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Data and Computer Communications

Chapter 9

Spread Spectrum

Spread Spectrum

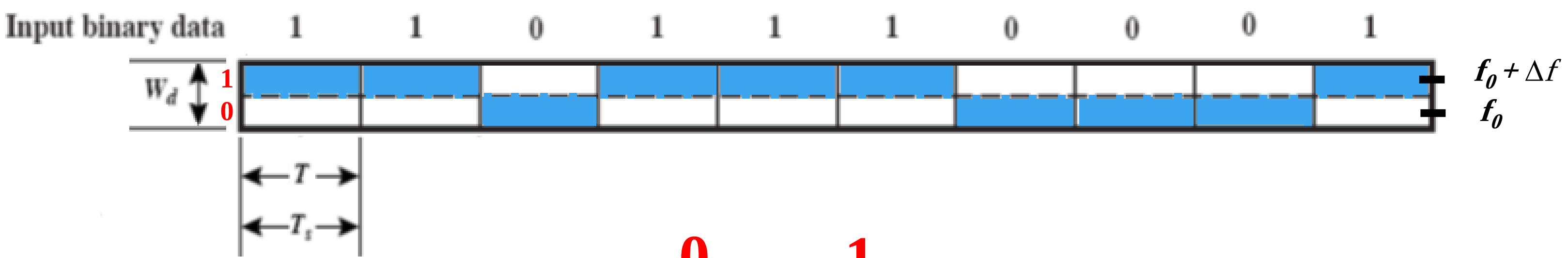
- important encoding method for **wireless** communications
- it was initially developed for military to make **jamming** and **interception harder**
- **analog & digital data** → **analog signal**
- **spreads data over wide bandwidth**
- two approaches, both in use:
  - Frequency Hopping Spread Spectrum (FHSS)
  - Direct Sequence Spread Spectrum (DSSS)

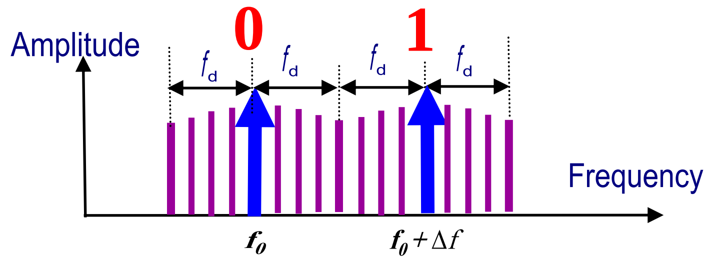
BFSK - Review

- The BFSK: $\boxed{s_d(t) = A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f)t)}$

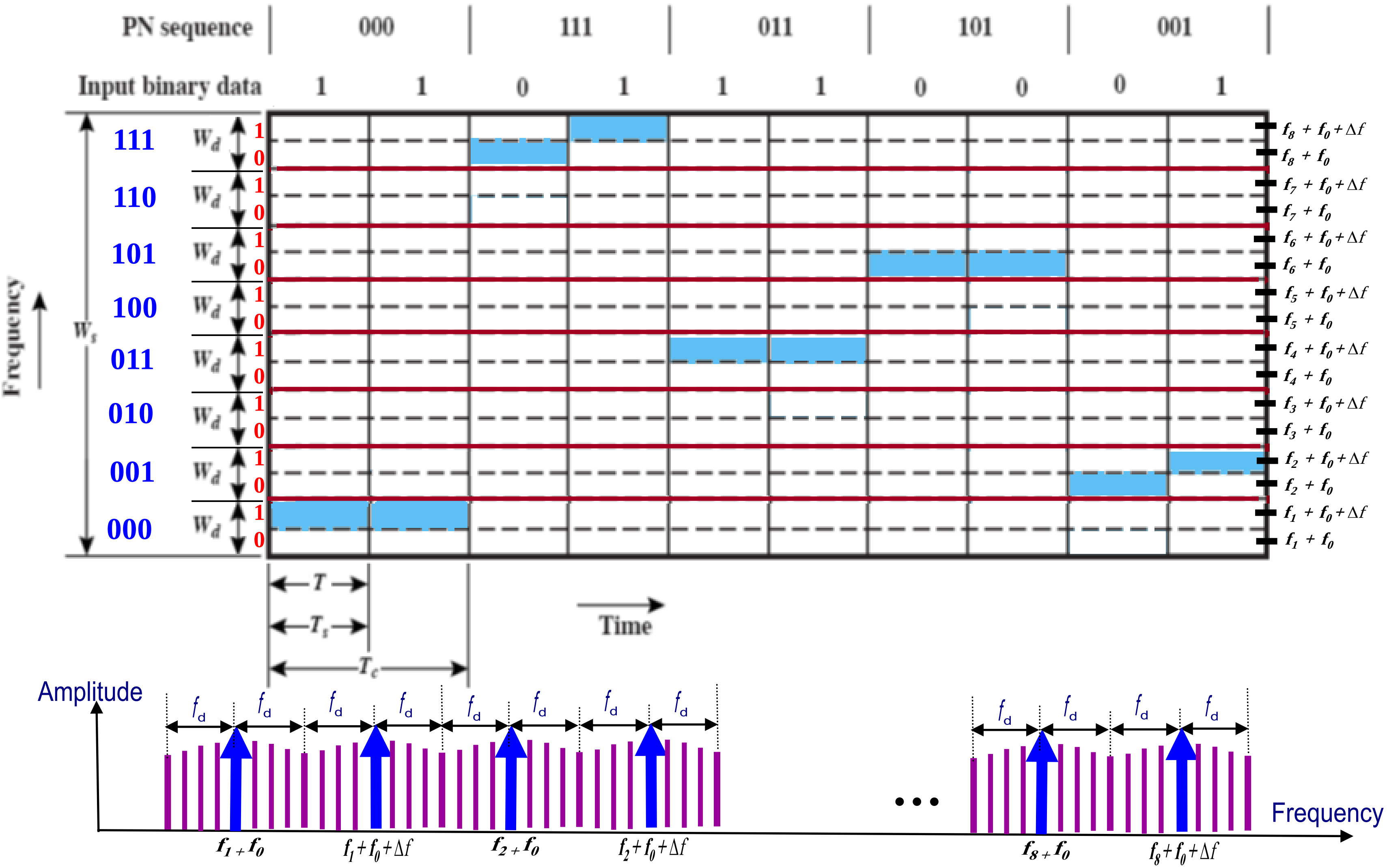
  where, A = amplitude of signal
  f_0 = base frequency
  b_i = value of the i^{th} bit of data (**+1 for binary 1 and -1 for binary 0**)
  Δf = frequency separation
  T = bit duration
  $1/T$ = data rate

- During the i^{th} bit interval, the frequency of data signal is ***f_0* if the data bit is -1** and ***$f_0+\Delta f$* if the data bit is +1**

