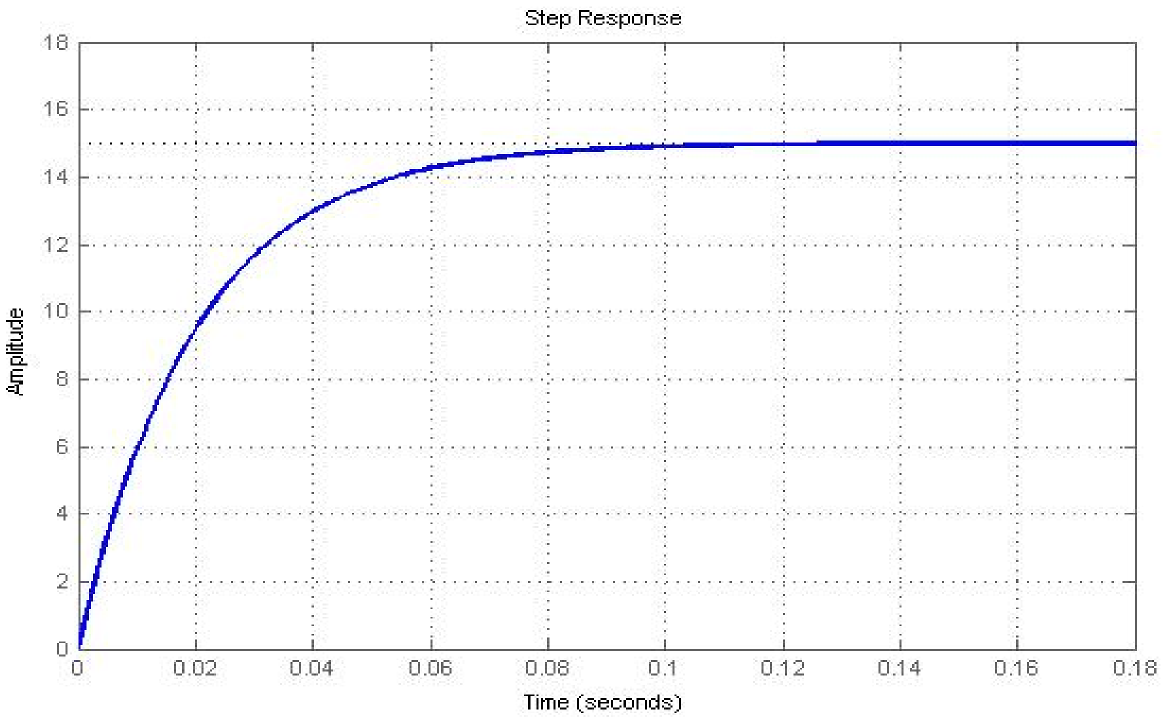
Exercise 1

For the unit step response shown in the following figure, find the transfer function of the system. Also find rise time and settling time.



Solution

Let, the transfer function $G(s)=\dfrac{K}{s+a}$

Final output = 15

63% of the final output is 15 x 0.63 = 9.45

Time to reach 9.45 is 0.02

Therefore, time constant = 0.02

$$a=\frac{1}{0.02}=50$$

$$\frac{K}{a}=15 \Rightarrow K=15\times 50=750$$

$$G(s)=\frac{750}{s+50}$$

Rise time, $T_r=\dfrac{2.2}{a}=0.044$ sec.

Settling time, $T_s=\dfrac{4}{a}=0.08$ sec.

Exercise 2

Find the ramp response for a system whose transfer function is:

$$G(s) = \frac{s}{(s+2)(s+3)}$$

Solution

Ramp input $\rightarrow$ $R(s) = \frac{1}{s^2}$,

we have $G(s) = \frac{C(s)}{R(s)}$ $\rightarrow$ The output $C(s) = R(s)G(s)$

$$C(s) = \frac{1}{s^2}\frac{s}{(s+2)(s+3)} = \frac{1}{s\,(s+2)(s+3)}$$

Using partial fraction:

$$C(s) = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+3)}$$

$$A = \left.\frac{1}{(s+2)(s+3)}\right|_{s=0} = {}^1/_6$$

$$B = \left.\frac{1}{s(s+3)}\right|_{s=-2} = -{}^1/_2$$

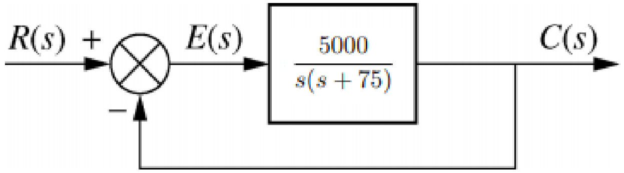
$$C = \left.\frac{1}{s(s+2)}\right|_{s=-3} = {}^1/_3$$

$$C(s) = \frac{{}^1/_6}{s} - \frac{{}^1/_2}{(s+2)} + \frac{{}^1/_3}{(s+3)}$$

The inverse Laplace Transform: $c(t) = \frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t}$

Exercise 3

For the unity feedback system with a unit step input, find:



1. Closed-loop transfer function.
2. Damping ratio, the natural frequency and the expected percent overshoot.
3. The settling time.

