# Reliability of Systems

## Overview

- Series System
- Parallel System (Redundant System)
- Combined Series-Parallel Systems
  - High-level versus Low-level Redundancy
  - k-out-of-n Redundancy
- Reliability Consideration in Design

# Motivation

**Before …**

Several probability models useful in describing different failure processes:

- Constant Failure Rate Model
  - Exponential Distribution
- Time-dependent Failure Models
  - Weibull Distribution
  - Normal Distribution
  - Lognormal Distribution

**Next …**

**System Structure or Configuration**

- We are interested in the System Reliability
- System reliability depends on both
  1. **Reliability of the components**
  2. **System structure or configuration**

# Goal

To study how system reliability depends on **component reliability** and **system structure**:

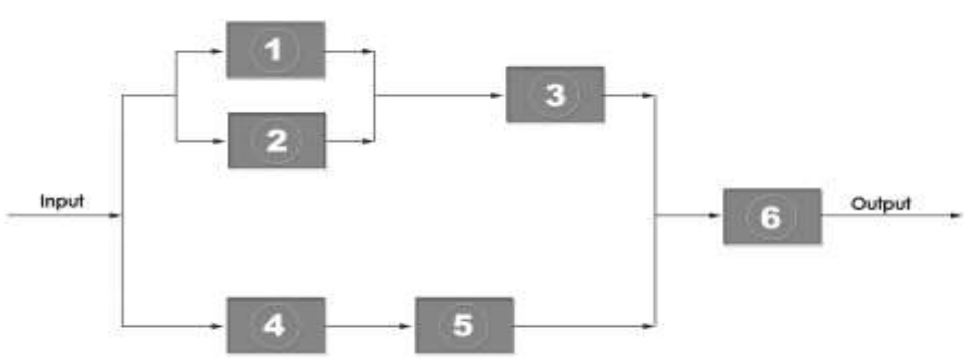
- Estimate system reliability
- Examine how to improve system reliability
  - Improve component reliability?
  - Change system structure?
  - Build in redundancy?

# Introduction

- System – a collection of components or subsystems assembled to perform a specific task.
- Typical System Structures:
  - Series System
  - Parallel System (Redundant System)
  - Combined Series-Parallel Systems
- Representations of System Structure

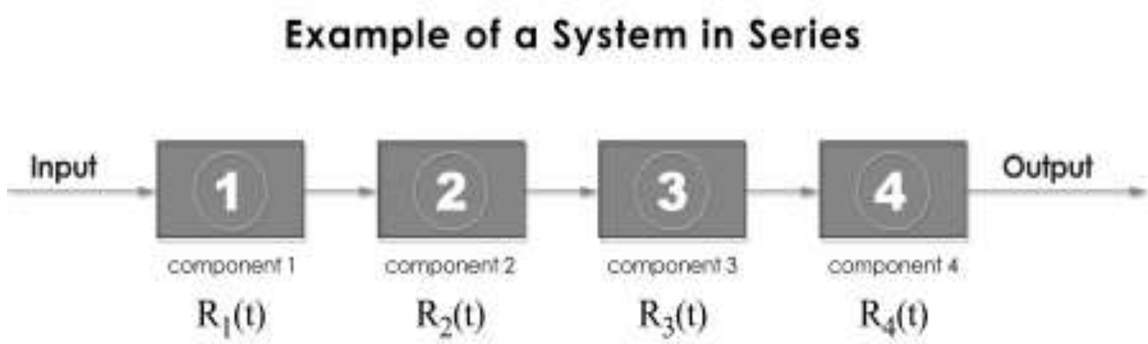
**Reliability Block Diagram**

– a logic diagram representing the arrangement of components



# Series System

In a series system, all components or subsystems must function for the system to function. (**A series system fails as long as any one of the components fails**.)




**Reliability Block Diagram**

**Applications:**

- Some computer networks
- Chains, Multi-cell batteries
- Decorative tree-lights

**Basic Equation for System Reliability:**

When components fail independently,

$$R_s(t) = R_1(t)R_2(t)R_3(t)\cdots R_n(t) = \prod_{i=1}^{n} R_i(t)$$

# Examples (Series System)

**Example 1:**

Suppose a system consists of 4 components arranged **in series**. The first two components have reliabilities of 0.9 at time $t$ = 1 year and the other two components have reliabilities of 0.8 at $t$ = 1 year. What is the overall reliability of the system at one year?

**Solution:**

The probability of the system operating successfully at the end of the 1st year is:

$$R_s(t) = R_1(t)R_2(t)R_3(t)R_4(t)$$

$$= (0.9)^2(0.8)^2 =$$

# Series System (cont.)

## Limit on System Reliability

$$R_s(t) \leq \min\{R_1(t), R_2(t), \cdots, R_n(t)\}$$

- The system reliability can not be greater than the smallest component reliability, why?

- It is important for all components to have a high reliability, especially when the system has a large number of components.

# Examples (Series System)

**Example 2:**

A truck cab assembly line is under development. The line will utilize five welding robots.

a) If each robot has 95 percent reliability, what is the total robot system reliability for the line?

b) To have a robot system reliability of 95 percent, what must each individual robot's reliability be?