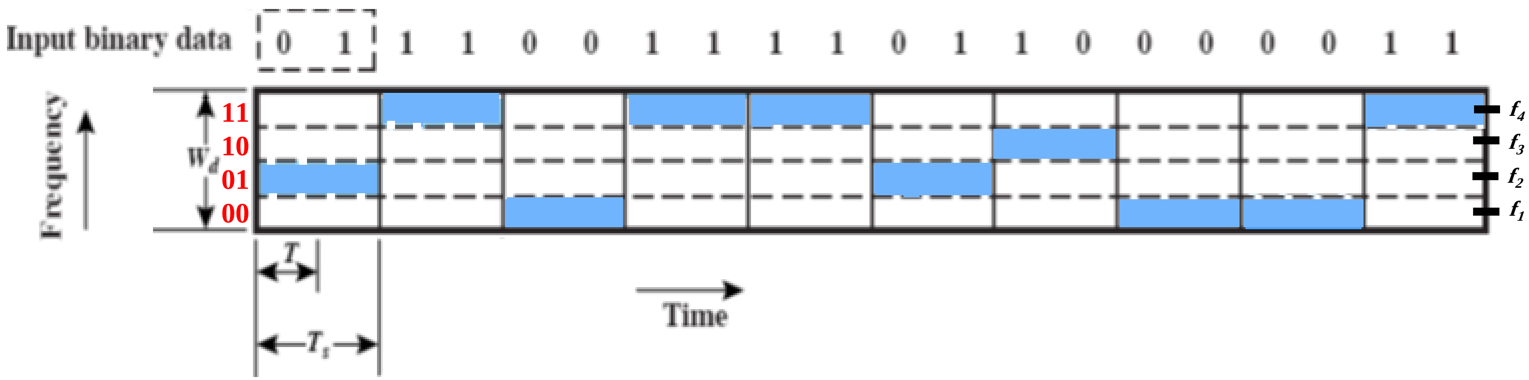


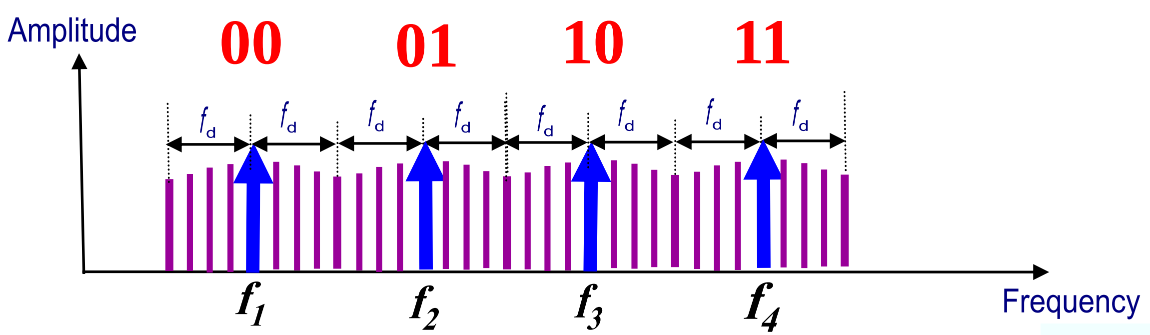


BFSK FHSS

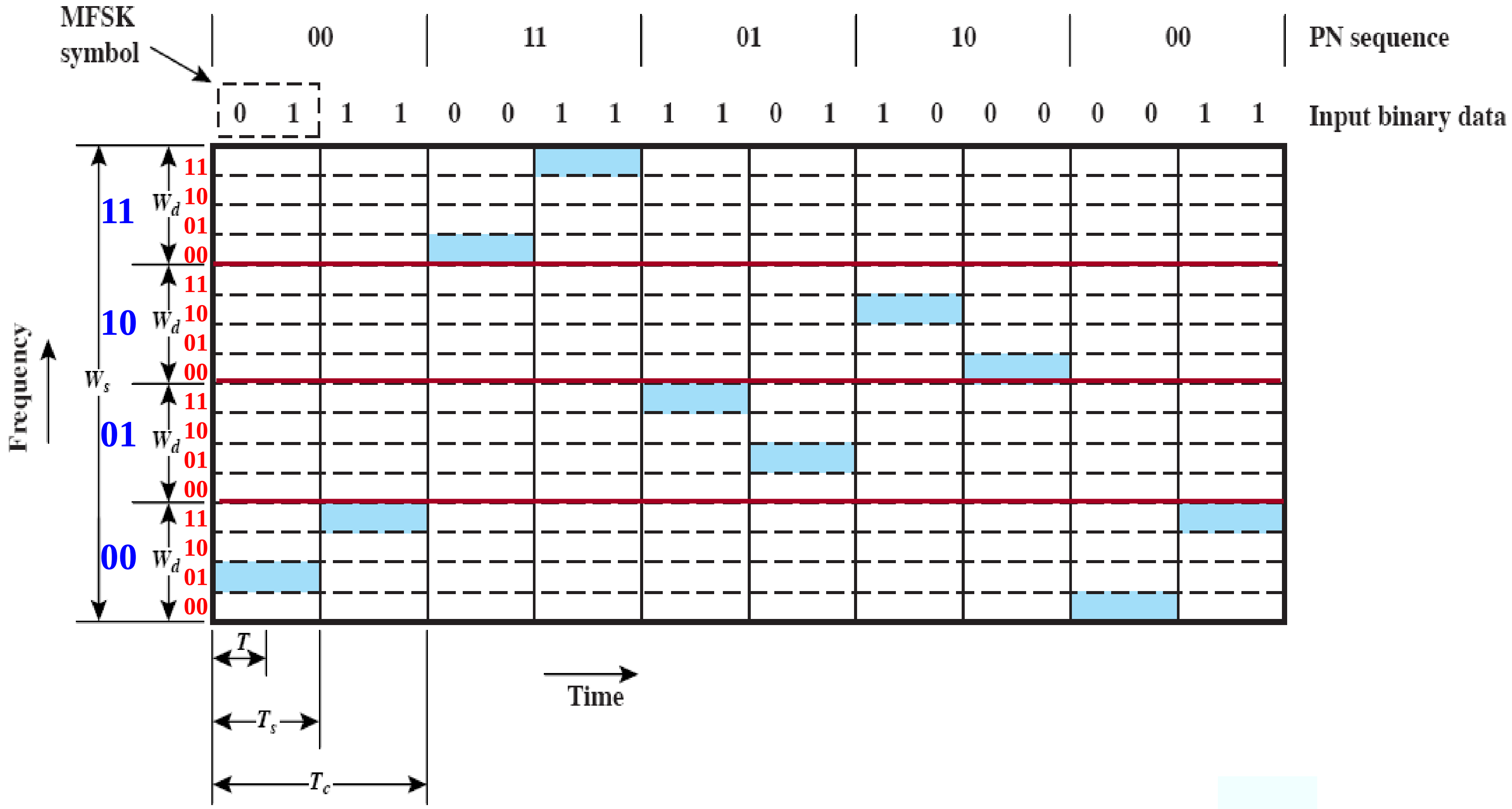

PN sequence
000
111
011
101
001
Input binary data
1 1 0 1 1 1 0 0 0 1
111
110
101
100
011
010
001
000
Frequency
Time
Amplitude
Frequency
$f_8 + f_0 + \Delta f$
$f_8 + f_0$
$f_7 + f_0 + \Delta f$
$f_7 + f_0$
$f_6 + f_0 + \Delta f$
$f_6 + f_0$
$f_5 + f_0 + \Delta f$
$f_5 + f_0$
$f_4 + f_0 + \Delta f$
$f_4 + f_0$
$f_3 + f_0 + \Delta f$
$f_3 + f_0$
$f_2 + f_0 + \Delta f$
$f_2 + f_0$
$f_1 + f_0 + \Delta f$
$f_1 + f_0$
f_d
$f_1 + f_0$
$f_1 + f_0 + \Delta f$
$f_2 + f_0$
$f_2 + f_0 + \Delta f$
$f_8 + f_0$
$f_8 + f_0 + \Delta f$

MFSK - Review

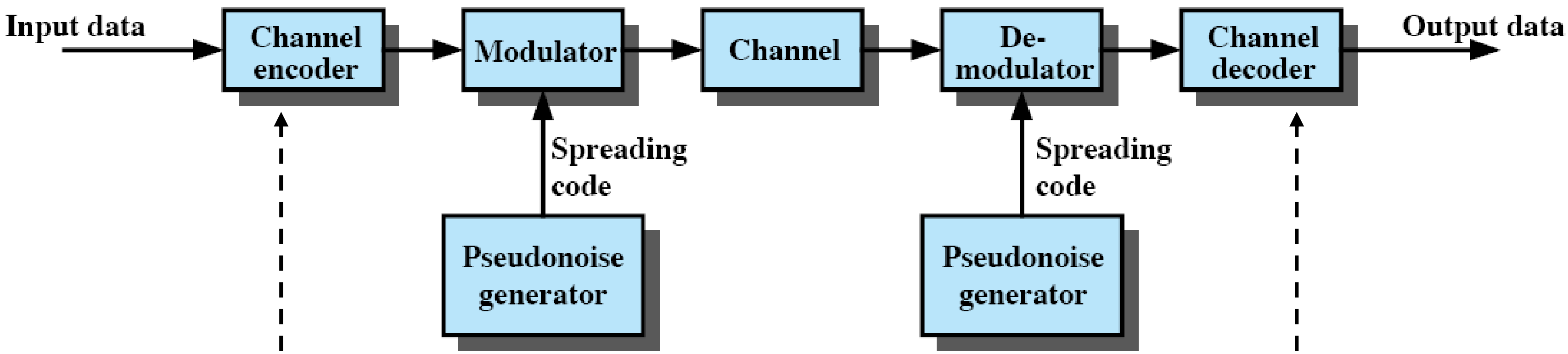

Input binary data
0 1 1 1 0 0 1 1 1 1 0 1 1 0 0 0 0 0 1 1
Frequency
W_d
11
10
01
00
f_4
f_3
f_2
f_1
T
T_s
Time


Amplitude
00
01
10
11
f_d
f_1
f_2
f_3
f_4
Frequency

MFSK FHSS


MFSK symbol
00
11
01
10
00
PN sequence
0 1 1 1 0 0 1 1 1 1 0 1 1 0 0 0 0 0 1 1
Input binary data
11
10
01
00
W_d
W_s
Frequency
T
T_s
T_c
Time

General Model of Spread Spectrum System



Input fed to channel encoder that produces an analog signal with a relatively narrow bandwidth around some center frequency

The signal is further modulated using spreading sequence (spreading code)

spreading code is generated using pseudorandom number generator

At the receiving end, the same spreading sequence is used to demodulate the spread spectrum signal

The signal is fed into a channel decoder to recover the data

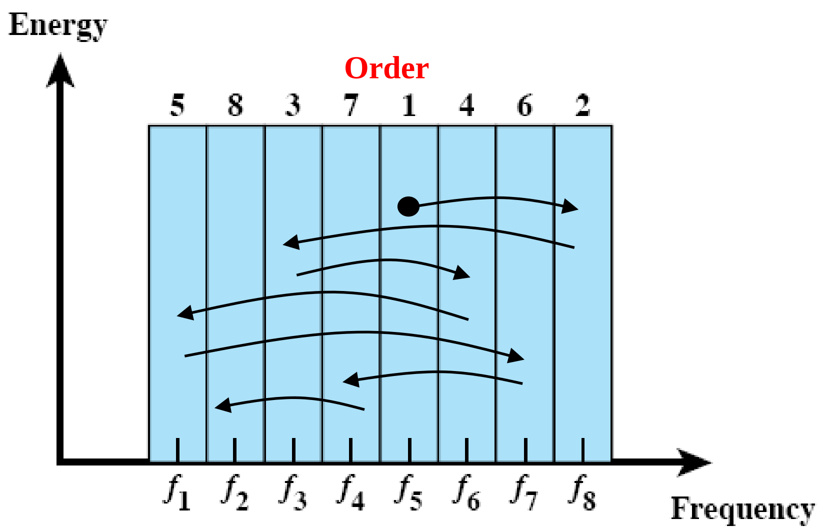
Concept of Spread Spectrum

- Input fed to **channel encoder** that produces an analog signal with a relatively **narrow bandwidth around some center frequency**
- The signal is further **modulated** using **spreading sequence** or **spreading code**
- **spreading code** is generated using **pseudorandom number generator**
- The effect of this modulation is to **increase significantly the bandwidth (spread the spectrum) of the signal to be transmitted**
- At the **receiving end**, the **same spreading sequence** is used to **demodulate** the spread spectrum signal.
- Finally, the signal is fed into a **channel decoder** to recover the data

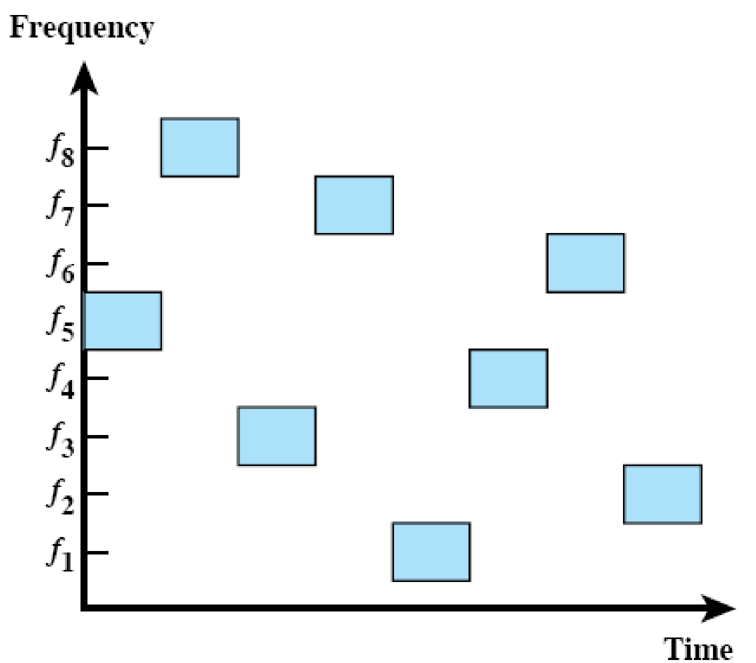
Spread Spectrum Advantages

- Several advantages can be gained from this apparent waste of spectrum:
  - **immunity from various kinds of noise** and multipath distortion
  - **Hiding and encryption signals.** Only a reception who knows the spreading code can recover the encoded information
  - **several users can share same higher bandwidth with little interference**
    - CDM/CDMA Mobile telephones