Solution

1. Closed-loop transfer function $T(s) = \frac{5000}{s^2 + 75s + 5000}$

The expected percent overshoot $\quad for\ second\ order\ system\ Canonical\ form\ :\ T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

From $T(s)$, we can check that $\omega_n = \sqrt{5000}$ = 70.711 and $2\zeta\omega_n = 75$. Thus, $\zeta = 0.53$ and %OS is given $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 14.01\%$

3 The settling time $T_s = \frac{4}{\zeta\omega_n} = 0.107$ sec.

Exercise 4

For the characteristic equations below, Find the range of gain K, that will cause the system to be stable, unstable, and marginally stable.

1. $s^4 + 6s^3 + 11s^2 + 6s + K = 0$
2. $s^3 + 2\,s^2 + 4\,s + K = 0$

Solution

System1: $s^4 + 6s^3 + 11s^2 + 6s + K = 0$

s^4	1	11	K
s^3	6	6	0
s^2	10	K	0
s^1	$\frac{60-6K}{10}$	0	
s^0	K		

The system is stable for $60 - 6K > 0 \rightarrow K < 10$

The system is unstable for $K > 10$

The system is marginally stable for $K = 10$ and $10s^2 + 10 = 0 \rightarrow s_{1,2} = \pm j$

System2: $s^3 + 2\,s^2 + 4\,s + K = 0$

s^3	1	4
s^2	2	K
s^1	$\frac{8-K}{2}$	0
s^0	K	0

The system is stable for $0 < K < 8$

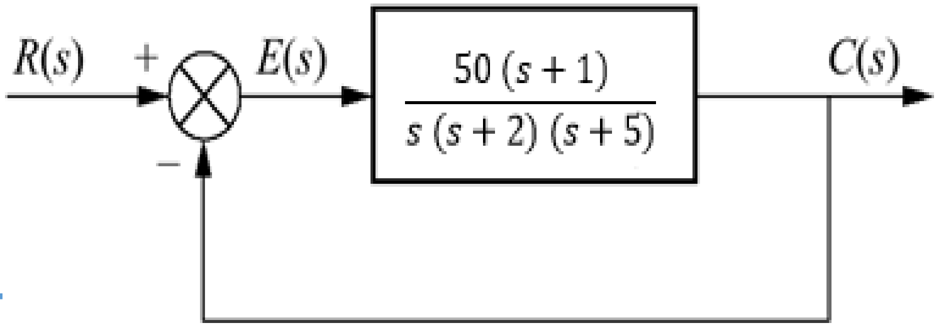
The system is unstable for $K > 8$

The system is marginally stable for $K = 8$

and $2s^2 + 8 = 0 \rightarrow s_{1,2} = \pm j2$

Exercise 5

Given the following non-unity feedback system, find the following:



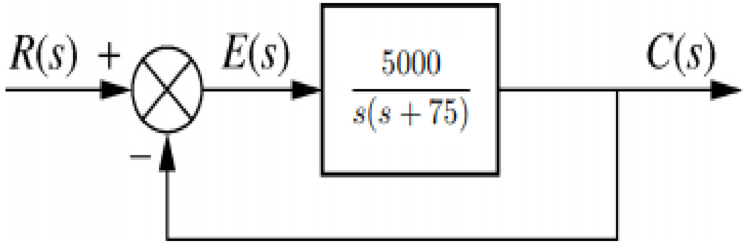
Evaluate the static error constants and find the expected error for the standard step, ramp, and parabolic inputs.

Solution

the static error constants	expected Steady state error
$K_p = \lim_{s \to 0} G(s) = \infty$	For step input: $e(\infty) = \frac{1}{1+K_p} = 0$
$K_v = \lim_{s \to 0} s\, G(s) = \frac{50 \times 1}{2 \times 5} = 5$	For ramp input: $e(\infty) = \frac{1}{K_v} = 0.2$
$K_a = \lim_{s \to 0} s^2 G(s) = 0$	For parabolic input: $e(\infty) = \frac{1}{K_a} = \infty$

Exercise 6

For the unity feedback system shown below, find:



a) The static error constants K_p, K_v, K_a

b) The steady-state error for an input of $u(t)$.

c) The steady-state error for an input of $2\, t\, u(t)$.

d) The steady-state error for an input of $3\, t^2\, u(t)$.

Solution

a) $K_p = \lim_{s\to 0} G(s) = \infty$, $K_v = \lim_{s\to 0} sG(s) = 66.7$, $K_a = \lim_{s\to 0} s^2 G(s) = 0$

b) for $r(t) = u(t) \;\rightarrow e(\infty) = \frac{1}{1+K_p} = 0$

c) for $r(t) = 2\, t\, u(t) \;\rightarrow e(\infty) = \frac{2}{K_v} = 0.03$

d) for $r(t) = 3\, t^2\, u(t) \;\rightarrow e(\infty) = \frac{6}{K_a} = \infty$