**Solution:**

**(a)**

$$R_s(t) = R_1(t)R_2(t)R_3(t)R_4(t)R_5(t)$$
$$= (0.95)^5 =$$

**(b)**

$$R_s(t) = 0.95 = R_i^5$$
$$R_i \quad = 0.9898$$

# Exponential Distribution
## (Series System)

- If the times-to-failure of the components behave according to the exponential p.d.f, then the overall p.d.f of times-to-failure is also exponential:

$$f_s(t) = (\lambda_1 + \lambda_2 + ...... + \lambda_n)\, e^{-(\lambda_1+\lambda_2+......+\lambda_n)\, t}$$
$$= \lambda_s\, e^{-\lambda_s t}$$

Where: $\boldsymbol{\lambda_s} = \sum_{i=1}^{n} \lambda_i$ **=** The hazard rate of the series system

- In addition, for exponentially distributed times to failure of unit ***i***, the unit reliability is:

$$R_i(t) = \exp(-\lambda_i\, t)$$

The mean time to failure in this case is:

$$MTTF_s = \int \exp(-\sum \lambda_i\, t)\, dt = 1/\sum \lambda_i = 1/\lambda_s$$

# Examples (Series System)

**Example 3**: ***Exponential case***

Five components in series are each distributed exponentially with a hazard rate of 0.2 failures per hour. What is the reliability and MTTF of the system?

**Solution**

$$\because \lambda_i = 0.2 \qquad \mathrm{i} = 1,2,3,4,5$$

$$\because \mathrm{R_s} = e^{-\lambda_S t}$$

$$\lambda_S = \sum_{i=1}^{5} \lambda_i = (0.2)^5 = 1$$

$$\therefore \mathrm{R_s} = e^{-t}$$

$$MTTF_s = \frac{1}{\lambda_s} = 1$$

# Examples (Series System)

**Example 4:** Consider a 4-component series system with iid failure distributions having CFR. Suppose Rs(100) = 0.95. What is individual component MTTF?

**Solution**

$$\because \mathrm{R_s} = e^{-\lambda_S t}$$

$$R_s(100) = e^{-\lambda_S * 100}$$

$$\lambda_s = \sum_{i=1}^{4} \lambda_i \Rightarrow \lambda_i = \frac{\lambda_s}{4} = 0.000128$$

$$MTTF_i = \frac{1}{\lambda_i} = 7812.5$$

# Series System (cont.)

- **What is <u>system hazard rate</u> *$h_s(t)$* for series system?**

$$h_s(t) = \sum h_i(t)$$

# *Weibull Distribution*

# (Series System)

$$h_i(t) = \left(\frac{\beta_i}{\theta_i}\right)\left(\frac{t}{\theta_i}\right)^{\beta_i - 1}$$

For n components with the same shape parameter β

$$h_s(t) = \sum_{i=1}^{n} h_i = \beta\, \mathrm{t}^{\beta\text{-}1}\, \theta_s$$

$$\theta_s = \left(\sum_{i=1}^{n} \frac{1}{\theta_i}\right)^{\frac{-1}{\beta}}$$

$$R_s(t) = e^{-(t/\theta_s)^{\beta}}$$

$$MTTF_s = \theta_s \Gamma(1+\beta)$$

$$Median\ \mathrm{TTF} = \beta_{50} = \theta_s (\ln 2)^{1/\beta}$$

# Examples (Series System)

**Example 6*: *Weibull case***

A jet engine consists of 5 modules each of which has a Weibull TTF distribution with shape parameter $\beta$= 1.5. The scale parameters (in operating cycles) are: 3600, 7200, 5850, 4780, 9300. Find the MTTF, median TTF, and the reliability of the engine.

**Solution**

$$\theta_s = [\frac{1}{(3600)^{1.5}} + \frac{1}{(7200)^{1.5}} + \frac{1}{(5850)^{1.5}} + \frac{1}{(4780)^{1.5}} + \frac{1}{(9300)^{1.5}}]^{-1/1.5} = 1842.7$$

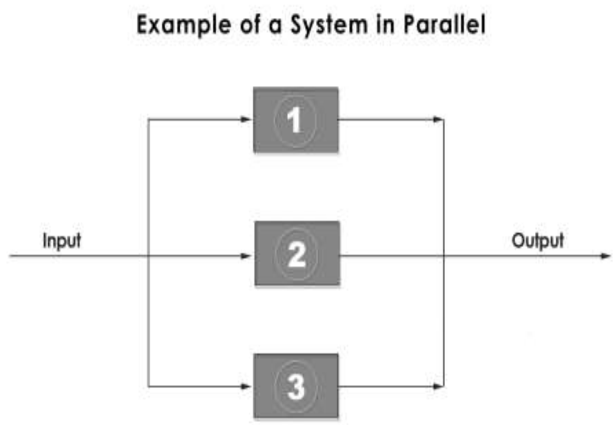
$$MTTF_s = 1842.7\Gamma(1+\frac{1}{1.5}) = 1842.7 * 0.90275 = 1663.5 \text{ Cycle}$$

$$Median = B_{50} = 1842.7(\ln 2)^{1/1.5} = 1443.2$$

$$R_s(t) = e^{-(\frac{t}{1842.7})^{1.5}}$$

# Parallel System
# (Redundant System)

In a parallel system, all components must fail for the system to fail. (A parallel system works as long as any one of the components works.)