Frequency Hopping Spread Spectrum (FHSS)



(a) Channel assignment



(b) Channel use

Frequency Hopping Spread Spectrum (FHSS)

- signal is broadcast over seemingly random series of frequencies
- **receiver hops from frequency to another over fixed intervals in synchronization with transmitter**
- **eavesdroppers** hear unintelligible blips
- **jamming** on one frequency affects only a few bits

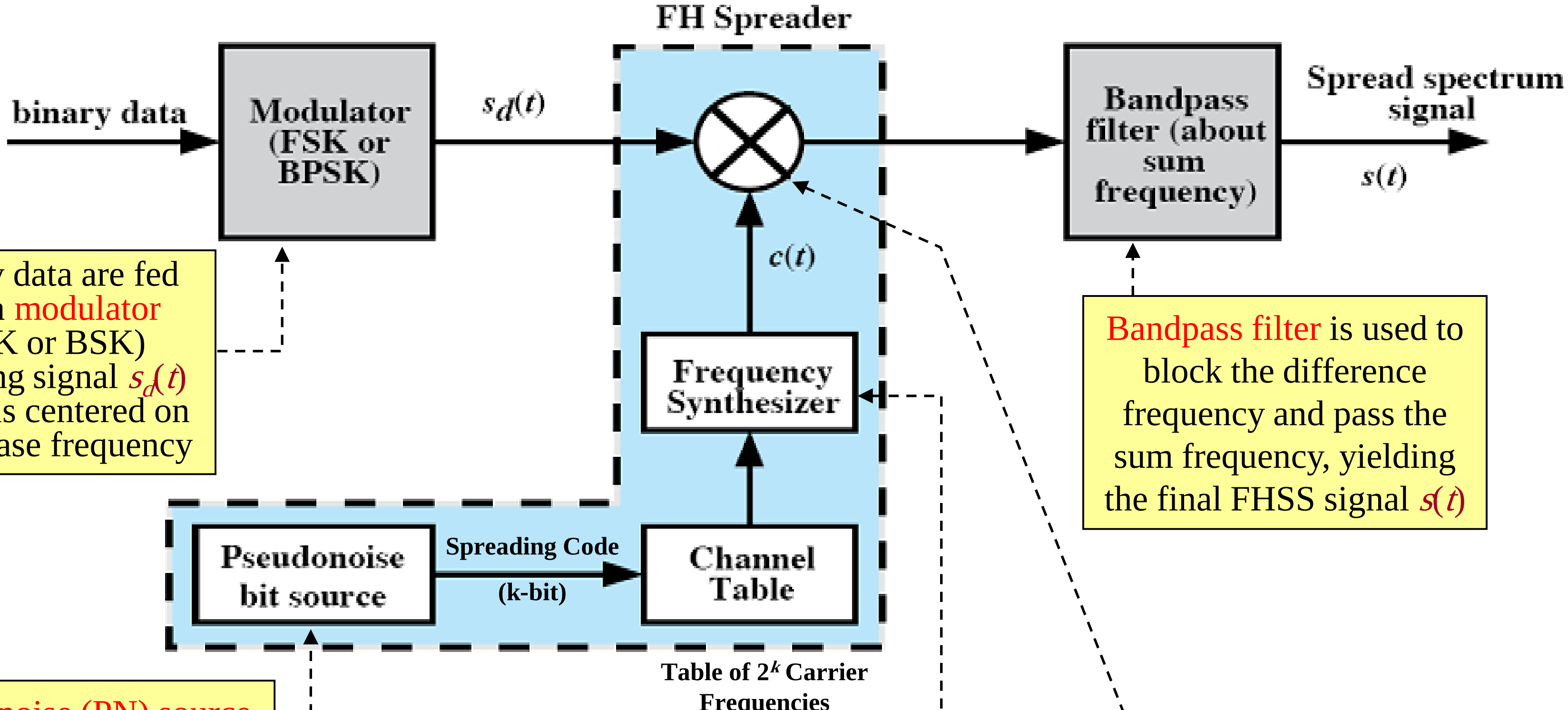
FHSS Basic Approach

- Number of **channels** allocated for a **frequency hopping** (FH) signal
- **2^k carrier frequencies** forming **2^k channels**
- spacing between carrier frequencies (i.e., the width of each channel) corresponds to the bandwidth of the input signal
- **transmitter operates in one channel at a time for a fixed interval**
- during that interval, some number of bits is transmitted using some encoding scheme
- spreading code dictates the sequence of channels used.
- **Both transmitter and receiver use the same code to tune into a sequence of channels in synchronization.**

FHSS (Transmitter)

- binary data are fed into a modulator using some digital-to-analog encoding scheme, such as FSK or BPSK resulting signal $s_d(t)$ which is centered on some base frequency
- pseudonoise (PN) source serves as an index into a table of frequencies
- each k bits of the PN source (i.e., spreading code) specifies one of the 2^k carrier frequencies
- at each successive interval, a new spreading code (k bits) is generated → a new carrier frequency is selected
- frequency synthesizer generates a constant-frequency tone whose frequency hops among a set of 2^k frequencies, with the hopping pattern determined by k bits from the PN sequence. It is known spreading or chipping signal $c(t)$
- $c(t)$ is then modulated by the signal produced from the initial modulator to produce a new signal with the same shape but now centered on the selected carrier frequency
- bandpass filter is used to block the difference frequency and pass the sum frequency, yielding the final FHSS signal $s(t)$

FHSS (Transmitter)



Binary data are fed into a modulator (FSK or BSK) resulting signal $s_d(t)$ which is centered on some base frequency

Bandpass filter is used to block the difference frequency and pass the sum frequency, yielding the final FHSS signal $s(t)$

Pseudonoise (PN) source generates k-bit spreading code which specifies one of the 2^k carrier frequencies from the channel table

Frequency synthesizer generates a constant-frequency tone which is called spreading or chipping signal $c(t)$

$c(t)$ is then modulated by the signal produced from the initial modulator to produce a new signal

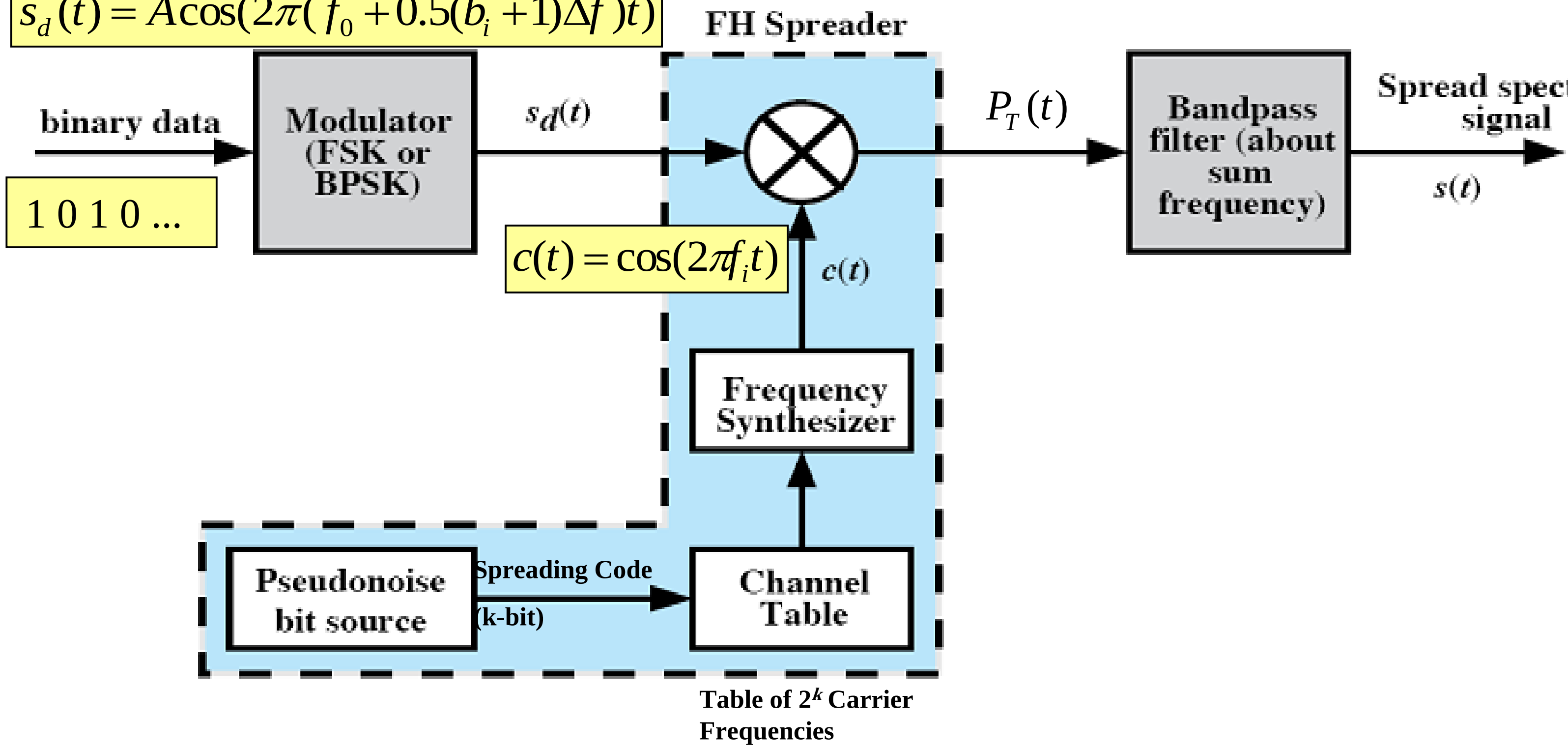
FHSS (Transmitter)

$$p_T(t) = s_d(t)c(t) = A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f)t)\cos(2\pi f_i t)$$

$$p_T(t) = 0.5A[\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f + f_i)t) + \cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f - f_i)t)]$$

