Latin Square Design

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The Latin square design is used to eliminate two nuisance sources of variability; that is, it systematically allows blocking in two directions. Thus, the rows and columns represent two restrictions on randomization.

A Latin square for p factors, or a p X p Latin square, is a square containing p rows and p columns. A significant assumption is that the three factors (treatments, nuisance factors) do not interact

The statistical model for a Latin square is

$$y_{ij} = \mu + \alpha_i + \beta_j + \tau_k + \varepsilon_{ijk} \qquad i = 1, 2, .. p; j = 1, 2, \dots p; k = 1, 2, .., p$$

where: μ represents the overall population mean

α_i represents the effect of the *ith* row

β_j represents the effect of the *jth* column

τ_k represents the f the *kth* treatment

ε_{ijk} represents the overall population mean

Latin Square Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}} = \frac{1}{p}\sum_{j=1}^{p} y_{.j.}^2 - \frac{y_{...}^2}{N}$	$p-1$	$\frac{SS_{\text{Treatments}}}{p-1}$	$F_0 = \frac{MS_{\text{Treatments}}}{MS_E}$
Rows	$SS_{\text{Rows}} = \frac{1}{p}\sum_{i=1}^{p} y_{i..}^2 - \frac{y_{...}^2}{N}$	$p-1$	$\frac{SS_{\text{Rows}}}{p-1}$	
Columns	$SS_{\text{Columns}} = \frac{1}{p}\sum_{k=1}^{p} y_{..k}^2 - \frac{y_{...}^2}{N}$	$p-1$	$\frac{SS_{\text{Columns}}}{p-1}$	
Error	SS_E (by subtraction)	$(p-2)(p-1)$	$\frac{SS_E}{(p-2)(p-1)}$	
Total	$SS_T = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{y_{...}^2}{N}$	p^2-1		

$$SS_T = SS_{\text{Rows}} + SS_{\text{Columns}} + SS_{\text{Treatments}} + SS_E$$

Rejection criteria

$$F_0 > F_{\alpha,(p-1),(p-1)(p-2)}$$

Latin Square Model Adequacy

As in any design problem, the experimenter should investigate the adequacy of the model by inspecting and plotting the residuals. For a Latin square, the residuals are given

$$\begin{aligned} e_{ijk} &= y_{ijk} - \hat{y}_{ijk} \\ &= y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...} \end{aligned}$$

The statistical model for a Latin square is

$$y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \epsilon_{ijk} \begin{cases} i = 1, 2, \ldots, p \\ j = 1, 2, \ldots, p \\ k = 1, 2, \ldots, p \end{cases}$$

where y_{ijk} is the observation in the *ith* row and *kth* column for the *jth* treatment

Example

Suppose that an experimenter is studying the effects of five different formulations of a rocket propellant used in aircrew escape systems on the observed burning rate. Each formulation is mixed from a batch of raw material that is only large enough for five formulations to be tested. Furthermore, the formulations are prepared by several operators, and there may be substantial differences in the skills and experience of the operators. Using Latin Square Design , the data would be

Batches of Raw Material	Operators 1	2	3	4	5
1	$A = 24$	$B = 20$	$C = 19$	$D = 24$	$E = 24$
2	$B = 17$	$C = 24$	$D = 30$	$E = 27$	$A = 36$
3	$C = 18$	$D = 38$	$E = 26$	$A = 27$	$B = 21$
4	$D = 26$	$E = 31$	$A = 26$	$B = 23$	$C = 22$
5	$E = 22$	$A = 30$	$B = 20$	$C = 29$	$D = 31$

The five formulations (or treatments) are denoted by the Latin letters A, B, C, D, and E

Example

After coding by subtracting 25 from each observation, we have the data in following table