**Reliability Block Diagram**

## Applications:

- Jet engines, Braking systems
- Tires in trucks, Projector light bulbs

## Basic Equation for System Reliability:

For parallel systems, it is easier to work with failure probability than reliability

$$F_s(t) = \prod_{i=1}^{n} F_i(t)$$

$$R_s(t) = 1 - \prod_{i=1}^{n} [1 - R_i(t)]$$

# Examples (Parallel System)

**Example 1:** Suppose a system consists of 4 components arranged in parallel. The first two components have reliabilities of 0.9 and time t = 1 year and the other two components have reliabilities of 0.8 at t = 1 year. What is the overall reliability of the system at one year?

**Solution**

$$R_s(t) = 1 - [(1-0.9)^2(1-0.8)^2] = 0.996$$

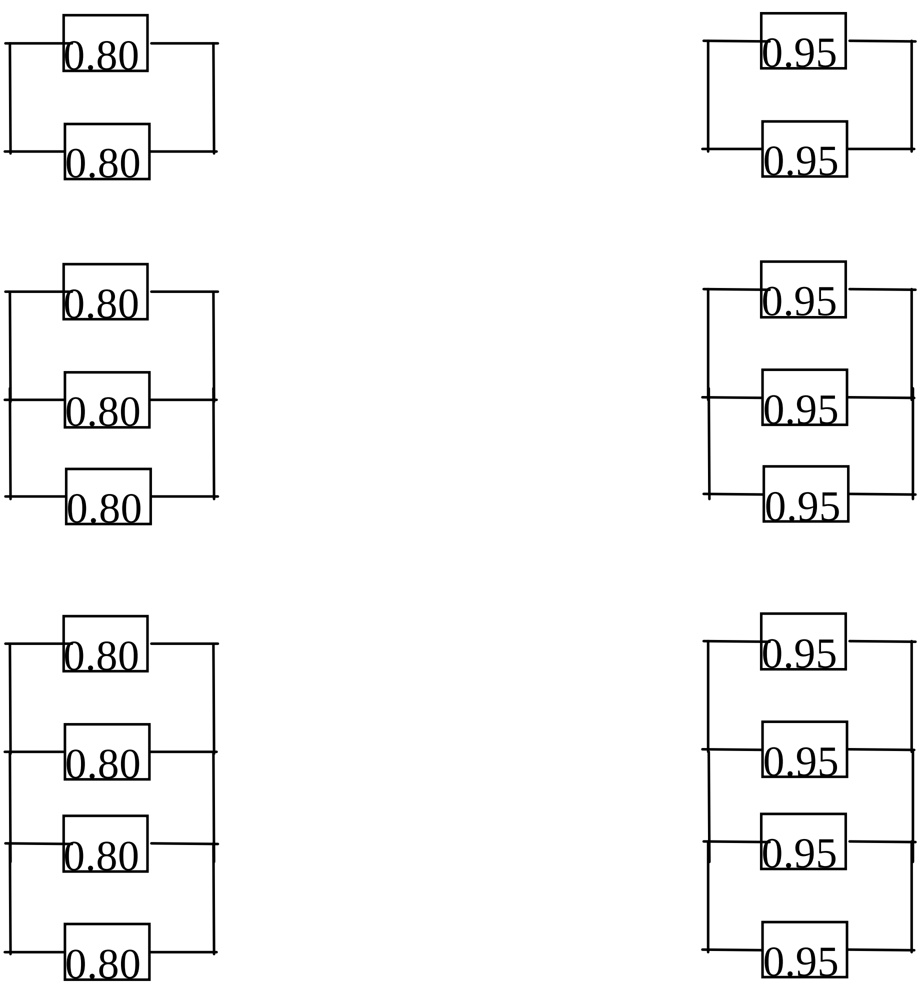
# Examples (Parallel System)

## Limit on System Reliability

$$R_s(t) \geq \max\{R_1(t), R_2(t), \cdots, R_n(t)\}$$

The system reliability can not be less than the largest component reliability, why?

## Example 2:



# Exponential Distribution
## (Parallel System)

If the times-to-failure of the components behave according to the exponential p.d.f, then the overall p.d.f of times-to-failure is not a simple exponential. For example, if the system compose of two component, then the system times-to-failure p.d.f :

$$f(t) = \lambda_1 \exp(-\lambda_1 t) + \lambda_2 \exp(-\lambda_2 t) - (\lambda_1 + \lambda_2) \exp(-(\lambda_1 + \lambda_2) t)$$

# Exponential Distribution
# (Parallel System)

For exponentially distribution times to failure of unit ***i***, the parallel system reliability is:

$$R_s = 1 - \Pi\,(1 - \exp(-\lambda_i\, t))$$

And for identical units ($\lambda_i = \lambda$) the reliability of the parallel system simplifies to:

$$R_s = 1 - (1 - \exp(-\lambda t))$$

And the mean time to failure for the identical unit parallel system is:

$$MTTF_s = \int [\,1 - (1 - \exp(-\lambda t))\,]\, dt = 1/\lambda \sum 1/i$$

The mean time to failure in this case:

$$MTTF = \int f(t)\, t\, dt$$

$$= 1/\lambda_1 + 1/\lambda_2 - 1/(\lambda_1 + \lambda_2)$$

# Parallel Systems

**Example**

A system composes of three components. These components have constant failure rates of 0.0004, 0.0005, 0.0003 failures per hour. The system will stop working, if any one of its components fails. Calculate the following:

1. The reliability of the system at 2500 hour running time?
2. The system hazard rate?
3. Mean time to failure of the system?