$$s(t) = 0.5A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f + f_i)t)$$

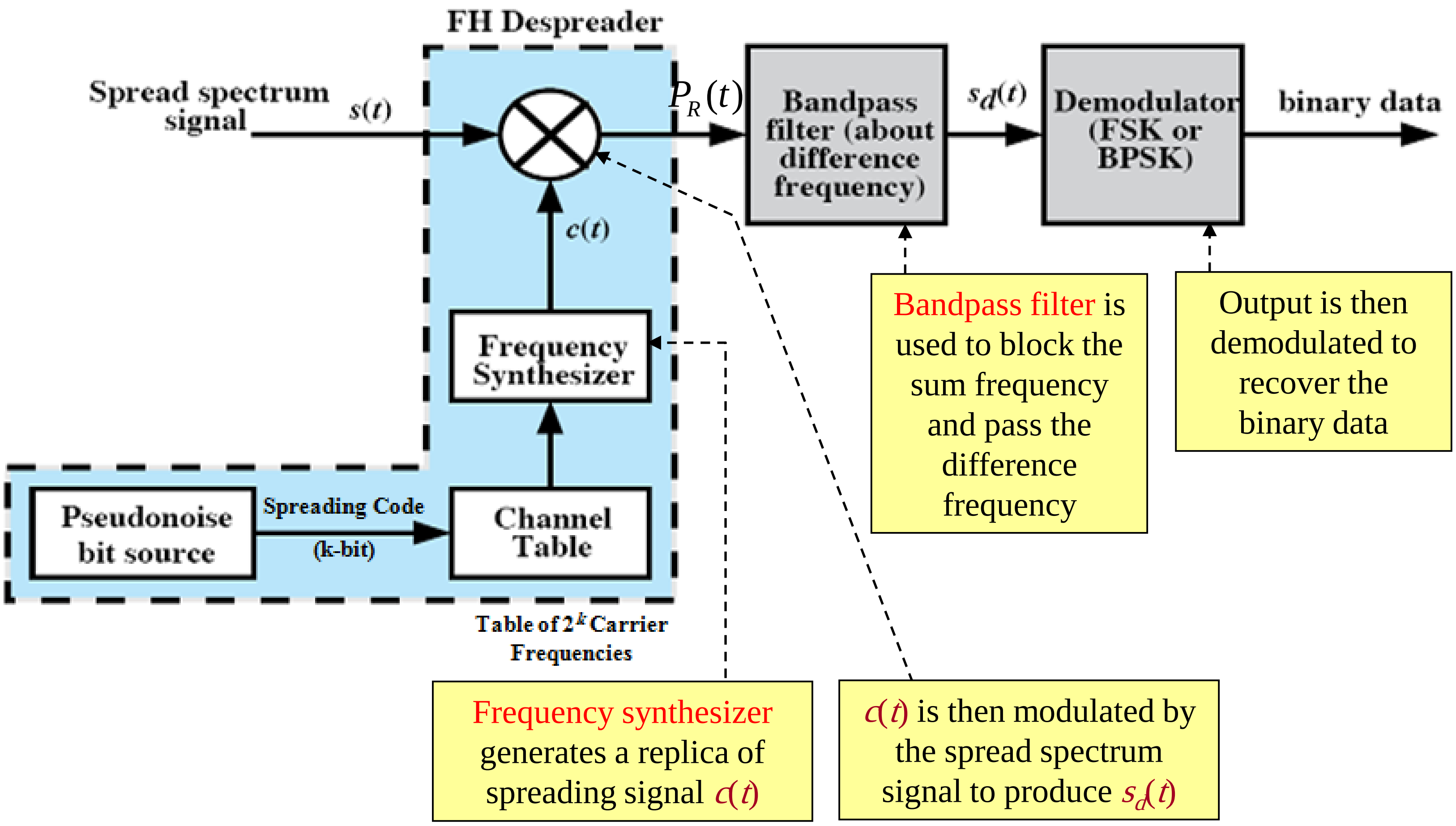
$$s_d(t) = A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f)t)$$



FHSS (Receiver)

- signal $s(t)$ is multiplied by a replica of the spreading signal $c(t)$ to yield a product signal $s_d(t)$
- **bandpass filter** is used to block the sum frequency and pass the difference frequency
- Output signal of bandpass filter is then demodulated to recover the binary data.

FHSS (Receiver)

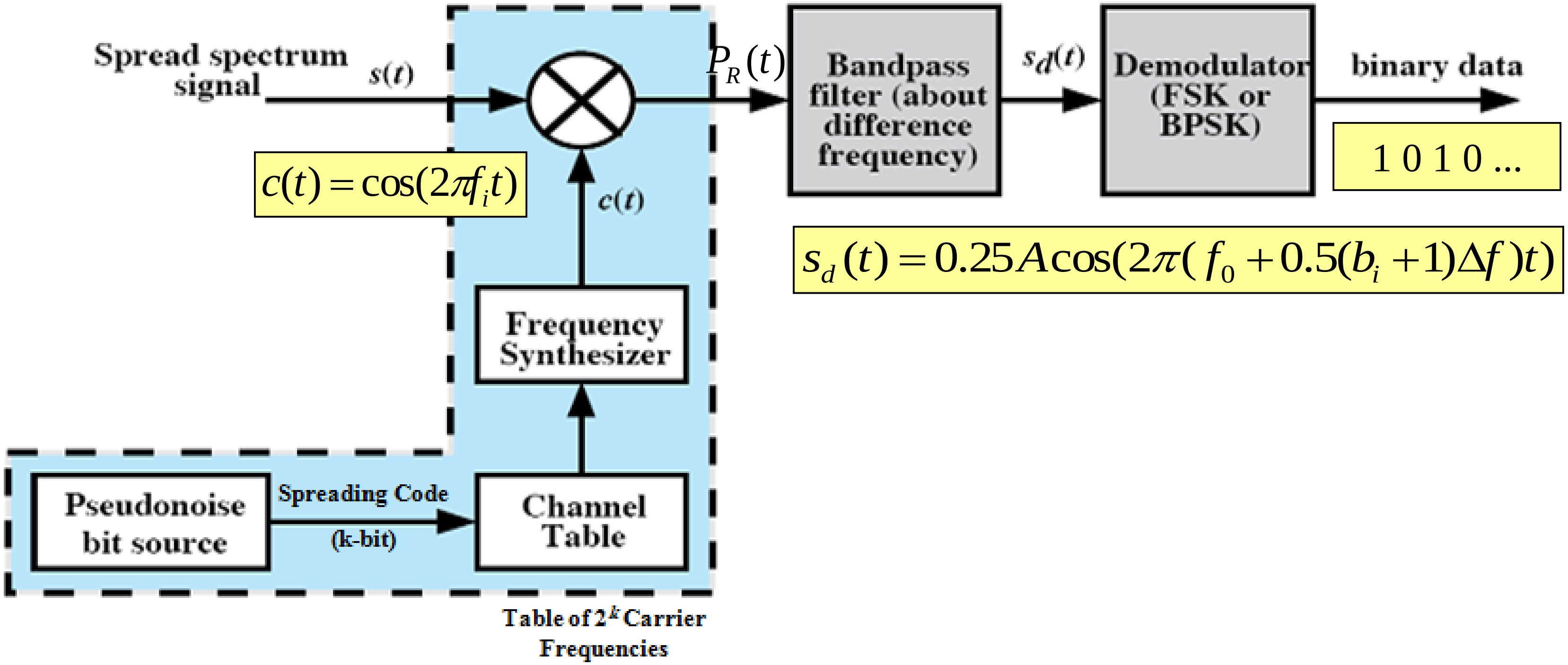


FHSS (Receiver)

$$p_R(t) = s(t)c(t) = 0.5A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f + f_i)t)\cos(2\pi f_i t)$$

$$p_R(t) = s(t)c(t) = 0.25A[\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f + f_i + f_i)t) + \cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f)t)]$$

$$s(t) = 0.5A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f + f_i)t)$$



FHSS (Transmitter)

- The BFSK input to FHSS system is:

$$\boxed{s_d(t) = A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f)t)} \qquad \text{for} \quad iT < t < (i+1)T$$

where,

A = amplitude of signal
f_0 = base frequency
b_i = value of the i^{th} bit of data (**+1 for binary 1 and -1 for binary 0**)
Δf = frequency separation
T = bit duration
$1/T$ = data rate

- During the i^{th} bit interval, the frequency of data signal is ***f_0* if the data bit is -1** and ***$f_0 + \Delta f$* if the data bit is +1**

FHSS (Transmitter)

- The transmitter product signal ($p_T(t)$) during i^{th} hop is:

$$p_T(t) = s_d(t)c(t) = A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f)t)\cos(2\pi f_i t)$$

where f_i is the frequency generated by the **frequency synthesizer** during the i^{th} hop.

- Using the trigonometric identity:

$$\cos(x)\cos(y) = \frac{1}{2}(\cos(x+y) + \cos(x-y))$$

- We have:

$$p_T(t) = 0.5A[\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f + f_i)t) + \cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f - f_i)t)]$$

- The **bandpass filter** is used to **block the differences frequency and pass the sum frequency**, yielding an FHSS signal:

$$s(t) = 0.5A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f + f_i)t)$$

- Thus, during the i^{th} bit interval, the frequency of data signal is **$f_0 + f_i$ if the data bit is -1 and $f_0 + f_i + \Delta f$ if the data bit is +1**

FHSS (Receiver)

⌘The receiver product signal ($p_R(t)$) during i^{th} hop is:

$$p_R(t) = s(t)c(t) = 0.5A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f + f_i)t)\cos(2\pi f_i t)$$

where f_i is the frequency generated by the **frequency synthesizer** during the i^{th} hop.

