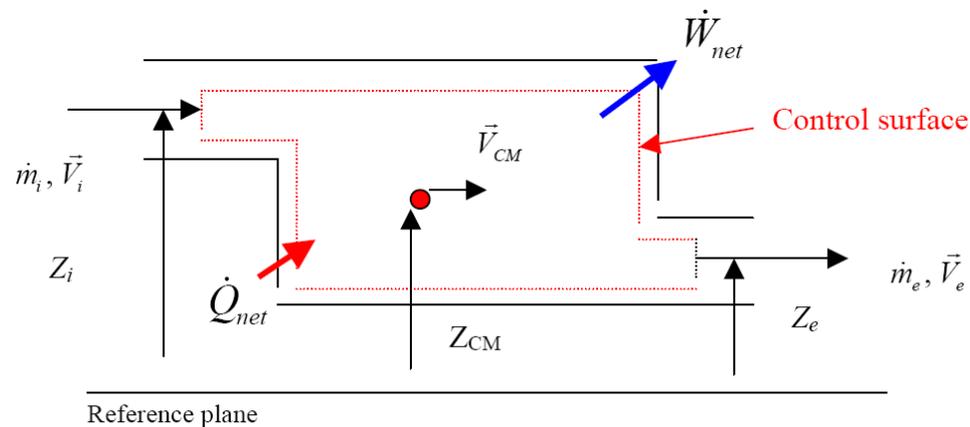


Chapter 5

Mass and Energy Analysis of Control Volumes

Conservation of Energy for Control volumes

The conservation of mass and the conservation of energy principles for open systems or control volumes apply to systems having mass crossing the system boundary or control surface. In addition to the heat transfer and work crossing the system boundaries, mass carries energy with it as it crosses the system boundaries. Thus, the mass and energy content of the open system may change when mass enters or leaves the control volume.



Typical control volume or open system

Thermodynamic processes involving control volumes can be considered in two groups: steady-flow processes and unsteady-flow processes. During a steady-flow process, the fluid flows through the control volume steadily, experiencing no change with time at a fixed position.

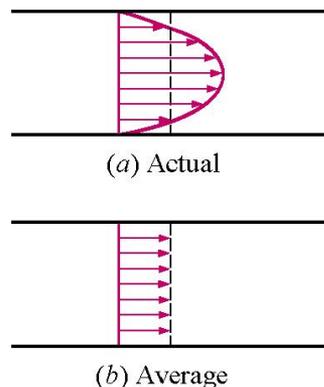
Let's review the concepts of mass flow rate and energy transport by mass. One should study the development of the general conservation of mass presented in the text. Here we present an overview of the concepts important to successful problem solving techniques.

Mass Flow Rate

Mass flow through a cross-sectional area per unit time is called the mass flow rate \dot{m} . Note the dot over the mass symbol indicates a time rate of change. It is expressed as

$$\dot{m} = \int_A \rho \vec{V}_n dA$$

where \vec{V}_n is the velocity normal to the cross-sectional flow area.



If the fluid density and velocity are constant over the flow cross-sectional area, the mass flow rate is

$$\dot{m} = \rho \vec{V}_{ave} A = \frac{\vec{V}_{ave} A}{v}$$

where ρ is the density, kg/m^3 ($= 1/v$), A is the cross-sectional area, m^2 ; and \vec{V}_{ave} is the average fluid velocity normal to the area, m/s .

Example 5-1

Refrigerant-134a at 200 kPa, 40% quality, flows through a 1.1-cm inside diameter, d , tube with a velocity of 50 m/s. Find the mass flow rate of the refrigerant-134a.

At $P = 200$ kPa, $x = 0.4$ we determine the specific volume from

$$v = v_f + xv_{fg}$$

$$= 0.0007533 + 0.4(0.0999 - 0.0007533)$$

$$= 0.0404 \frac{\text{m}^3}{\text{kg}}$$

$$\dot{m} = \frac{\vec{V}_{ave} A}{v} = \frac{\vec{V}_{ave}}{v} \frac{\pi d^2}{4}$$

$$= \frac{50 \text{ m/s}}{0.0404 \text{ m}^3/\text{kg}} \frac{\pi(0.011 \text{ m})^2}{4}$$

$$= 0.117 \frac{\text{kg}}{\text{s}}$$

The fluid volume flowing through a cross-section per unit time is called the volume flow rate. The volume flow rate is given by integrating the product of the velocity normal to the flow area and the differential flow area over the flow area. If the velocity over the flow area is a constant, the volume flow rate is given by (note we are dropping the “ave” subscript on the velocity)

$$\dot{V} = \vec{V}A \quad (m^3 / s)$$

The mass and volume flow rate are related by

$$\dot{m} = \rho\dot{V} = \frac{\dot{V}}{v} \quad (kg / s)$$

Example 5-2

Air at 100 kPa, 50°C, flows through a pipe with a volume flow rate of 40 m³/min. Find the mass flow rate through the pipe, in kg/s.

Assume air to be an ideal gas, so

$$\begin{aligned} v &= \frac{RT}{P} = 0.287 \frac{kJ}{kg \cdot K} \frac{(50 + 273)K}{100kPa} \frac{m^3 kPa}{kJ} \\ &= 0.9270 \frac{m^3}{kg} \end{aligned}$$

$$\dot{m} = \frac{\dot{V}}{v} = \frac{40m^3 / \text{min}}{0.9270m^3 / \text{kg}} \frac{1 \text{ min}}{60s}$$

$$= 0.719 \frac{\text{kg}}{s}$$

Conservation of Mass for General Control Volume

The conservation of mass principle for the open system or control volume is expressed as

$$\left[\begin{array}{l} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{into control volume} \end{array} \right] - \left[\begin{array}{l} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{from control volume} \end{array} \right] = \left[\begin{array}{l} \text{Time rate change} \\ \text{of mass inside} \\ \text{control volume} \end{array} \right]$$

or

$$\sum \dot{m}_{in} - \sum \dot{m}_{out} = \Delta \dot{m}_{system} \quad (\text{kg} / \text{s})$$

Steady-State, Steady-Flow Processes

Most energy conversion devices operate steadily over long periods of time. The rates of heat transfer and work crossing the control surface are constant with time. The states of the mass streams crossing the control surface or boundary are constant with time. Under these conditions the mass and energy content of the control volume are constant with time.

$$\frac{dm_{CV}}{dt} = \Delta \dot{m}_{CV} = 0$$

Steady-state, Steady-Flow Conservation of Mass:

Since the mass of the control volume is constant with time during the steady-state, steady-flow process, the conservation of mass principle becomes

$$\left[\begin{array}{l} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{into control volume} \end{array} \right] = \left[\begin{array}{l} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{from control volume} \end{array} \right]$$

or

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \quad (kg / s)$$

Special Case: Steady Flow of an Incompressible Fluid

The mass flow rate is related to volume flow rate and fluid density by

$$\dot{m} = \rho \dot{V}$$

For one entrance, one exit steady flow control volume, the mass flow rates are related by

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \quad (\text{kg/s})$$

$$\rho_{\text{in}} \dot{V}_{\text{in}} = \rho_{\text{out}} \dot{V}_{\text{out}}$$

$$\rho_{\text{in}} = \rho_{\text{out}} \quad \text{incompressible assumption}$$

$$\dot{V}_{\text{in}} = \dot{V}_{\text{out}}$$

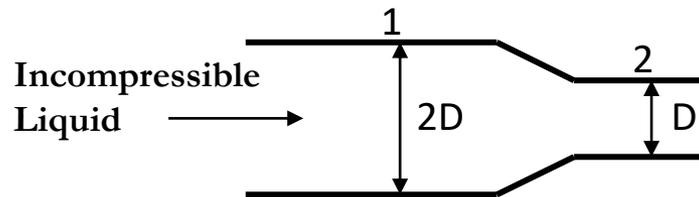
$$\vec{V}_{\text{in}} A_{\text{in}} = \vec{V}_{\text{out}} A_{\text{out}}$$

Word of caution: This result applies only to incompressible fluids. Most thermodynamic systems deal with processes involving compressible fluids such as ideal gases, steam, and the refrigerants for which the above relation **will not apply**.

