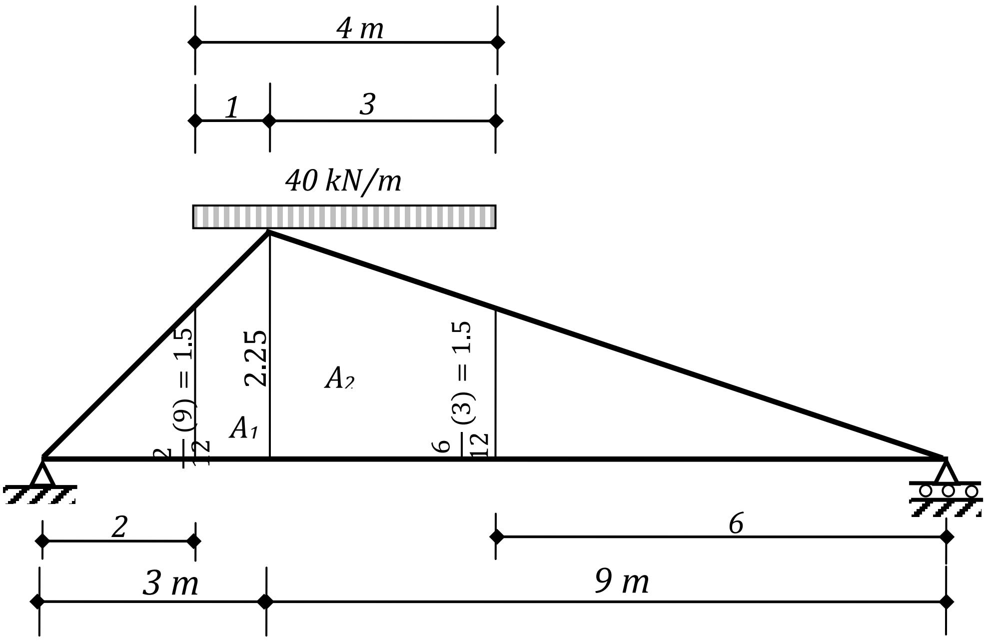
From the previous example, find the maximum bending moment due to uniformly distributed load cross the beam.



$$r = \frac{a}{L - a} = \frac{3}{9} = 0.333$$

$$S_1 = \frac{r\,S}{1 + r} = \frac{0.333(4)}{1 + 0.333} = 1$$

$$S_2 = 4 - 1 = 3$$



$$A_1 = \frac{1}{2}(1.5 + 2.25)(1) = 1.875\ m$$

$$A_2 = \frac{1}{2}(1.5 + 2.25)(3) = 5.625\ m$$

$$M_{max} = 40(1.875 + 5.625) = 300\ kN.m$$