⌘Using the trigonometric identity:

$$\cos(x)\cos(y) = \frac{1}{2}(\cos(x+y) + \cos(x-y))$$

⌘ We have:

$$p_R(t) = s(t)c(t) = 0.25A[\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f + f_i + f_i)t) + \cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f)t)]$$

⌘ The **bandpass filter** is used to **block the sum frequency and pass the difference frequency**, yielding a signal of the form $s_d(t)$:

$$s_d(t) = 0.25A\cos(2\pi(f_0 + 0.5(b_i + 1)\Delta f)t)$$

Pseudorandom Numbers (PN)

- generated by **algorithm using initial seed** by a algorithm
  - Deterministic, not actually random
  - Same seed produces same number
  - However, if algorithm good, results pass reasonable tests of randomness
- starting from an initial seed
- **need to know algorithm and seed to predict sequence**
- hence only receiver can decode signal

FHSS Using MFSK

- commonly use **multiple FSK (MFSK)**
- **have frequency shifted every T_c seconds**
- **for data rate R**
  - **bit duration $T_b = 1/R$ sec**
  - **signal element duration $T_s = mT_b$**
- if T_c is greater than or equal to T_s, the spreading modulation is referred to as **slow-frequency-hop spread spectrum**; otherwise it is known as ***fast-frequency-hop spread spectrum***

Slow-frequency-hop spread spectrum	$T_c \geq T_s$
Fast-frequency-hop spread spectrum	$T_c < T_s$

- FHSS quite resistant to noise or jamming
  - with fast FHSS giving better performance

FHSS Using MFSK

- MFSK commonly used with FHSS
- For **one signal element MFSK**

$$s(t) = A\cos(2\pi f_i t), \qquad 1 \le i \le M$$

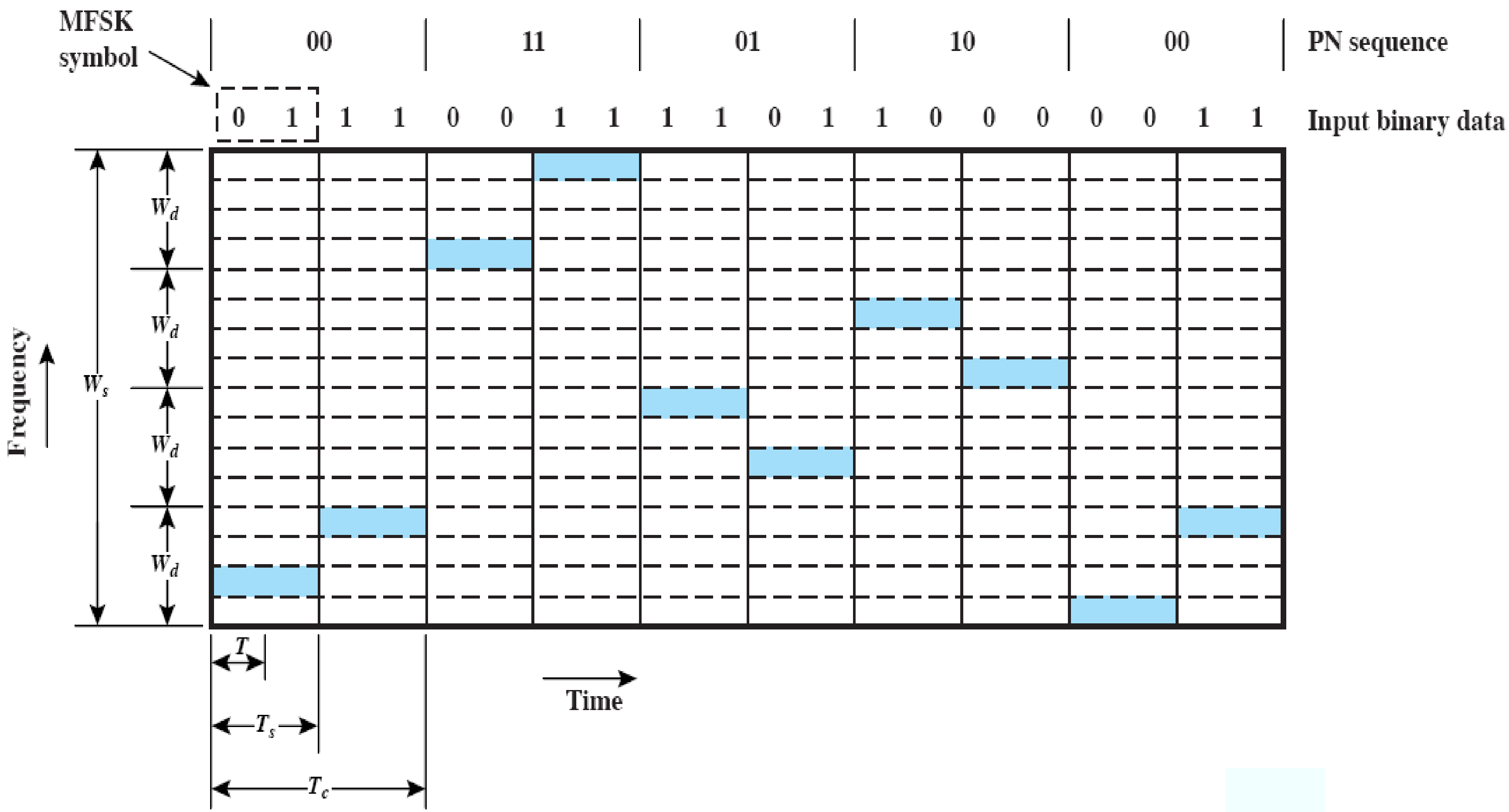
$$f_i = f_c + (2i - 1 - M) f_d$$

  - f_c = carrier frequency
  - f_d = difference frequency
  - M = number of different signal elements = 2^m
  - m = number of bits per signal element

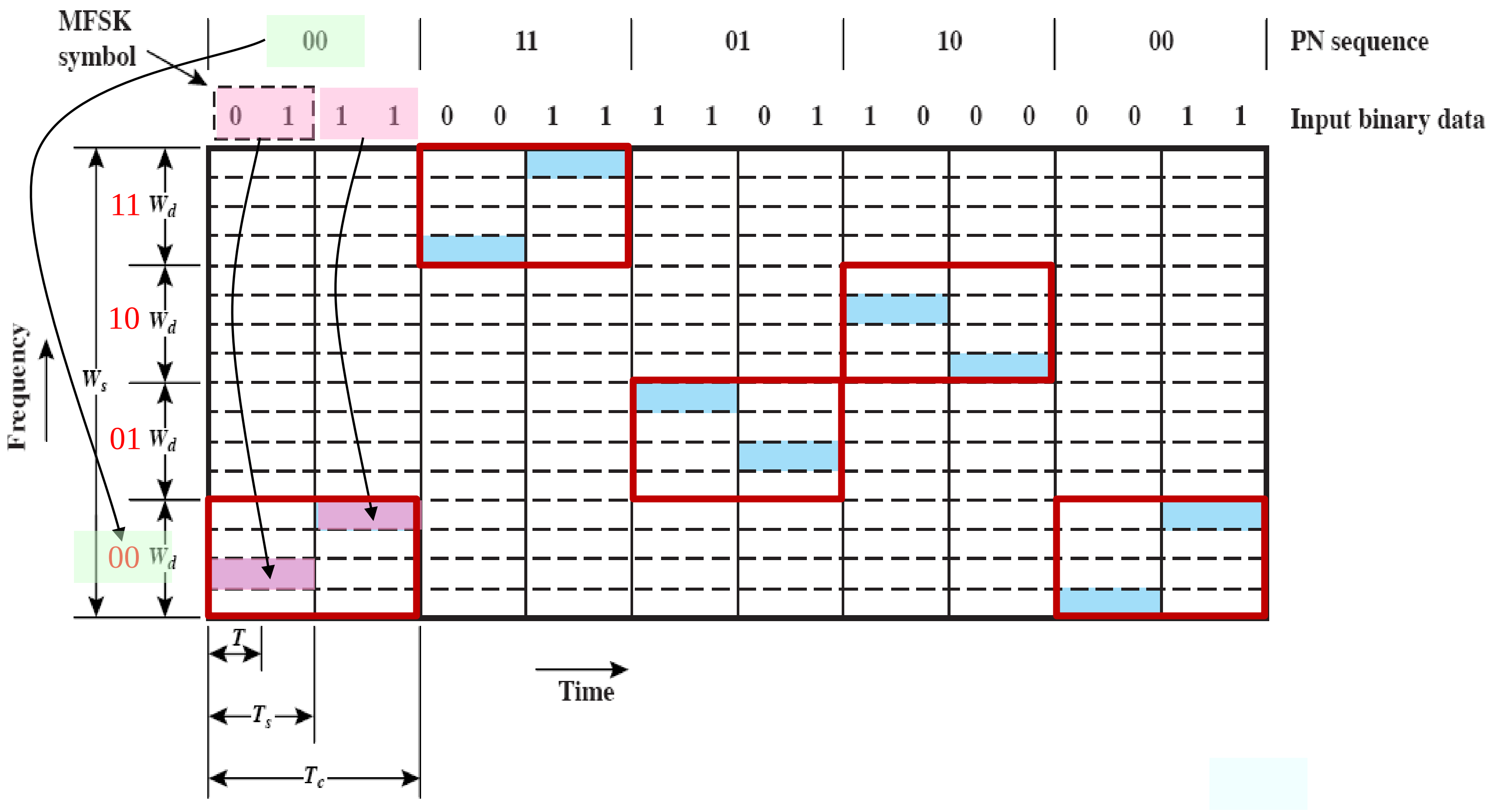
FHSS Using MFSK - Example

- $M = 4$ frequencies encode 2 bits at a time
- **MFSK bandwidth** $W_d = 2M f_d$
- Using **FHSS** with $k = 2$, $2^k = 4$ **channels**
- Each channel with bandwidth W_d
- Total **bandwidth for FHSS**: $W_s = 2^k W_d$
- Slow FHSS: $T_c = 2T_s = 4T_b$
  - Each 2 bits of the PN sequence is used to select one of the four channels
  - channel held for duration of two signal elements, or four bits
- Fast FHSS: $T_s = 2T_c = 2T_b$
  - signal element represented in two channels