Batches of Raw Material	Operators 1	2	3	4	5	$y_{i..}$
1	$A = -1$	$B = -5$	$C = -6$	$D = -1$	$E = -1$	-14
2	$B = -8$	$C = -1$	$D = 5$	$E = 2$	$A = 11$	9
3	$C = -7$	$D = 13$	$E = 1$	$A = 2$	$B = -4$	5
4	$D = 1$	$E = 6$	$A = 1$	$B = -2$	$C = -3$	3
5	$E = -3$	$A = 5$	$B = -5$	$C = 4$	$D = 6$	7
$y_{..k}$	-18	18	-4	5	9	$10 = y_{...}$

Example

$$SS_T = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{y_{...}^2}{N}$$

$$= 680 - \frac{(10)^2}{25} = 676.00$$

$$SS_{\text{Batches}} = \frac{1}{p} \sum_{i=1}^{p} y_{i..}^2 - \frac{y_{...}^2}{N}$$

$$= \frac{1}{5}[(-14)^2 + 9^2 + 5^2 + 3^2 + 7^2] - \frac{(10)^2}{25} = 68.00$$

$$SS_{\text{Operators}} = \frac{1}{p} \sum_{k=1}^{p} y_{..k}^2 - \frac{y_{...}^2}{N}$$

$$= \frac{1}{5}[(-18)^2 + 18^2 + (-4)^2 + 5^2 + 9^2] - \frac{(10)^2}{25} = 150.00$$

$$SS_{\text{Formulations}} = \frac{1}{p} \sum_{j=1}^{p} y_{.j.}^2 - \frac{y_{...}^2}{N}$$

$$= \frac{18^2 + (-24)^2 + (-13)^2 + 24^2 + 5^2}{5} - \frac{(10)^2}{25} = 330.00$$

The error sum of squares is found by subtraction:

$$SS_E = SS_T - SS_{\text{Batches}} - SS_{\text{Operators}} - SS_{\text{Formulations}}$$
$$= 676.00 - 68.00 - 150.00 - 330.00 = 128.00$$

Example

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Formulations	330.00	4	82.50	7.73
Batches of raw material	68.00	4	17.00	
Operators	150.00	4	37.50	
Error	128.00	12	10.67	
Total	676.00	24		