Example 5-3 Geometry Effects on Fluid Flow

An incompressible liquid flows through the pipe shown in the figure. The velocity at location

2 is: A) $\frac{1}{4} \vec{V}_1$ B) $\frac{1}{2} \vec{V}_1$ C) $2\vec{V}_1$ D) $4\vec{V}_1$



Solution:

$$\dot{m} = \rho \dot{V}$$

$$\sum_{\text{Inlets}} \dot{m}_{in} = \sum_{\text{Outlets}} \dot{m}_{out}$$

$$\rho \dot{V}_1 = \rho \dot{V}_2$$

$$\dot{V}_1 = \dot{V}_2$$

$$A_1 \vec{V}_1 = A_2 \vec{V}_2$$

$$\vec{V}_2 = \frac{A_1}{A_2} \vec{V}_1 = \frac{\pi D_1^2 / 4}{\pi D_2^2 / 4} \vec{V}_1$$

$$\vec{V}_2 = \left(\frac{D_1}{D_2} \right)^2 \vec{V}_1 = \left(\frac{2D}{D} \right)^2 \vec{V}_1$$

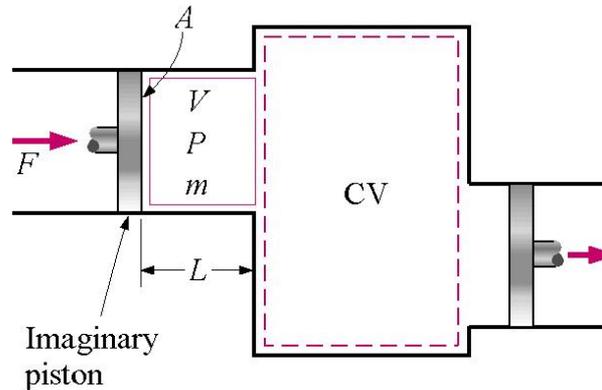
$$\vec{V}_2 = 4\vec{V}_1$$

Answer: D

Flow work and the energy of a flowing fluid

Energy flows into and from the control volume with the mass. The energy required to push the mass into or out of the control volume is known as the flow work or flow energy.

The fluid upstream of the control surface acts as a piston to push a unit of mass into or out of the control volume. Consider the unit of mass entering the control volume shown below.



As the fluid upstream pushes mass across the control surface, work done on that unit of mass is

$$W_{flow} = FL = FL \frac{A}{A} = PV = Pmv$$

$$w_{flow} = \frac{W_{flow}}{m} = Pv$$

The term Pv is called the flow work done on the unit of mass as it crosses the control surface.

The total energy of flowing fluid

The total energy carried by a unit of mass as it crosses the control surface is the sum of the internal energy, flow work, potential energy, and kinetic energy.

$$\begin{aligned}\theta &= u + Pv + \frac{\vec{V}^2}{2} + gz \\ &= h + \frac{\vec{V}^2}{2} + gz\end{aligned}$$

Here we have used the definition of enthalpy, $h = u + Pv$.

Energy transport by mass

Amount of energy transport across a control surface:

$$E_{mass} = m\theta = m \left(h + \frac{\vec{V}^2}{2} + gz \right) \quad (\text{kJ})$$

Rate of energy transport across a control surface:

$$\dot{E}_{mass} = \dot{m}\theta = \dot{m} \left(h + \frac{\vec{V}^2}{2} + gz \right) \quad (kW)$$

Conservation of Energy for General Control Volume

The conservation of energy principle for the control volume or open system has the same word definition as the first law for the closed system. Expressing the energy transfers on a rate basis, the control volume first law is

$$\left[\begin{array}{l} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{into control volume} \end{array} \right] - \left[\begin{array}{l} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{from control volume} \end{array} \right] = \left[\begin{array}{l} \text{Time rate change} \\ \text{of energy inside} \\ \text{control volume} \end{array} \right]$$

or

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}_{\text{Rate change in internal, kinetic, potential, etc., energies}} \quad (kW)$$

Considering that energy flows into and from the control volume with the mass, energy enters because net heat is transferred to the control volume, and energy leaves because the control volume does net work on its surroundings, the open system, or control volume, the first law becomes

$$\dot{Q}_{net} + \sum_{\text{for each inlet}} \underbrace{\dot{m}_i \theta_i}_{\text{for each inlet}} - \dot{W}_{net} - \sum_{\text{for each exit}} \underbrace{\dot{m}_e \theta_e}_{\text{for each exit}} = \frac{dE_{CV}}{dt} \quad (kW)$$

where θ is the energy per unit mass flowing into or from the control volume. The energy per unit mass, θ , flowing across the control surface that defines the control volume is composed of four terms: the internal energy, the kinetic energy, the potential energy, and the flow work.

The total energy carried by a unit of mass as it crosses the control surface is

$$\theta = u + Pv + \frac{\vec{V}^2}{2} + gz$$

$$= h + \frac{\vec{V}^2}{2} + gz$$

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{CV}$$

$$\dot{Q}_{net} + \sum_{\text{for each inlet}} \underbrace{\dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} - \dot{W}_{net} - \sum_{\text{for each exit}} \underbrace{\dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}} = \Delta \dot{E}_{CV}$$

Where the time rate change of the energy of the control volume has been written as $\Delta \dot{E}_{CV}$

Steady-State, Steady-Flow Processes

Most energy conversion devices operate steadily over long periods of time. The rates of heat transfer and work crossing the control surface are constant with time. The states of the mass streams crossing the control surface or boundary are constant with time. Under these conditions the mass and energy content of the control volume are constant with time.

$$\frac{dm_{CV}}{dt} = \Delta \dot{m}_{CV} = 0$$

$$\frac{dE_{CV}}{dt} = \Delta \dot{E}_{CV} = 0$$

Steady-state, Steady-Flow Conservation of Mass:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \quad (kg / s)$$

Steady-state, steady-flow conservation of energy

Since the energy of the control volume is constant with time during the steady-state, steady-flow process, the conservation of energy principle becomes

$$\left[\begin{array}{l} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{into control volume} \end{array} \right] = \left[\begin{array}{l} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{from control volume} \end{array} \right]$$

or

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\overset{0}{\Delta E}_{system}}_{\text{Rate change in internal, kinetic, potential, etc., energies}} \quad (kW)$$

or

$$\underbrace{\dot{E}_{in}}_{\text{Rate of net energy transfer by heat, work, and mass into the system}} = \underbrace{\dot{E}_{out}}_{\text{Rate of energy transfer by heat, work, and mass from the system}}$$