**Solution**

# Parallel Systems

**Example**

Another system composes of four items and each one of these items has constant failure rate of 0.0008 failures per hour. These items when they work the system will work. However, when any one of the system's items fails, the whole system will come to a complete halt. Calculate the following:

1. The reliability of the system at 2000 hour running time?
2. The system hazard rate?
3. Mean time to failure of the system?

**Solution**

# Parallel Systems

**Example**

A machine has two independent cutting parts. Any one of these cutting parts is sufficient to operate the machine. The hazard rates for the two parts of this machine are constant and they are 0.001 and 0.0015 failures per hour. Calculate:

1. The survival probability of the machine at 300 hour of running time?
2. The mean time to failure for this machine?

**Solution**

# Examples (Parallel System)

**Example 4:** System has two identical components in parallel with CFR of λ. We want Rs(1000) = 0.95. What should component MTTF be?

**Solution**

$$R_s(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

$$= 2e^{-\lambda t} + e^{-2\lambda t} = 0.95$$

$$0.95 = 2e^{-\lambda(1000)} + e^{-2\lambda(1000)} \Rightarrow \lambda = 0.000253$$

$$MTTF = \frac{1}{\lambda} = 3951.204$$

# Parallel Systems

The advantages of connecting equipment in parallel are:

1. To improve reliability of the system by making some of the equipment redundant to the other.

2. Extensive preventive maintenance can be pursued with no loss in plant availability since the separate parallel units can be isolated.

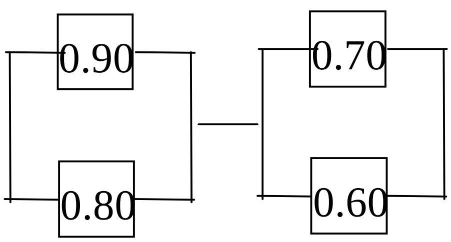
3. In the event of failure corrective maintenance can be arranged under less pressure from production or from competing maintenance tasks.

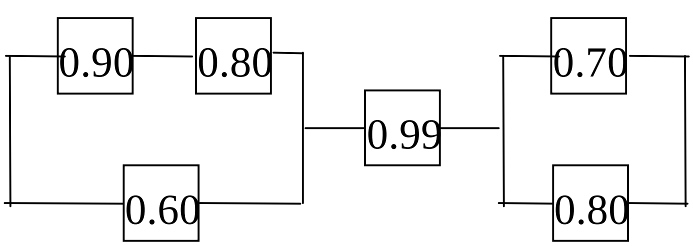
In the case of very high reliability units, there are only very marginal increments in reliability to be gained by installing redundant capacity unless the safety factors become evident.

# Combined Series-Parallel Systems

Series and parallel structures are the basis for building more complicated structures which use ***redundancy*** to increase system reliability.