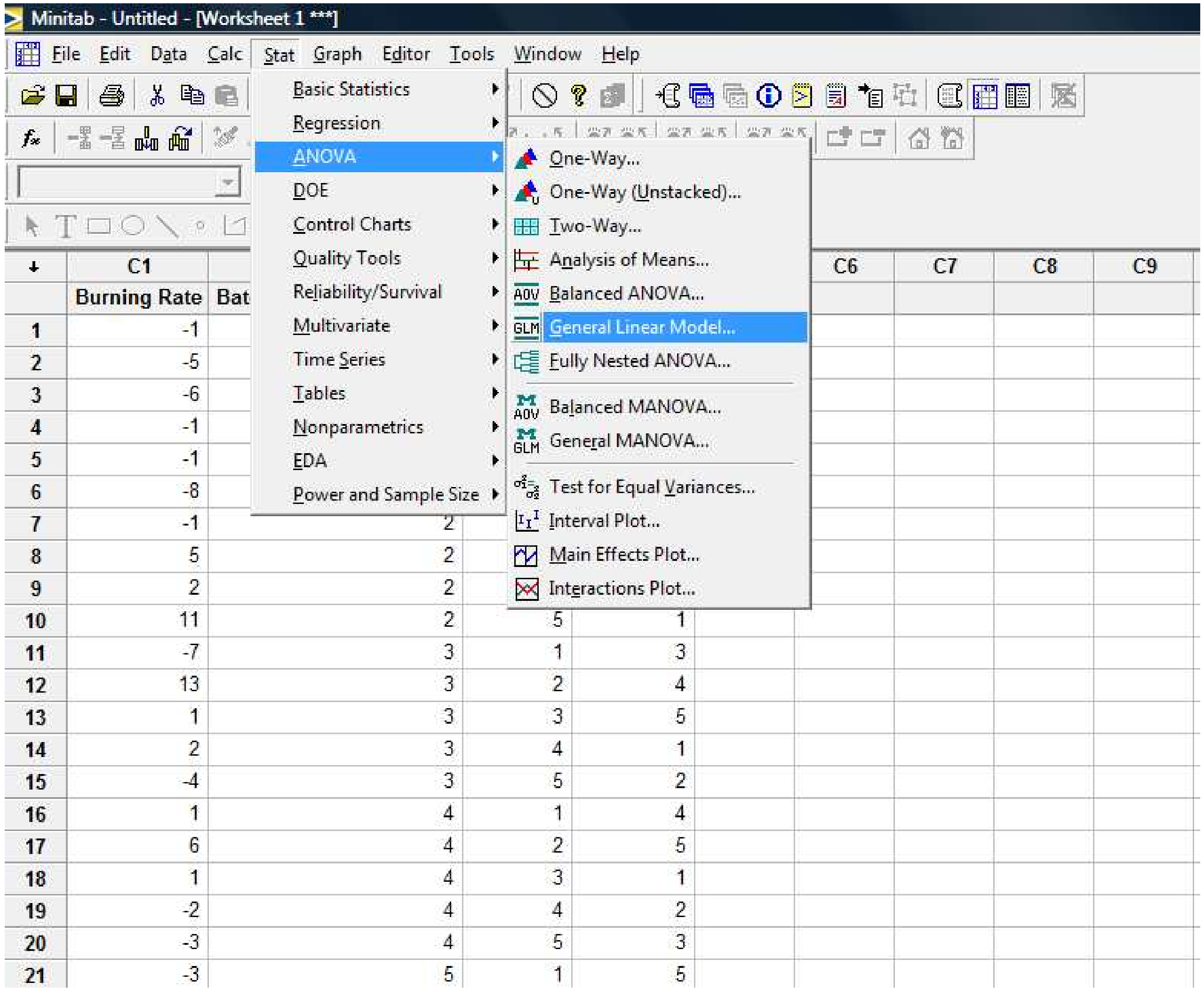
The analysis of variance is summarized in Table 4-12. We conclude that there is a significant difference in the mean burning rate generated by the different rocket propellant formulations. There is also an indication that there are differences between operators, so blocking on this factor was a good precaution

Example

↓	C1	C2	C3	C4	C5	C6
	Burning Rate	Batches of raw materials	Operators	Treatments		
1	-1	1	1	1		
2	-5	1	2	2		
3	-6	1	3	3		
4	-1	1	4	4		
5	-1	1	5	5		
6	-8	2	1	2		
7	-1	2	2	3		
8	5	2	3	4		
9	2	2	4	5		
10	11	2	5	1		
11	-7	3	1	3		
12	13	3	2	4		
13	1	3	3	5		
14	2	3	4	1		
15	-4	3	5	2		
16	1	4	1	4		
17	6	4	2	5		
18	1	4	3	1		
19	-2	4	4	2		
20	-3	4	5	3		
21	-3	5	1	5		
22	5	5	2	1		
23	-5	5	3	2		
24	4	5	4	3		

Current Worksheet: Worksheet 1

Example


Minitab - Untitled - [Worksheet 1 ***]
File Edit Data Calc Stat Graph Editor Tools Window Help
Basic Statistics
Regression
ANOVA
DOE
Control Charts
Quality Tools
Reliability/Survival
Multivariate
Time Series
Tables
Nonparametrics
EDA
Power and Sample Size
One-Way...
One-Way (Unstacked)...
Two-Way...
Analysis of Means...
Balanced ANOVA...
General Linear Model...
Fully Nested ANOVA...
Balanced MANOVA...
General MANOVA...
Test for Equal Variances...
Interval Plot...
Main Effects Plot...
Interactions Plot...
C1
Burning Rate
C6
C7
C8
C9