Considering that energy flows into and from the control volume with the mass, energy enters because heat is transferred to the control volume, and energy leaves because the control volume does work on its surroundings, the steady-state, steady-flow first law becomes

$$\dot{Q}_{in} + \dot{W}_{in} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

Often this result is written as

$$\dot{Q}_{net} - \dot{W}_{net} = \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}} - \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}}$$

where

$$\dot{Q}_{net} = \sum \dot{Q}_{in} - \sum \dot{Q}_{out}$$

$$\dot{W}_{net} = \sum \dot{W}_{out} - \sum \dot{W}_{in}$$

Steady-state, steady-flow for one entrance and one exit

A number of thermodynamic devices such as pumps, fans, compressors, turbines, nozzles, diffusers, and heaters operate with one entrance and one exit. The steady-state, steady-flow conservation of mass and first law of thermodynamics for these systems reduce to

$$\dot{m}_1 = \dot{m}_2 \quad (kg / s)$$

$$\frac{1}{v_1} \vec{V}_1 A_1 = \frac{1}{v_2} \vec{V}_2 A_2$$

$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1) \right] \quad (kW)$$

where the entrance to the control volume is state 1 and the exit is state 2 and \dot{m} is the mass flow rate through the device.

When can we neglect the kinetic and potential energy terms in the first law?

Consider the kinetic and potential energies per unit mass.

$$ke = \frac{\vec{V}^2}{2}$$

$$\text{For } \vec{V} = 45 \frac{\text{m}}{\text{s}} \quad ke = \frac{(45 \text{ m/s})^2}{2} \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} = 1 \frac{\text{kJ}}{\text{kg}}$$

$$\vec{V} = 140 \frac{\text{m}}{\text{s}} \quad ke = \frac{(140 \text{ m/s})^2}{2} \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} = 10 \frac{\text{kJ}}{\text{kg}}$$

$$pe = gz$$

$$\text{For } z = 100 \text{ m} \quad pe = 9.8 \frac{\text{m}}{\text{s}^2} 100 \text{ m} \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} = 0.98 \frac{\text{kJ}}{\text{kg}}$$

$$z = 1000 \text{ m} \quad pe = 9.8 \frac{\text{m}}{\text{s}^2} 1000 \text{ m} \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} = 9.8 \frac{\text{kJ}}{\text{kg}}$$

When compared to the enthalpy of steam ($h \cong 2000$ to 3000 kJ/kg) and the enthalpy of air ($h \cong 200$ to 6000 kJ/kg), the kinetic and potential energies are often neglected.

When the kinetic and potential energies can be neglected, the conservation of energy equation becomes

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1) \quad (kW)$$

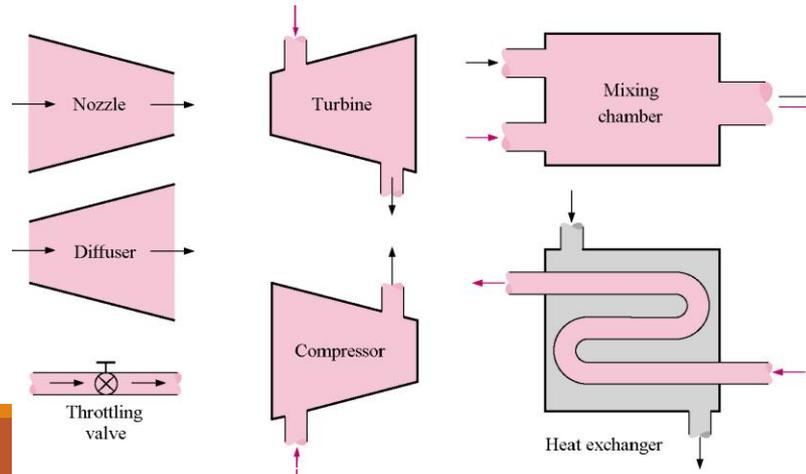
We often write this last result per unit mass flow as

$$q - w = (h_2 - h_1) \quad (kJ / kg)$$

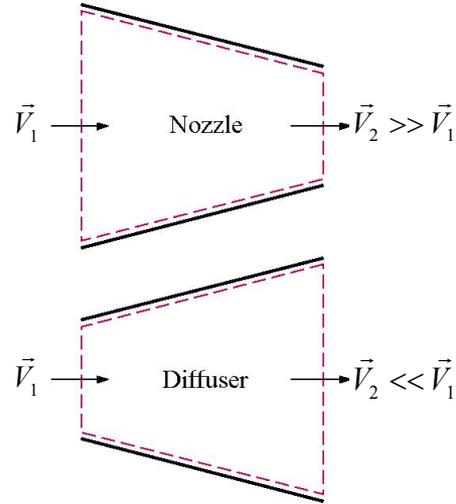
where $q = \frac{\dot{Q}}{\dot{m}}$ and $w = \frac{\dot{W}}{\dot{m}}$

Some Steady-Flow Engineering Devices

Below are some engineering devices that operate essentially as steady-state, steady-flow control volumes.



Nozzles and Diffusers



For flow through nozzles, the heat transfer, work, and potential energy are normally neglected, and nozzles have one entrance and one exit. The conservation of energy becomes

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

$$\dot{m} \left(h_1 + \frac{\vec{V}_1^2}{2} \right) = \dot{m} \left(h_2 + \frac{\vec{V}_2^2}{2} \right)$$

Solving for \vec{V}_2

$$\vec{V}_2 = \sqrt{2(h_1 - h_2) + \vec{V}_1^2}$$

Example 5-4

Steam at 0.4 MPa, 300°C, enters an adiabatic nozzle with a low velocity and leaves at 0.2 MPa with a quality of 90%. Find the exit velocity, in m/s.

Control Volume: The nozzle

Property Relation: Steam tables

Process: Assume adiabatic, steady-flow

Conservation Principles:

Conservation of mass:

For one entrance, one exit, the conservation of mass becomes

$$\begin{aligned}\sum \dot{m}_{in} &= \sum \dot{m}_{out} \\ \dot{m}_1 &= \dot{m}_2 = \dot{m}\end{aligned}$$

Conservation of energy:

According to the sketched control volume, mass crosses the control surface, but no work or heat transfer crosses the control surface. Neglecting the potential energies, we have

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m} \left(h_1 + \frac{\vec{V}_1^2}{2} \right) = \dot{m} \left(h_2 + \frac{\vec{V}_2^2}{2} \right)$$

Neglecting the inlet kinetic energy, the exit velocity is

$$\vec{V}_2 = \sqrt{2(h_1 - h_2)}$$

Now, we need to find the enthalpies from the steam tables.

$$\left. \begin{array}{l} \text{Superheated} \\ T_1 = 300^\circ C \\ P_1 = 0.4 MPa \end{array} \right\} h_1 = 3067.1 \frac{kJ}{kg}$$
$$\left. \begin{array}{l} \text{Saturated Mix.} \\ P_2 = 0.2 MPa \\ x_2 = 0.90 \end{array} \right\} h_2$$

At 0.2 MPa $h_f = 504.7$ kJ/kg and $h_{fg} = 2201.6$ kJ/kg.