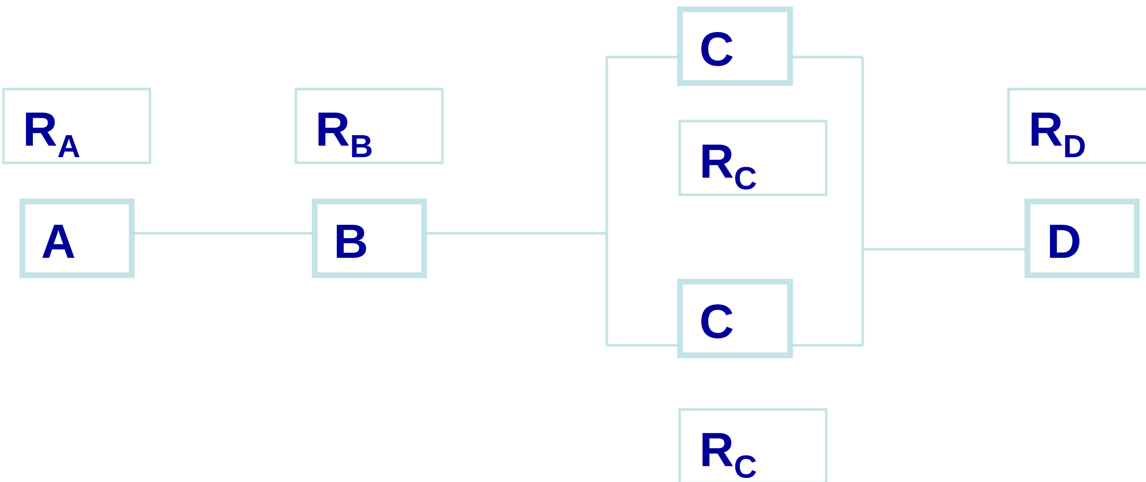
**Example**



# Series and Parallel System Reliability

## Combination of serial and parallel structures:

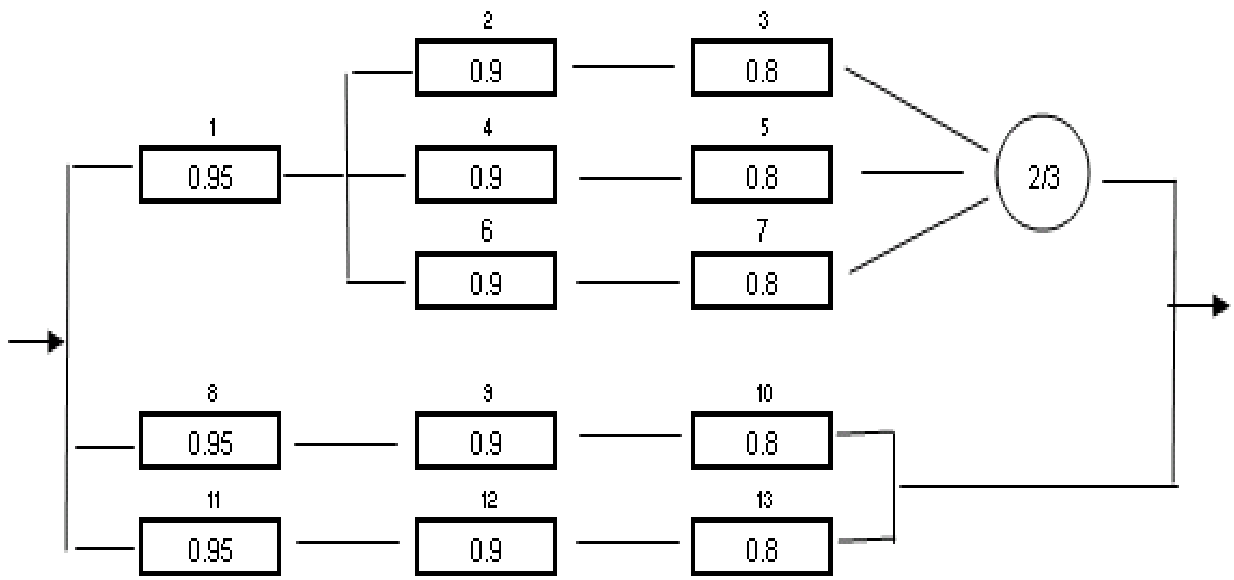



## Convert to equivalent series system

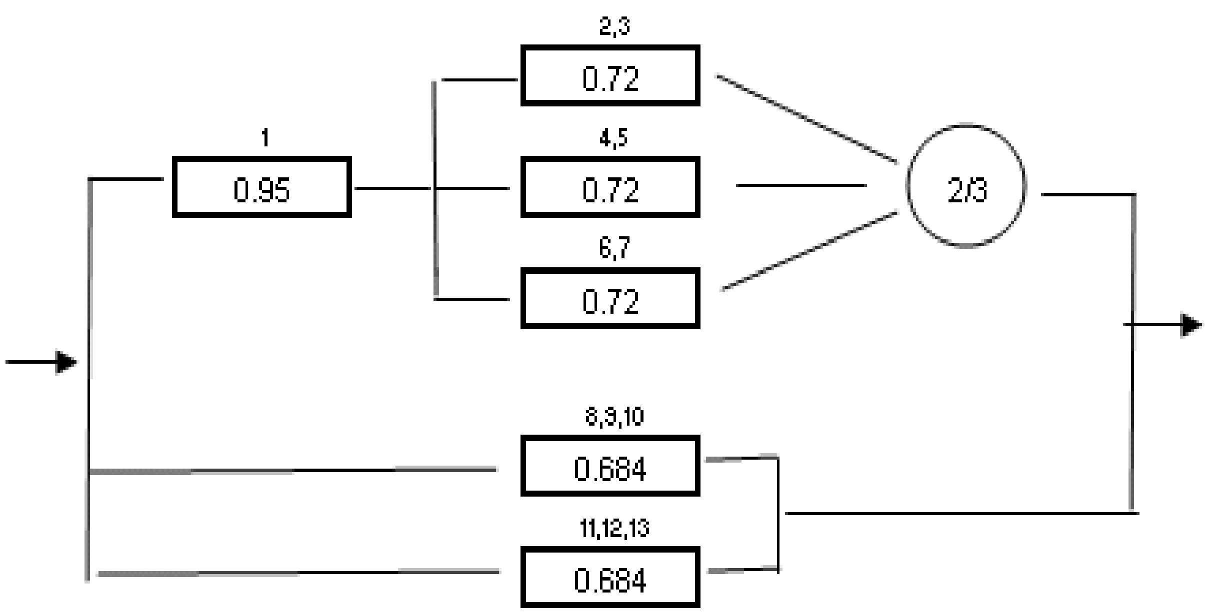

$$R_{C'} = 1 - (1-R_C)(1-R_C)$$

# Series and Parallel System Reliability

## Example

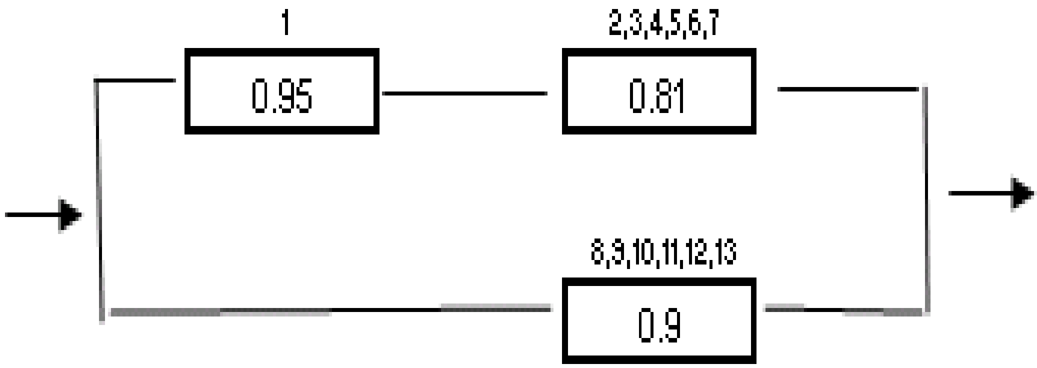


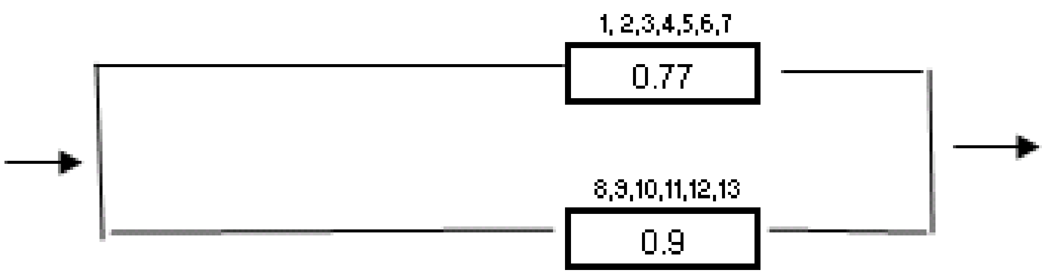

## Solution



# Series and Parallel System Reliability

## Solution




A(system) = 1- (1-0.77)*(1-0.9) = 0.98

# Series and Parallel System Reliability

**Example**

A system is made up of three components. **Component #1** has a normal life distribution with a mean of 1000 hours and a standard deviation of 250 hours. **Component #2** has a Weibull distribution with $\theta = 1100$ hours and $\beta = 0.85$, and **component # 3** has an Exponential distribution with a mean of 1400 hours. **The system fails when the first component fails.