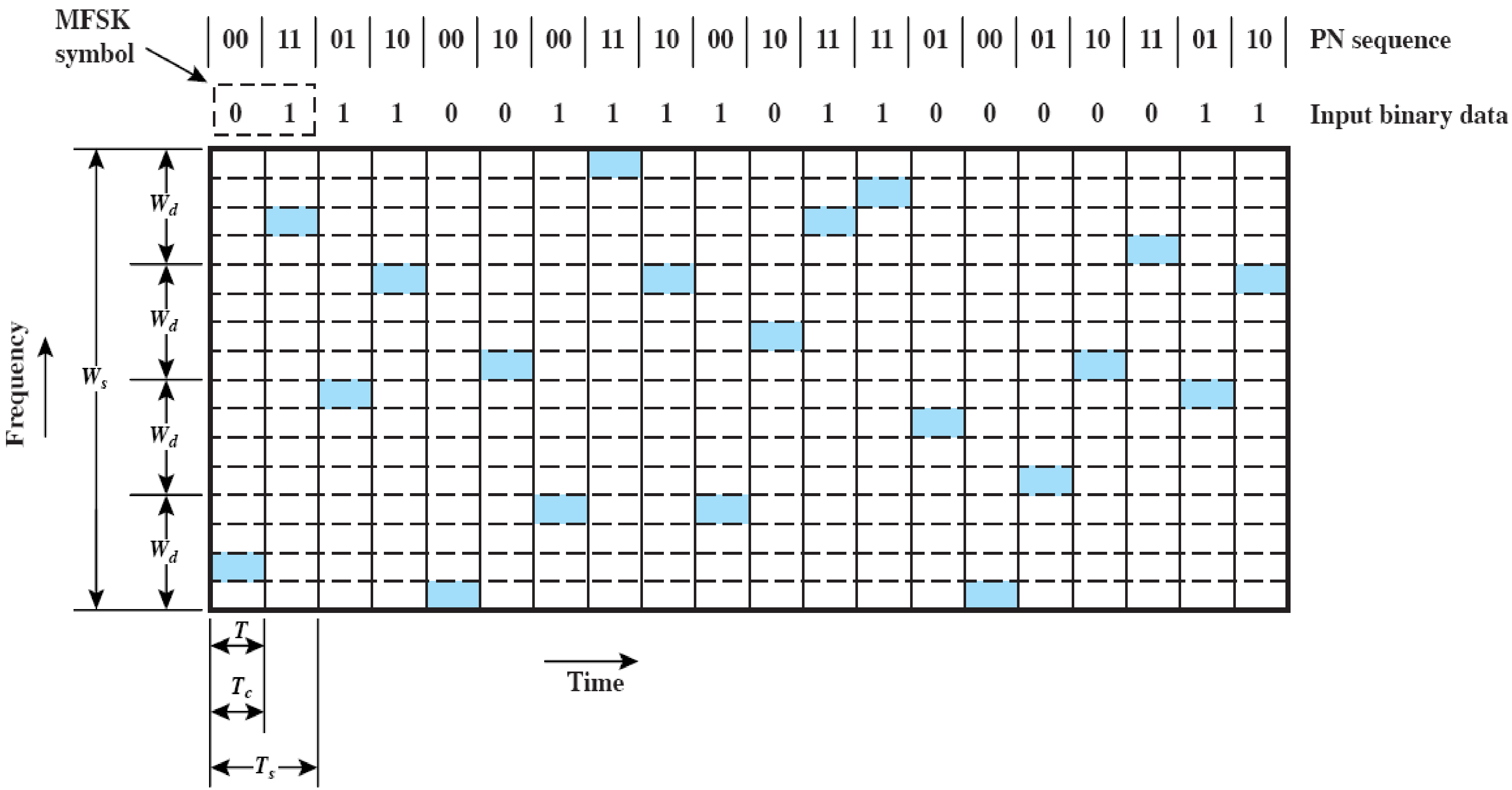
Slow MFSK FHSS



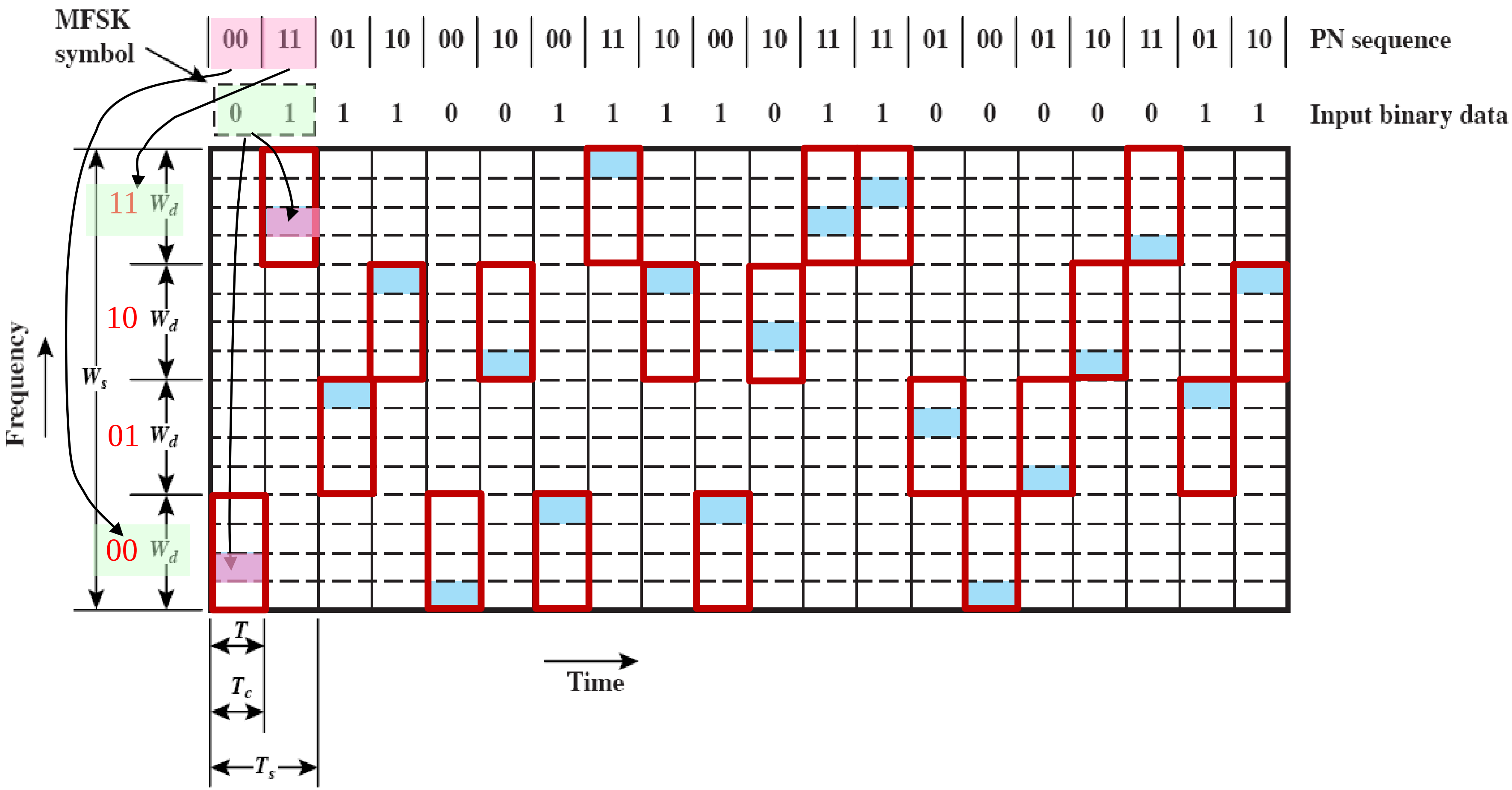
Slow MFSK FHSS



Fast MFSK FHSS



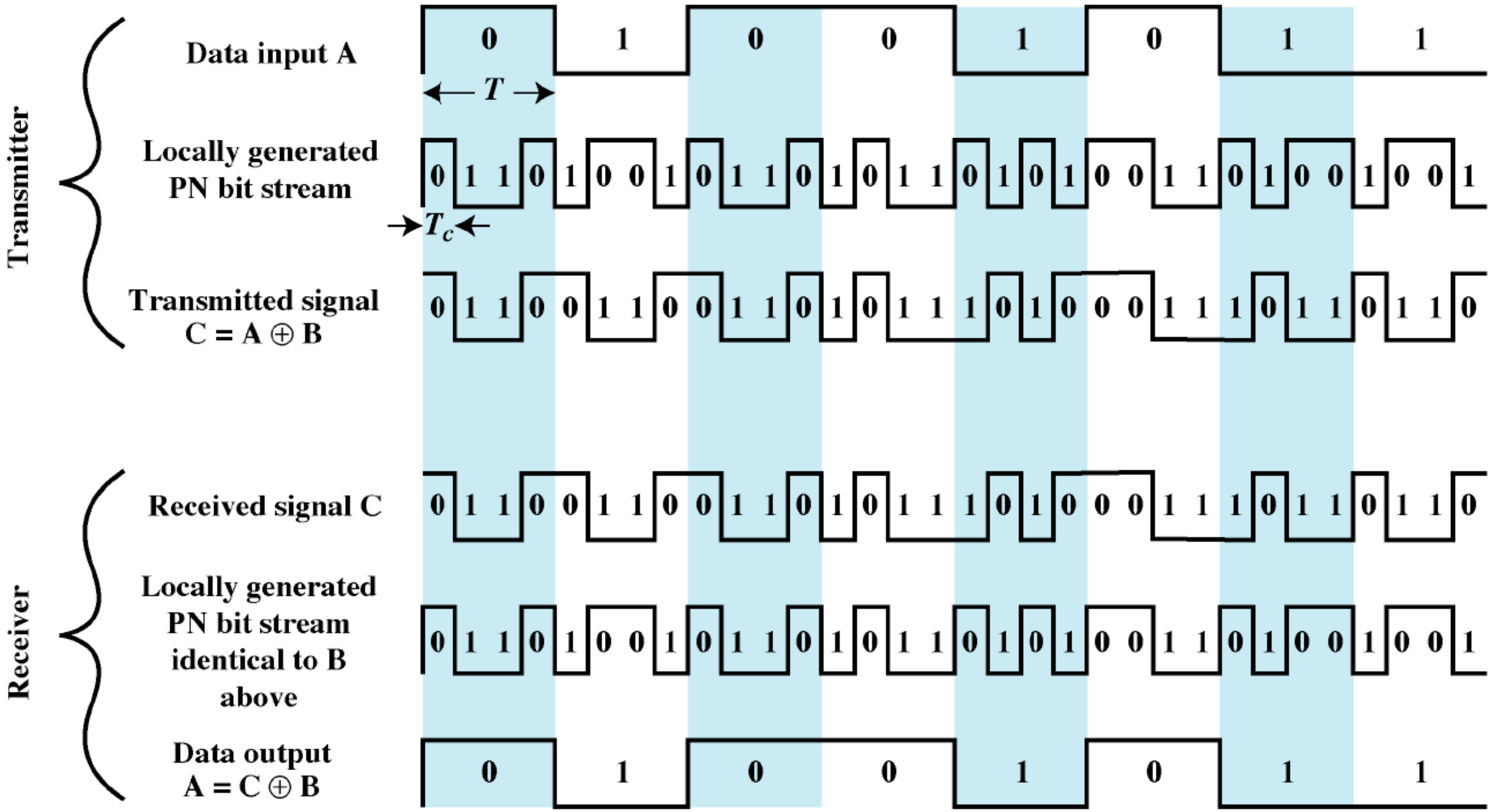
Fast MFSK FHSS



Direct Sequence Spread Spectrum (DSSS)

- **each bit is represented by multiple bits** using a spreading code
- this spreads signal across a wider frequency band
- frequency band of signal is proportional to number of bits
  - 10-bit spreading code → spreads the signal across the frequency band 10 times greater than a 1-bit spreading code
- Input is combined with spread code using **XOR**
  - **input 0: spreading code unchanged**
  - **input 1: spreading code inverted**

Direct Sequence Spread Spectrum Example



Direct Sequence Spread Spectrum (DSSS)