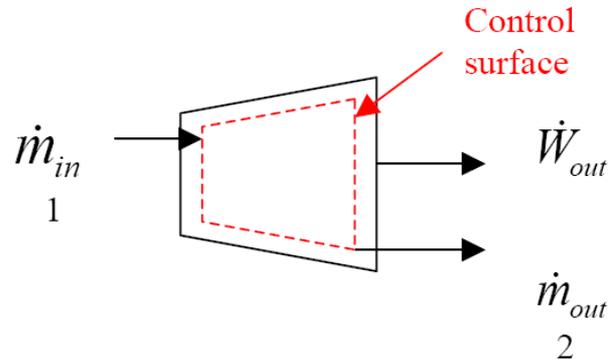
$$h_2 = h_f + x_2 h_{fg}$$

$$= 504.7 + (0.90)(2201.6) = 2486.1 \frac{\text{kJ}}{\text{kg}}$$

$$\vec{V}_2 = \sqrt{2(3067.1 - 2486.1) \frac{\text{kJ}}{\text{kg}} \frac{1000 \text{ m}^2 / \text{s}^2}{\text{kJ} / \text{kg}}}$$

$$= 1078.0 \frac{\text{m}}{\text{s}}$$

Turbines



Turbine control volume

If we neglect the changes in kinetic and potential energies as fluid flows through an adiabatic turbine having one entrance and one exit, the conservation of mass and the steady-state, steady-flow first law becomes

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

$$\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{W}_{out}$$

$$\dot{W}_{out} = \dot{m}(h_1 - h_2)$$

Example 5-5

High pressure air at 1300 K flows into an aircraft gas turbine and undergoes a steady-state, steady-flow, adiabatic process to the turbine exit at 660 K. Calculate the work done per unit mass of air flowing through the turbine when

- Temperature-dependent data are used.
- $C_{p,ave}$ at the average temperature is used.
- C_p at 300 K is used.

Control Volume: The turbine.

Property Relation: Assume air is an ideal gas and use ideal gas relations.

Process: Steady-state, steady-flow, adiabatic process

Conservation Principles:

Conservation of mass:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$
$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

$$\dot{Q}_{in} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{out} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

According to the sketched control volume, mass and work cross the control surface. Neglecting kinetic and potential energies and noting the process is adiabatic, we have

$$0 + \dot{m}_1 h_1 = \dot{W}_{out} + \dot{m}_2 h_2$$

$$\dot{W}_{out} = \dot{m}(h_1 - h_2)$$

The work done by the air per unit mass flow is

$$w_{out} = \frac{\dot{W}_{out}}{\dot{m}} = h_1 - h_2$$

Notice that the work done by a fluid flowing through a turbine is equal to the enthalpy decrease of the fluid.

(a) Using the air tables, Table A-17

$$\text{at } T_1 = 1300 \text{ K, } h_1 = 1395.97 \text{ kJ/kg}$$

$$\text{at } T_2 = 660 \text{ K, } h_2 = 670.47 \text{ kJ/kg}$$

$$w_{out} = h_1 - h_2$$

$$= (1395.97 - 670.47) \frac{\text{kJ}}{\text{kg}}$$

$$= 725.5 \frac{\text{kJ}}{\text{kg}}$$

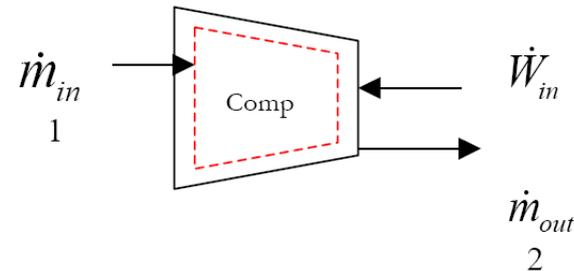
(b) Using Table A-2(c) at $T_{ave} = 980 \text{ K}$, $C_{p, ave} = 1.138 \text{ kJ/kg}\cdot\text{K}$

$$\begin{aligned}w_{out} &= h_1 - h_2 = C_{p, ave} (T_1 - T_2) \\&= 1.138 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} (1300 - 660) \text{K} \\&= 728.3 \frac{\text{kJ}}{\text{kg}}\end{aligned}$$

(c) Using Table A-2(a) at $T = 300 \text{ K}$, $C_p = 1.005 \text{ kJ/kg}\cdot\text{K}$

$$\begin{aligned}w_{out} &= h_1 - h_2 = C_p (T_1 - T_2) \\&= 1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} (1300 - 660) \text{K} \\&= 643.2 \frac{\text{kJ}}{\text{kg}}\end{aligned}$$

Compressors and fans



Steady-Flow Compressor

Compressors and fans are essentially the same devices. However, compressors operate over larger pressure ratios than fans. If we neglect the changes in kinetic and potential energies as fluid flows through an adiabatic compressor having one entrance and one exit, the steady-state, steady-flow first law or the conservation of energy equation becomes

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

$$-\dot{W}_{net} = \dot{m}(h_2 - h_1)$$

$$-(-\dot{W}_{in}) = \dot{m}(h_2 - h_1)$$

$$\dot{W}_{in} = \dot{m}(h_2 - h_1)$$

Example 5-6

Nitrogen gas is compressed in a steady-state, steady-flow, adiabatic process from 0.1 MPa, 25°C. During the compression process the temperature becomes 125°C. If the mass flow rate is 0.2 kg/s, determine the work done on the nitrogen, in kW.

Control Volume: The compressor (see the compressor sketched above)

Property Relation: Assume nitrogen is an ideal gas and use ideal gas relations

Process: Adiabatic, steady-flow

Conservation Principles:

Conservation of mass:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$
$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

According to the sketched control volume, mass and work cross the control surface.

Neglecting kinetic and potential energies and noting the process is adiabatic, we have for one entrance and one exit

$$0 + \dot{m}_1 (h_1 + 0 + 0) = (-\dot{W}_{in}) + \dot{m}_2 (h_2 + 0 + 0)$$

$$\dot{W}_{in} = \dot{m}(h_2 - h_1)$$

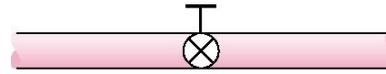
The work done **on** the nitrogen is related to the enthalpy rise of the nitrogen as it flows through the compressor. The work done on the nitrogen per unit mass flow is

$$w_{in} = \frac{\dot{W}_{in}}{\dot{m}} = h_2 - h_1$$

Assuming constant specific heats at 300 K from Table A-2(a), we write the work as

$$\begin{aligned}w_{in} &= C_p (T_2 - T_1) \\&= 1.039 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (125 - 25) \text{K} \\&= 103.9 \frac{\text{kJ}}{\text{kg}} \\ \dot{W}_{in} &= \dot{m} w_{in} = 0.2 \frac{\text{kg}}{\text{s}} \left(103.9 \frac{\text{kJ}}{\text{kg}} \right) \\&= 20.78 \frac{\text{kJ}}{\text{s}} = 20.78 \text{ kW}\end{aligned}$$

Throttling devices



(a) An adjustable valve



(b) A porous plug



(c) A capillary tube

Consider fluid flowing through a one-entrance, one-exit porous plug. The fluid experiences a pressure drop as it flows through the plug. No net work is done by the fluid. Assume the process is adiabatic and that the kinetic and potential energies are neglected; then the conservation of mass and energy equations become

$$\dot{m}_i = \dot{m}_e$$
$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

$$\dot{m}_i h_i = \dot{m}_e h_e$$

$$h_i = h_e$$

This process is called a throttling process. What happens when an ideal gas is throttled?

$$h_i = h_e$$

$$h_e - h_i = 0$$

$$\int_i^e C_p(T) dT = 0$$

or

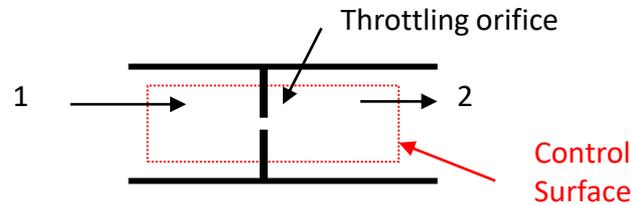
$$T_e = T_i$$

When throttling an ideal gas, the temperature does not change. We will see later in Chapter 11 that the throttling process is an important process in the refrigeration cycle.