** What is the reliability of the system at 900 hours?

**Solution**

# Combined Series-Parallel Systems

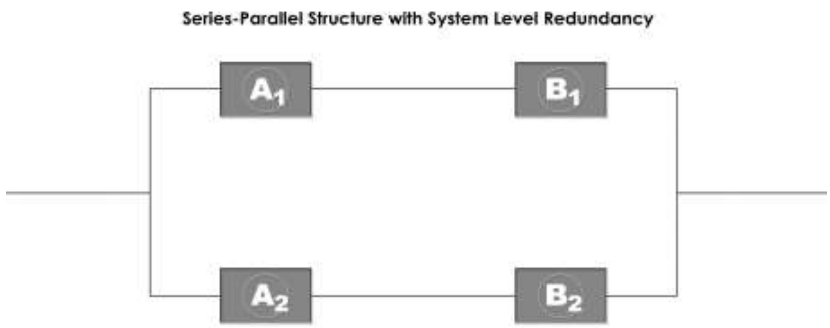
Redundancy in systems can be accomplished either at system level (high level) or at component level (low level) or some combination:

- High-level (system-level) Redundancy
- Low-level (component-level) Redundancy
- k-out-of-n Redundancy

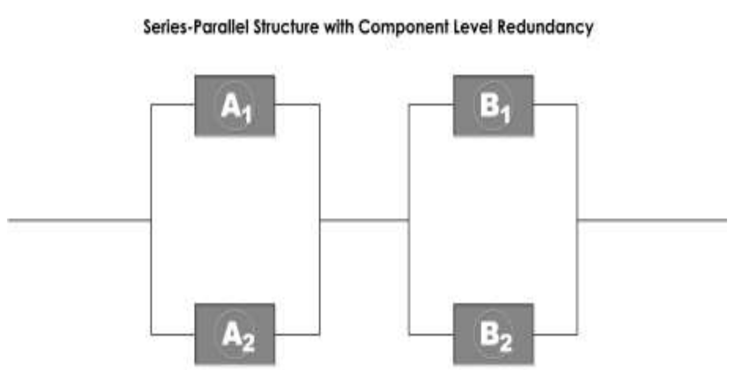
# High-level versus Low-level Redundancy

For a simple system with two components, ***A*** and ***B***.

- A combined series-parallel structure with **system-level (or high-level) redundancy**

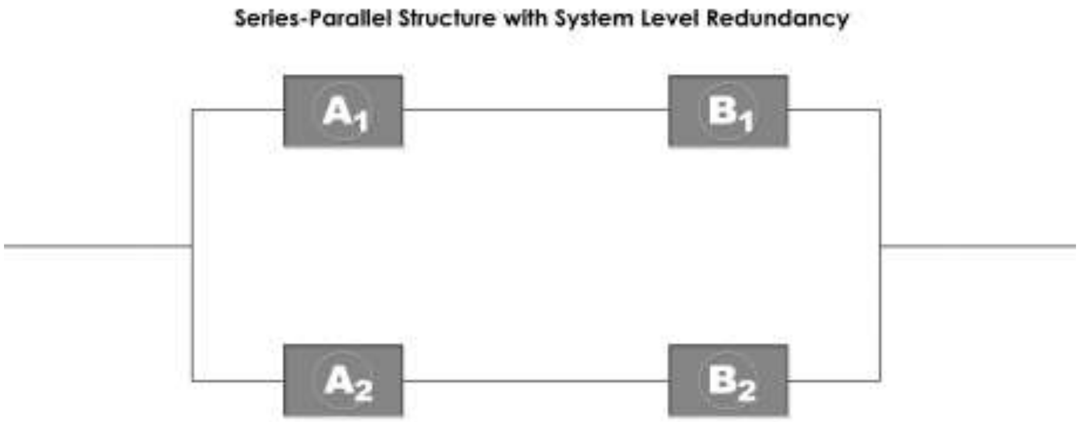



- A combined series-parallel structure with **component-level (low-level) redundancy**