⌘The BPSK signal is represented as:

$$\boxed{s_d(t) = Ad(t)\cos(2\pi f_c t)}$$

where,

A = amplitude of signal

f_c = carrier frequency

$d(t)$ = discrete function that takes on the value of +1 for one bit time if the corresponding bit in the bit stream is 1 and the value -1 for one bit if the corresponding bit in the bit stream is 0

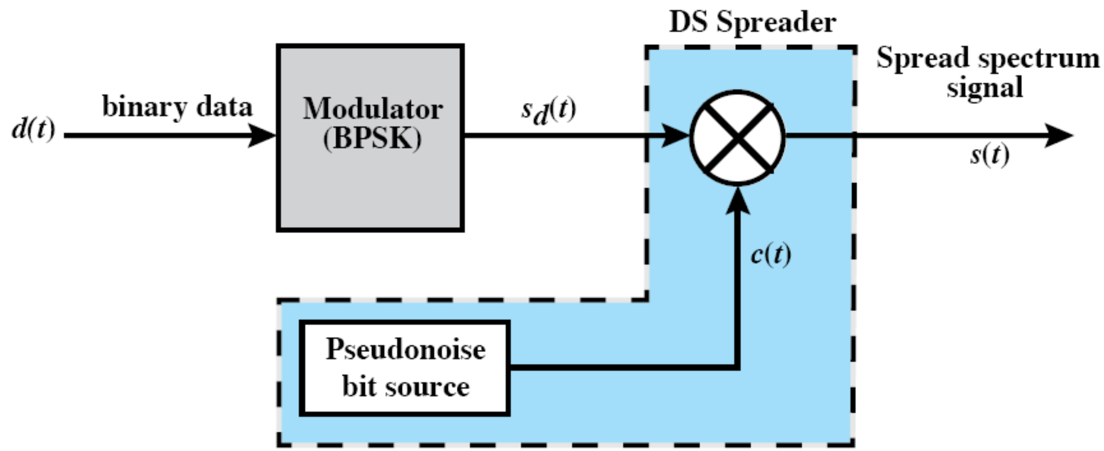
⌘ To produce the DSSS signal, we multiply $d(t)$ by $c(t)$, which is the PN sequence taking on values of +1 and -1:

$$\boxed{s(t) = s_d(t)c(t) = Ad(t)c(t)\cos(2\pi f_c t)}$$

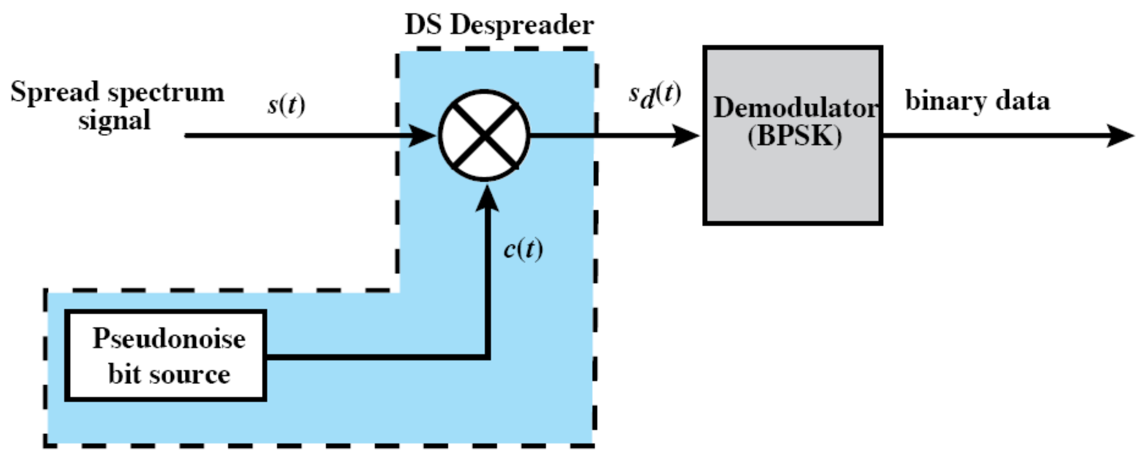
⌘ At the receive, the incoming signal is multiplied again by $c(t)$, but $c(t)\times c(t)$=1 and therefore, the original signal is recovered:

$$\boxed{s(t)c(t) = Ad(t)c(t)c(t)\cos(2\pi f_c t) = s_d(t)}$$

Direct Sequence Spread Spectrum System

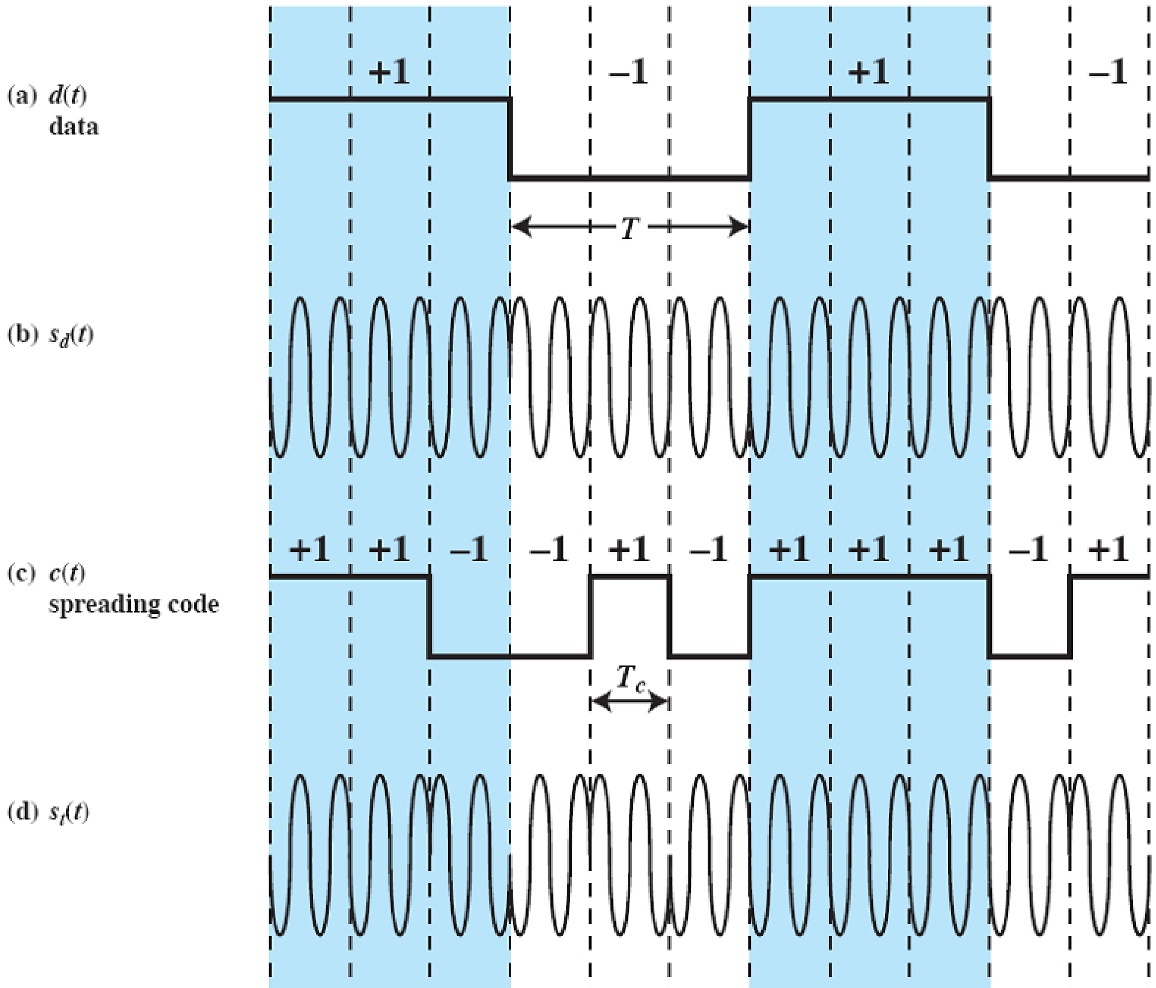


(a) Transmitter



(b) Receiver

DSSS Example Using BPSK



Code Division Multiple Access (CDMA)

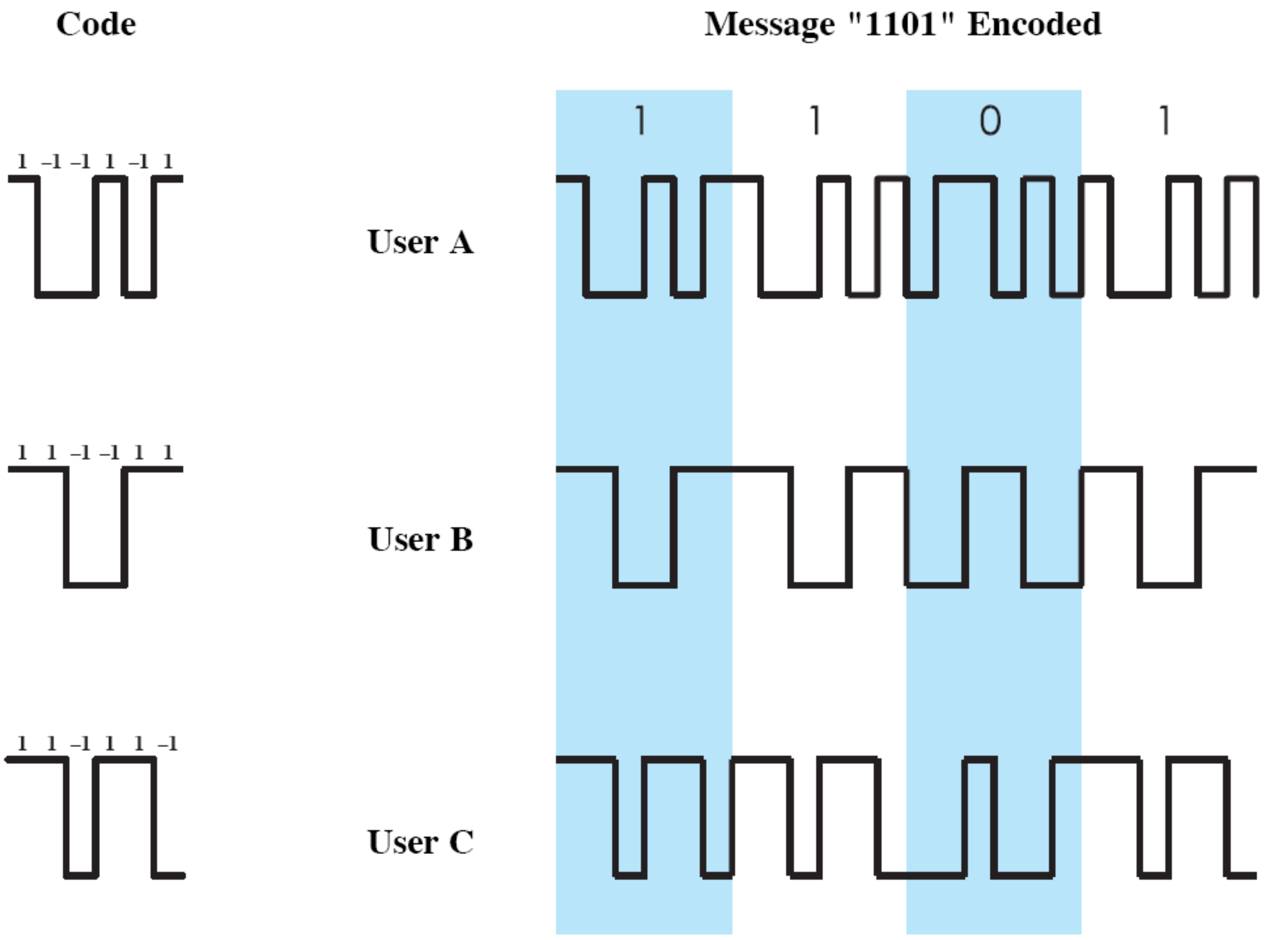
- a multiplexing technique used with spread spectrum
- given a **data signal rate *D***
- **break each bit into *k* chips** according to a fixed chipping code specific to each user
  - Pattern unique for each user (user code)
- resulting new channel has **chip data rate *kD* chips per second**