A throttling device may be used to determine the enthalpy of saturated steam. The steam is throttled from the pressure in the pipe to ambient pressure in the calorimeter. The pressure drop is sufficient to superheat the steam in the calorimeter. Thus, the temperature and pressure in the calorimeter will specify the enthalpy of the steam in the pipe.

Example 5-7

One way to determine the quality of saturated steam is to throttle the steam to a low enough pressure that it exists as a superheated vapor. Saturated steam at 0.4 MPa is throttled to 0.1 MPa, 100°C. Determine the quality of the steam at 0.4 MPa.



Control Volume: The throttle

Property Relation: The steam tables

Process: Steady-state, steady-flow, no work, no heat transfer, neglect kinetic and potential energies, one entrance, one exit

Conservation Principles:

Conservation of mass:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$
$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

According to the sketched control volume, mass crosses the control surface. Neglecting kinetic and potential energies and noting the process is adiabatic with no work, we have for one entrance and one exit

$$0 + \dot{m}_1(h_1 + 0 + 0) = 0 + \dot{m}_2(h_2 + 0 + 0)$$

$$\dot{m}_1 h_1 = \dot{m}_2 h_2$$

$$h_1 = h_2$$

$$\left. \begin{array}{l} T_2 = 100^\circ C \\ P_2 = 0.1 MPa \end{array} \right\} h_2 = 2675.8 \frac{kJ}{kg}$$

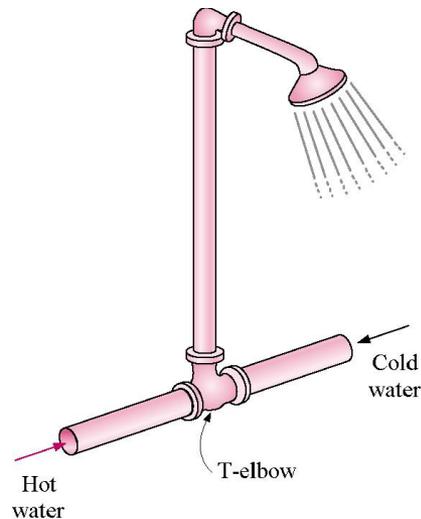
Therefore,

$$h_1 = h_2 = 2675.8 \frac{kJ}{kg}$$
$$= \left(h_f + x_1 h_{fg} \right)_{@ P_1 = 0.4 MPa}$$

$$\begin{aligned}x_1 &= \frac{h_1 - h_f}{h_{fg}} \\ &= \frac{2675.8 - 604.66}{2133.4} \\ &= 0.971\end{aligned}$$

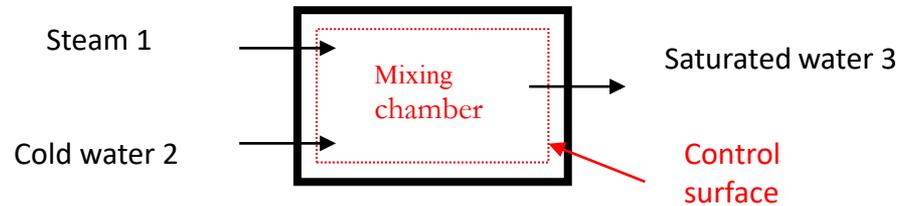
Mixing chambers

The mixing of two fluids occurs frequently in engineering applications. The section where the mixing process takes place is called a mixing chamber. The ordinary shower is an example of a mixing chamber.



Example 5-8

Steam at 0.2 MPa, 300°C, enters a mixing chamber and is mixed with cold water at 20°C, 0.2 MPa, to produce 20 kg/s of saturated liquid water at 0.2 MPa. What are the required steam and cold water flow rates?



Control Volume: The mixing chamber

Property Relation: Steam tables

Process: Assume steady-flow, adiabatic mixing, with no work

Conservation Principles:

Conservation of mass:

$$\begin{aligned}\sum \dot{m}_{in} &= \sum \dot{m}_{out} \\ \dot{m}_1 + \dot{m}_2 &= \dot{m}_3 \\ \dot{m}_2 &= \dot{m}_3 - \dot{m}_1\end{aligned}$$

Conservation of energy:

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

According to the sketched control volume, mass crosses the control surface. Neglecting kinetic and potential energies and noting the process is adiabatic with no work, we have for two entrances and one exit

$$\begin{aligned}\dot{m}_1 h_1 + \dot{m}_2 h_2 &= \dot{m}_3 h_3 \\ \dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1) h_2 &= \dot{m}_3 h_3 \\ \dot{m}_1 (h_1 - h_2) &= \dot{m}_3 (h_3 - h_2) \\ \dot{m}_1 &= \dot{m}_3 \frac{(h_3 - h_2)}{(h_1 - h_2)}\end{aligned}$$

Now, we use the steam tables to find the enthalpies:

$$\left. \begin{array}{l} T_1 = 300^\circ C \\ P_1 = 0.2 MPa \end{array} \right\} h_1 = 3072.1 \frac{kJ}{kg}$$

$$\left. \begin{array}{l} T_2 = 20^\circ C \\ P_2 = 0.2 MPa \end{array} \right\} h_2 \approx h_{f@20^\circ C} = 83.91 \frac{kJ}{kg}$$

$$\left. \begin{array}{l} P_3 = 0.2 MPa \\ \text{Sat. liquid} \end{array} \right\} h_3 = h_{f@0.2 MPa} = 504.7 \frac{kJ}{kg}$$

$$\begin{aligned} \dot{m}_1 &= \dot{m}_3 \frac{(h_3 - h_2)}{(h_1 - h_2)} \\ &= 20 \frac{kg}{s} \frac{(504.7 - 83.91) kJ / kg}{(3072.1 - 83.91) kJ / kg} \end{aligned}$$