Computing reliabilities: Which has higher reliability? High or low level redundancy?

# High-level Redundancy



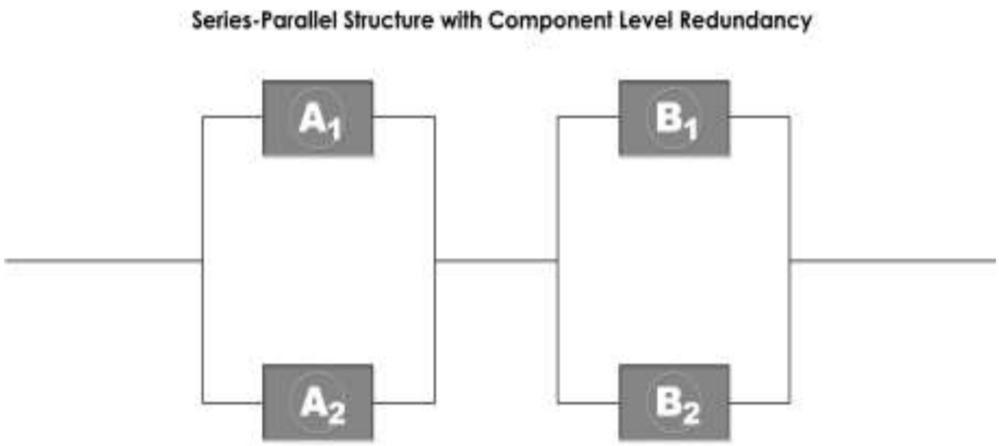

Examples:

- Dual central processors for critical switching communication systems
- Automobile brake systems
- Transatlantic undersea transmission cables

Reliability of the system:

$$R_{high} = 1 - (1 - R_A R_B)(1 - R_A R_B)$$

# Low-level Redundancy




Examples:

- Dual repeaters in undersea cables
- Human body (kidneys and lungs) …

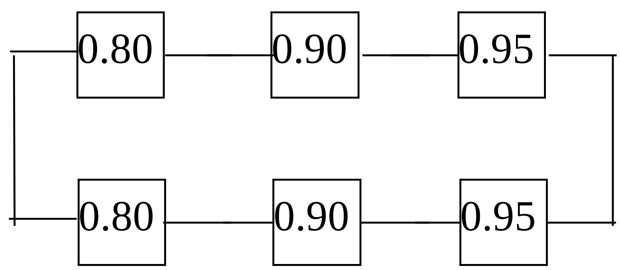
Reliability of the system:

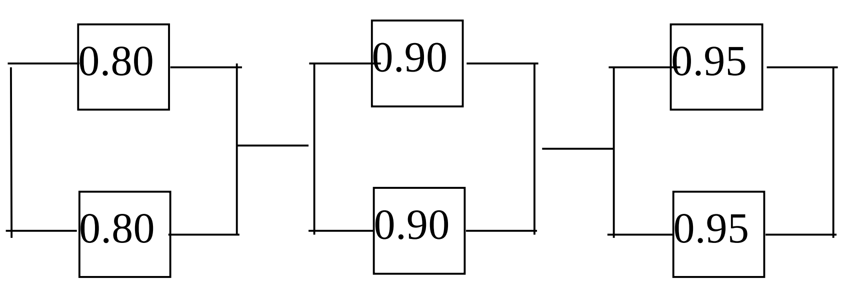
$$R_{low} = \left(1-(1-R_A)^2\right)\left(1-(1-R_B)^2\right)$$

# Examples (High or Low Level)

**Which has higher reliability? High or low level? Why?**

**Example:** A radio consists of 3 major components: power supply, receiver, and amplifier, with R of 0.8, 0.9, and 0.95 respectively for some design life L. Compute system reliability for both high-level and low-level redundancy with two parallel components. Which would be higher?




**If redundancy is to be used to improve reliability, it should be applied at the lowest possible level in the design.**

# k-out-of-n Redundancy

- The system works if *k* or more of the *n* components work
- Series and parallel systems are special cases of k-out-of-n systems:
  - parallel systems: 1-out-of-n system (k=1)
  - series systems: n-out-of-n system (k=n)
- Examples:
  - A bicycle wheel that needs only k out of its n spokes to function
  - An automobile engine that needs only k of its n cylinders to run
  - Satellite battery systems which continue to operate as long as there are 6 out of 10 batteries working

# k-out-of-n Redundancy

- For independent identical distributed components ( $R_i = R, i = 1, \cdots, n$ ), the reliability of k-out-of-n system (the probability of *k* or more successes from among the *n* components) is:

$$R_s = \sum_{x=k}^{n} \binom{n}{x} R^x (1-R)^{n-x}$$

**Example 1:** For a 2-out-of-3 system (a special case of k-out-of-n system), each component has reliability of 0.9. What is the system reliability?

$$R_s = P(X = 2) + P(X = 3)$$

$$= \binom{3}{2}(0.9)^2(0.1)^1 + \binom{3}{3}(0.9)^3(0.1)^0 =$$

# Examples (k-out-of-n)

**Example 2:**

A space shuttle requires 3-out-of-4 of its main engines to achieve orbit. Each engine has a reliability of 0.97. What is the probability of achieving orbit?