CDMA – Example

- User A code c_A = <1, -1, -1, 1, -1, 1>
- User B code c_B = <1, 1, -1, -1, 1, 1>
- User C code c_C = <1, 1, -1, 1, 1, -1>

- If A wants to send bit 1:
  - transmit chip code <1, -1, -1, 1, -1, 1>
- If A wants to send bit 0:
  - transmit chip code <-1, 1, 1, -1, 1, -1>
  - i.e. 1's complement (1, -1 inverted)

CDMA - Example



CDMA – Example

- If a receiver R receives a chip pattern $d=<d_1,d_2,d_3,d_4,d_5,d_6>$ and the receiver is seeking to communicate with a user u so that it has at hand u's code $<c_1,c_2,c_3,c_4,c_5,c_6>$, the receiver performs the following decoding function:

$$S_u(d) = d_1 \times c_1 + d_2 \times c_2 + d_3 \times c_3 + d_4 \times c_4 + d_5 \times c_5 + d_6 \times c_6$$

- If u is actually user A, then
  - If A sends 1:
    - $d = <1, -1, -1, 1, -1, 1>$
    - $S_A = 1\times1+(-1\times-1)+(-1\times-1)+1\times1+(-1\times-1)+1\times1= 6$
  - If A sends 0:
    - $d = <-1, 1, 1, -1, 1, -1>$
    - $S_A = -1\times1+1\times-1+-1\times1+1\times-1+1\times-1+-1\times1= -6$

CDMA – Example

- If user B send 1, receiver using S_A
  - $d = <1, 1, -1, -1, 1, 1>$
  - $c_A = <1, -1, -1, 1, -1, 1>$
  - $S_A(d) = S_A(1, 1, -1, -1, 1, 1)$
    $= 1 \times 1 + 1 \times -1 + -1 \times -1 + -1 \times 1 + 1 \times -1 + 1 \times 1 = 0$
- Same result if B sends 0

Orthogonal Codes

- If A, B transmit same time, S_A is used
  - only A signal is received, B is ignored
- If A, B transmit same time, S_B is used
  - only B signal is received, A is ignored
- $S_A(c_B) = S_B(c_A) = 0$
- Codes of A, B are called orthogonal

Orthogonal Codes

- Orthogonal codes are not always available
- More commonly, $S_X(c_Y)$ is small if $X \neq Y$
- Thus, can distinguish when $X = Y$, $X \neq Y$
- In the previous example
  - $S_A(c_C) = S_C(C_A) = 0$
  - $S_B(c_C) = S_C(c_B) = 2$
  - signal makes small contribution instead of 0
- Receiver can identify signal of user even if other users transmitting at same time

CDMA – Example

(a) User's codes

User A	1	–1	–1	1	–1	1
User B	1	1	–1	–1	1	1
User C	1	1	–1	1	1	–1

(b) Transmission from A

Transmit (data bit = 1)	1	–1	–1	1	–1	1	
Receiver codeword	1	–1	–1	1	–1	1	
Multiplication	1	1	1	1	1	1	= 6

Transmit (data bit = 0)	–1	1	1	–1	1	–1	
Receiver codeword	1	–1	–1	1	–1	1	
Multiplication	–1	–1	–1	–1	–1	–1	= –6

CDMA – Example

(c) Transmission from B, receiver attempts to recover A's transmission

Transmit (data bit = 1)	1	1	–1	–1	1	1	
Receiver codeword	1	–1	–1	1	–1	1	
Multiplication	1	–1	1	–1	–1	1	= 0

(d) Transmission from C, receiver attempts to recover B's transmission

Transmit (data bit = 1)	1	1	–1	1	1	–1	
Receiver codeword	1	1	–1	–1	1	1	
Multiplication	1	1	1	–1	1	–1	= 2

(e) Transmission from B and C, receiver attempts to recover B's transmission

B (data bit = 1)	1	1	–1	–1	1	1	
C (data bit = 1)	1	1	–1	1	1	–1	
Combined signal	2	2	–2	0	2	0	
Receiver codeword	1	1	–1	–1	1	1	
Multiplication	2	2	2	0	2	0	= 8

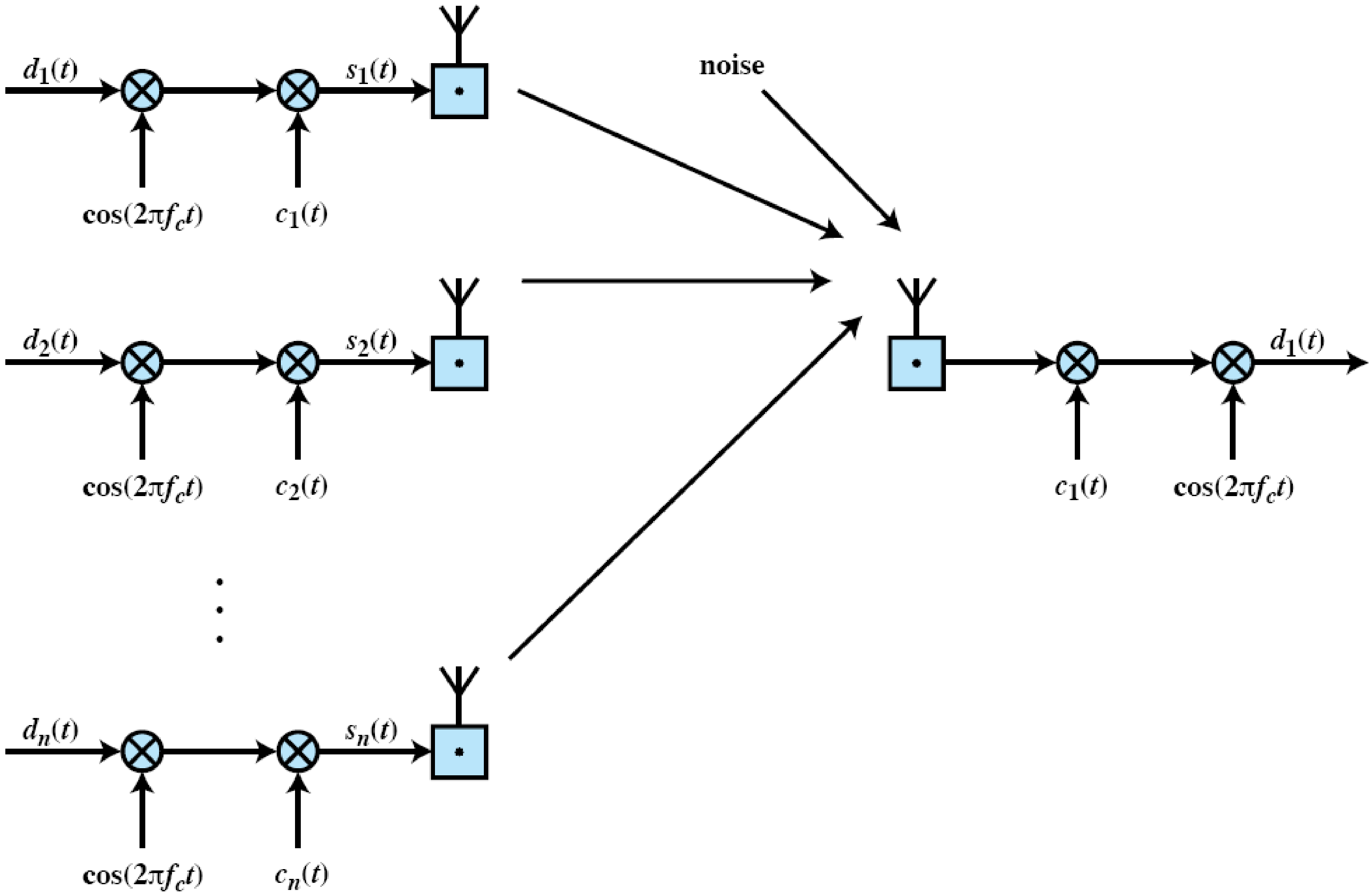
CDMA Limitations

- Receiver can filter unwanted users
  - either 0 or low-level noise
- However, system will break down if
  - many users compete for channel
  - signal power from some users is too high because some users are very near to receiver

CDMA for DSSS

- There are *n* users, each transmitting using different PN sequence
- For each user, data stream $d_i(t)$ is BPSK modulated to produce signal with bandwidth W_d and then multiplied by spreading code for that user $c_i(t)$
- All of the signals, plus noise, are received at the receiver's antenna
- Suppose that the receiver is attempting to recover the data of user 1. The incoming signal is multiplied by the spreading code of user 1 ($c_1(t)$) and then demodulated.
- → Narrow the bandwidth of that portion of the incoming signal corresponding to user 1 to the original bandwidth of the unspread signal
- Incoming signals from other users are not despread by the spreading code from user 1 and hence retain their bandwidth of Ws
- Unwanted signal energy remains spread over a large bandwidth and the wanted signal is concentrated in a narrow bandwidth
- Bandpass filter at the demodulator can therefore recover the desired signal

CDMA for DSSS



Summary

- looked at use of spread spectrum techniques:
- FHSS
- DSSS
- CDMA