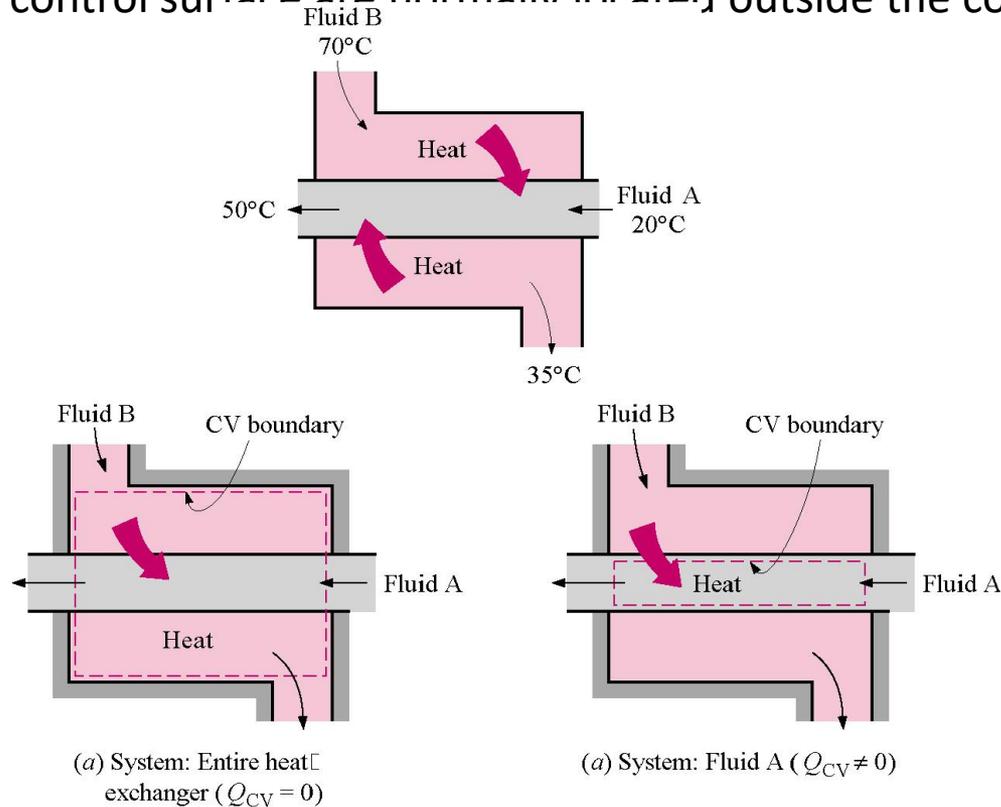
$$= 2.82 \frac{kg}{s}$$

$$\begin{aligned} \dot{m}_2 &= \dot{m}_3 - \dot{m}_1 \\ &= (20 - 2.82) \frac{kg}{s} \end{aligned}$$

$$= 17.18 \frac{kg}{s}$$

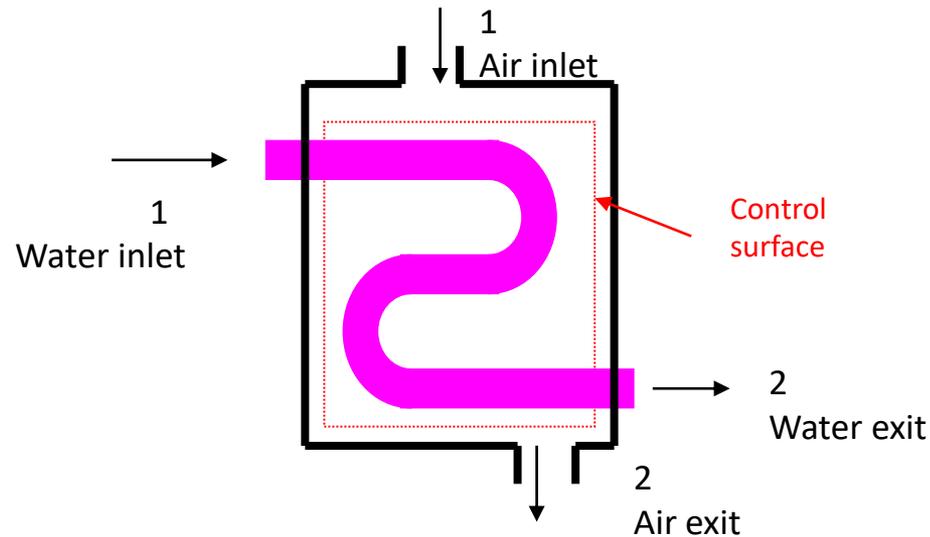
Heat exchangers

Heat exchangers are normally well-insulated devices that allow energy exchange between hot and cold fluids without mixing the fluids. The pumps, fans, and blowers causing the fluids to flow across the control surface are normally located outside the control surface.



Example 5-9

Air is heated in a heat exchanger by hot water. The water enters the heat exchanger at 45°C and experiences a 20°C drop in temperature. As the air passes through the heat exchanger, its temperature is increased by 25°C. Determine the ratio of mass flow rate of the air to mass flow rate of the water.



Control Volume: The heat exchanger

Property Relation: Air: ideal gas relations

Water: steam tables or incompressible liquid results

Process: Assume adiabatic, steady-flow

Conservation Principles:

Conservation of mass:

$$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{system}^{0(\text{steady})} \quad (\text{kg} / \text{s})$$

For two entrances, two exits, the conservation of mass becomes

$$\dot{m}_{in} = \dot{m}_{out}$$
$$\dot{m}_{air,1} + \dot{m}_{w,1} = \dot{m}_{air,2} + \dot{m}_{w,2}$$

For two fluid streams that exchange energy but do not mix, it is better to conserve the mass for the fluid streams separately.

$$\dot{m}_{air,1} = \dot{m}_{air,2} = \dot{m}_{air}$$
$$\dot{m}_{w,1} = \dot{m}_{w,2} = \dot{m}_w$$

Conservation of energy:

According to the sketched control volume, mass crosses the control surface, but no work or heat transfer crosses the control surface. Neglecting the kinetic and potential energies, we have for steady-flo

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\overset{0(\text{steady})}{\Delta \dot{E}_{system}}}_{\text{Rate change in internal, kinetic, potential, etc., energies}} \quad (kW)$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_{air,1} h_{air,1} + \dot{m}_{w,1} h_{w,1} = \dot{m}_{air,2} h_{air,2} + \dot{m}_{w,2} h_{w,2}$$

$$\dot{m}_{air} (h_{air,1} - h_{air,2}) = \dot{m}_w (h_{w,2} - h_{w,1})$$

$$\frac{\dot{m}_{air}}{\dot{m}_w} = \frac{(h_{w,2} - h_{w,1})}{(h_{air,1} - h_{air,2})}$$

We assume that the air has constant specific heats at 300 K, Table A-2(a) (we don't know the actual temperatures, just the temperature difference). Because we know the initial and final temperatures for the water, we can use either the incompressible fluid result or the steam tables for its properties.