**Solution**

n = 4 k = 3 R = 0.97

$$R_s = P(X = 3) + P(X = 4)$$

$$= \binom{4}{3}(0.97)^3(0.03)^1 + \binom{4}{4}(0.97)^4(0.03)^0 = 0.9948$$

# k-out-of-n Redundancy

## Exponential failures

$$R_s(t) = \sum_{x=k}^{n} \binom{n}{x} e^{-\lambda t x}(1-e^{-\lambda t})^{n-x}$$

$$MTTF_s = \frac{1}{\lambda}\sum_{x=k}^{n}\frac{1}{x}$$

**Example 3:** Suppose each engine has CFR= 0.0038074. Again assuming 3-out-of-4 structure, what is system MTTF?

**Solution**

n = 4 k = 3 λ = 0.0038074

$$R_s(t) = \sum_{x=3}^{4} \binom{4}{x} e^{-\lambda t x}(1-e^{-\lambda t})^{4-x}$$

$$= \binom{4}{3} e^{-3\lambda t}(1-e^{-\lambda t})^{1} + \binom{4}{4} e^{-4\lambda t}(1-e^{-\lambda t})^{0}$$

$$= 4(e^{-3\lambda t} - e^{-4\lambda t}) + e^{-4\lambda t} = 4e^{-3\lambda t} - 3e^{-4\lambda t}$$

$$MTTF = \int_0^{\infty} R_s(t)dt = \frac{4}{3\lambda} - \frac{3}{4\lambda} = \frac{7}{12\lambda}$$

# Reliability Design

- The reliability level of a system is determined during the design process.

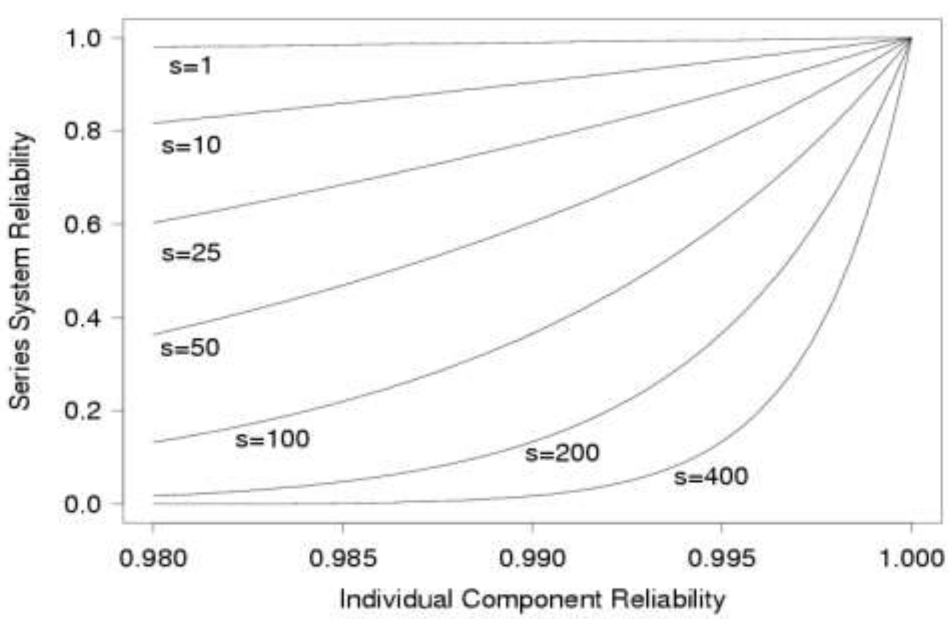
- The design process dictates the system configuration, which influences the reliability level.

- The designer should be familiar with the basic reliability analysis concepts that can be used to evaluate the design.

# Reliability Consideration in Series-System Design

- Two determinants of the system reliability of a series system:
  - The component reliability level
  - The number of components
- The reliability of a series system can be improved by
  - Increasing the component reliability
  - Decreasing the number of series components

Reliability of a System with $s$ Identical Independent Components in Series



# Reliability Consideration in Parallel-System Design

Parallel-system is a means for increasing reliability.

- Advantages:
  - Quickest solution if time is of major concern.
  - An easy and natural solution in some design situations
  - The only solution if the reliability requirement is beyond the state-of-the-art.
- Disadvantages:
  - May not be feasible or possible from a design standpoint
  - Switching mechanism may be needed and may defeat reliability gains
  - In highly reliable systems the gains will be small
  - May exceed weight and size limitations
  - Cost may be prohibitive
  - Power consumption may be increased
  - Repair frequency may increase

# Reliability Consideration in Parallel-System Design

Reliability of a System with $s$ Identical Independent Components in Parallel