Using the incompressible fluid approach for the water, Table A-3,
 $C_{p,w} = 4.18 \text{ kJ/kg}\cdot\text{K}$.

$$\begin{aligned}
\frac{\dot{m}_{air}}{\dot{m}_w} &= \frac{C_{p,w}(T_{w,2} - T_{w,1})}{C_{p,air}(T_{air,1} - T_{air,2})} \\
&= \frac{4.18 \frac{kJ}{kg_w \cdot K} (-20 K)}{1.005 \frac{kJ}{kg_{air} \cdot K} (-25 K)} \\
&= 3.33 \frac{kg_{air} / s}{kg_w / s}
\end{aligned}$$

A second solution to this problem is obtained by determining the heat transfer rate from the hot water and noting that this is the heat transfer rate to the air. Considering each fluid separately for steady-flow, one entrance, and one exit, and neglecting the kinetic and potential energies, the first law, or conservation of energy, equations become

$$\dot{E}_{in} = \dot{E}_{out}$$

$$air: \dot{m}_{air,1} h_{air,1} + \dot{Q}_{in,air} = \dot{m}_{air,2} h_{air,2}$$

$$water: \dot{m}_{w,1} h_{w,1} = \dot{Q}_{out,w} + \dot{m}_{w,2} h_{w,2}$$

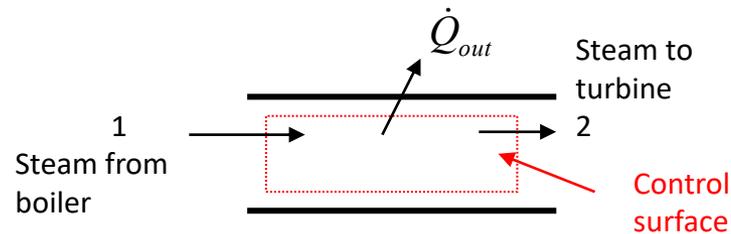
$$\dot{Q}_{in,air} = \dot{Q}_{out,w}$$

Pipe and duct flow

The flow of fluids through pipes and ducts is often a steady-state, steady-flow process. We normally neglect the kinetic and potential energies; however, depending on the flow situation, the work and heat transfer may or may not be zero.

Example 5-10

In a simple steam power plant, steam leaves a boiler at 3 MPa, 600°C, and enters a turbine at 2 MPa, 500°C. Determine the in-line heat transfer from the steam per kilogram mass flowing in the pipe between the boiler and the turbine.



Control Volume: Pipe section in which the heat loss occurs.

Property Relation: Steam tables

Process: Steady-flow

Conservation Principles:

Conservation of mass:

$$\dot{m}_{in} - \dot{m}_{out} = \overset{0(\text{steady})}{\Delta \dot{m}_{system}} \quad (\text{kg} / \text{s})$$

For one entrance, one exit, the conservation of mass becomes

$$\dot{m}_{in} = \dot{m}_{out}$$
$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

According to the sketched control volume, heat transfer and mass cross the control surface, but no work crosses the control surface. Neglecting the kinetic and potential energies, we have for steady-flow

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \overset{0(\text{steady})}{\underbrace{\Delta \dot{E}_{system}}_{\text{Rate change in internal, kinetic, potential, etc., energies}}} \quad (\text{kW})$$

We determine the heat transfer rate per unit mass of flowing steam as: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{Q}_{out}$

$$\dot{Q}_{out} = \dot{m}(h_1 - h_2)$$

$$q_{out} = \frac{\dot{Q}_{out}}{\dot{m}} = h_1 - h_2$$

We use the steam tables to determine the enthalpies at the two states as

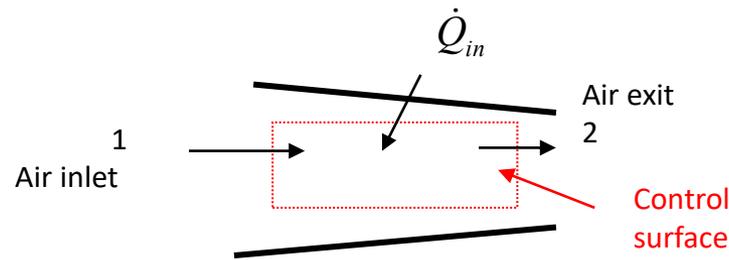
$$\left. \begin{array}{l} T_1 = 600^\circ C \\ P_1 = 3 \text{ MPa} \end{array} \right\} h_1 = 3682.8 \frac{\text{kJ}}{\text{kg}}$$

$$\left. \begin{array}{l} T_2 = 500^\circ C \\ P_2 = 2 \text{ MPa} \end{array} \right\} h_2 = 3468.3 \frac{\text{kJ}}{\text{kg}}$$

$$\begin{aligned} q_{out} &= h_1 - h_2 \\ &= (3682.8 - 3468.3) \frac{\text{kJ}}{\text{kg}} \\ &= 214.5 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Example 5-11

Air at 100°C, 0.15 MPa, 40 m/s, flows through a converging duct with a mass flow rate of 0.2 kg/s. The air leaves the duct at 0.1 MPa, 113.6 m/s. The exit-to-inlet duct area ratio is 0.5. Find the required rate of heat transfer to the air when no work is done by the air.



Control Volume: The converging duct

Property Relation: Assume air is an ideal gas and use ideal gas relations

Process: Steady-flow

Conservation Principles:

Conservation of mass:

$$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{system}^{0(\text{steady})} \quad (\text{kg} / \text{s})$$

For one entrance, one exit, the conservation of mass becomes

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

According to the sketched control volume, heat transfer and mass cross the control surface, but no work crosses the control surface. Here keep the kinetic energy and still neglect the potential energies, we have for steady-state, steady-flow process

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\overset{0(\text{steady})}{\Delta \dot{E}}_{system}}_{\text{Rate change in internal, kinetic, potential, etc., energies}} \quad (kW)$$

$$\dot{m}_1 \left(h_1 + \frac{\vec{V}_1^2}{2} \right) + \dot{Q}_{in} = \dot{m}_2 \left(h_2 + \frac{\vec{V}_2^2}{2} \right)$$
$$\dot{Q}_{in} = \dot{m} \left((h_2 - h_1) + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} \right)$$

In the first law equation, the following are known: P_1 , T_1 (and h_1), \vec{V}_1 , \vec{V}_2 , \dot{m} , and A_2/A_1 . The unknowns are \dot{Q}_{in} , and h_2 (or T_2). We use the first law and the conservation of mass equation to solve for the two unknowns.

$$\dot{m}_1 = \dot{m}_2 \quad (\text{kg} / \text{s})$$

$$\frac{1}{v_1} \vec{V}_1 A_1 = \frac{1}{v_2} \vec{V}_2 A_2$$

$$\vec{V}_1 A_1 \frac{P_1}{RT_1} = \vec{V}_2 A_2 \frac{P_2}{RT_2}$$

Solving for T_2

$$\begin{aligned} T_2 &= T_1 \frac{P_2}{P_1} \frac{A_2}{A_1} \frac{\vec{V}_2}{\vec{V}_1} \\ &= (100 + 273) \text{K} \left(\frac{0.1 \text{ MPa}}{0.15 \text{ MPa}} \right) (0.5) \left(\frac{113.6 \text{ m/s}}{40 \text{ m/s}} \right) \\ &= 353.1 \text{ K} \end{aligned}$$

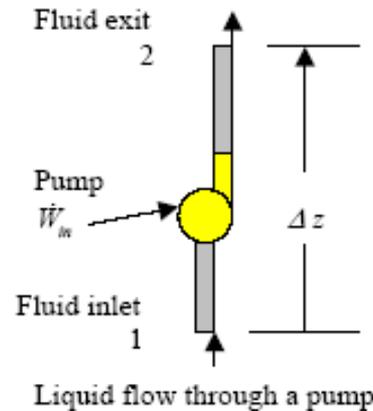
Assuming $C_p = \text{constant}$, $h_2 - h_1 = C_p(T_2 - T_1)$

$$\begin{aligned} \dot{Q}_m &= \dot{m} \left(C_p(T_2 - T_1) + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} \right) \\ &= 0.2 \frac{\text{kg}}{\text{s}} \left(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (353.1 - 373) \text{K} \right. \\ &\quad \left. + \frac{(113.6^2 - 40^2) \text{m}^2 / \text{s}^2}{2} \frac{\text{kJ} / \text{kg}}{1000 \text{m}^2 / \text{s}^2} \right) \\ &= -2.87 \frac{\text{kJ}}{\text{s}} = -2.87 \text{ kW} \end{aligned}$$

Looks like we made the wrong assumption for the direction of the heat transfer. The heat is really leaving the flow duct. (What type of device is this anyway?)

$$\dot{Q}_{out} = -\dot{Q}_{in} = 2.87 \text{ kW}$$

Liquid pumps



The work required when pumping an incompressible liquid in an adiabatic steady-state, steady-flow process is given by

$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1) \right] \quad (\text{kW})$$

The enthalpy difference can be written as

$$h_2 - h_1 = (u_2 - u_1) + (Pv)_2 - (Pv)_1$$

For incompressible liquids we assume that the density and specific volume are constant. The pumping process for an incompressible liquid is essentially isothermal, and the internal energy change is approximately zero (we will see this more clearly after introducing the second law). Thus, the enthalpy difference reduces to the difference in the pressure-specific volume products. Since $v_2 = v_1 = v$ the work input to the pump becomes

$$-\dot{W} = \dot{m} \left[v(P_2 - P_1) + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1) \right] \quad (kW)$$

\dot{W} is the net work done by the control volume, and it is noted that work is input to the pump; so,

$$\dot{W} = -\dot{W}_{in, pump}$$

If we neglect the changes in kinetic and potential energies, the pump work becomes

$$-(-\dot{W}_{in, pump}) = \dot{m} v(P_2 - P_1) \quad (kW)$$

$$\dot{W}_{in, pump} = \dot{m} v(P_2 - P_1)$$

We use this result to calculate the work supplied to boiler feedwater pumps in steam power plants.

If we apply the above energy balance to a pipe section that has no pump ($\dot{W} = 0$), we obtain.

$$-\dot{W} = \dot{m} \left[v(P_2 - P_1) + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1) \right] \quad (kW)$$

$$0 = \dot{m} \left[v(P_2 - P_1) + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1) \right]$$

$$v = \frac{1}{\rho}$$

$$\frac{P_2}{\rho} + \frac{\vec{V}_2^2}{2g} + z_2 = \frac{P_1}{\rho} + \frac{\vec{V}_1^2}{2g} + z_1$$

This last equation is the famous Bernoulli's equation for frictionless, incompressible fluid flow through a pipe.

Uniform-State, Uniform-Flow Problems

During unsteady energy transfer to or from open systems or control volumes, the system may have a change in the stored energy and mass. Several unsteady thermodynamic problems may be treated as uniform-state, uniform-flow problems. The assumptions for uniform-state, uniform-flow are

- The process takes place over a specified time period.
- The state of the mass within the control volume is uniform at any instant of time but may vary with time.
- The state of mass crossing the control surface is uniform and steady. The mass flow may be different at different control surface locations.

To find the amount of mass crossing the control surface at a given location, we integrate the mass flow rate over the time period.

$$\text{Inlets: } m_i = \int_0^t \dot{m}_i dt \quad \text{Exits: } m_e = \int_0^t \dot{m}_e dt$$

The change in mass of the control volume in the time period is

$$(m_2 u_2 - m_1 u_1)_{CV} = \int_0^t \frac{dU}{dt} \Big|_{CV} dt$$

The uniform-state, uniform-flow conservation of mass becomes

$$\sum m_i - \sum m_e = (m_2 - m_1)_{CV}$$

The change in internal energy for the control volume during the time period is

$$Q = \int_0^t \dot{Q} dt \quad \text{and} \quad W = \int_0^t \dot{W} dt$$

The energy crossing the control surface with the mass in the time period is

$$\sum m_j \theta_j = \int_0^t \dot{m}_j \left(h_j + \frac{\vec{V}_j^2}{2} + gz_j \right) dt$$

where

j = i, for inlets
e, for exits

The first law for uniform-state, uniform-flow becomes

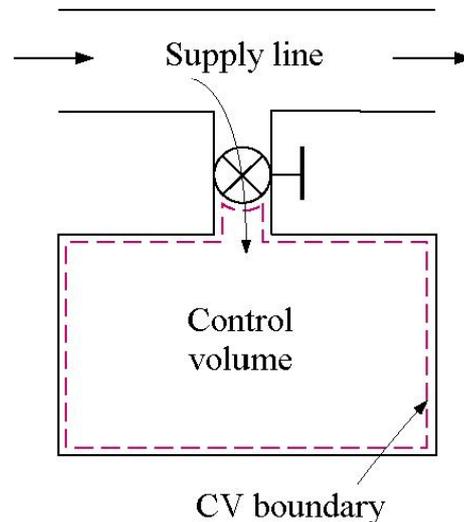
$$E_{in} - E_{out} = \Delta E_{CV}$$
$$Q - W = \sum m_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right) - \sum m_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) + (m_2 e_2 - m_1 e_1)_{CV}$$

When the kinetic and potential energy changes associated with the control volume and the fluid streams are negligible, it simplifies to

$$Q - W = \sum m_e h_e - \sum m_i h_i + (m_2 u_2 - m_1 u_1)_{CV} \quad (kJ)$$

Example 5-12

Consider an evacuated, insulated, rigid tank connected through a closed valve to a high-pressure line. The valve is opened and the tank is filled with the fluid in the line. If the fluid is an ideal gas, determine the final temperature in the tank when the tank pressure equals that of the line.



Control Volume: The tank

Property Relation: Ideal gas relations

Process: Assume uniform-state, uniform-flow

Conservation Principles:

Conservation of mass:

$$\sum m_i - \sum m_e = (m_2 - m_1)_{CV}$$

Or, for one entrance, no exit, and initial mass of zero, this becomes

$$m_i = (m_2)_{CV}$$

Conservation of energy:

For an insulated tank Q is zero and for a rigid tank with no shaft work W is zero. For a one-inlet mass stream and no-exit mass stream and neglecting changes in kinetic and potential energies, the uniform-state, uniform-flow conservation of energy reduces to

$$Q - W = \sum m_e h_e - \sum m_i h_i + (m_2 u_2 - m_1 u_1)_{CV} \quad (kJ)$$

$$0 = -m_i h_i + (m_2 u_2)_{CV}$$

or

$$m_i h_i = (m_2 u_2)_{CV}$$

$$h_i = u_2$$

$$u_i + P_i v_i = u_2$$

$$u_2 - u_i = P_i v_i$$

$$C_v (T_2 - T_i) = P_i v_i$$

$$C_v(T_2 - T_i) = RT_i$$
$$T_2 = \frac{C_v + R}{C_v} T_i = \frac{C_p}{C_v} T_i$$
$$= kT_i$$

If the fluid is air, $k = 1.4$ and the absolute temperature in the tank at the final state is 40 percent higher than the fluid absolute temperature in the supply line. The internal energy in the full tank differs from the internal energy of the supply line by the amount of flow work done to push the fluid from the line into the tank.

Extra Assignment

Rework the above problem for a 10 m^3 tank initially open to the atmosphere at 25°C and being filled from an air supply line at 90 psig, 25°C , until the pressure inside the tank is 70 psig.