Signals & Systems (CNET - 221)

Signals

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Text Book & Online Reference

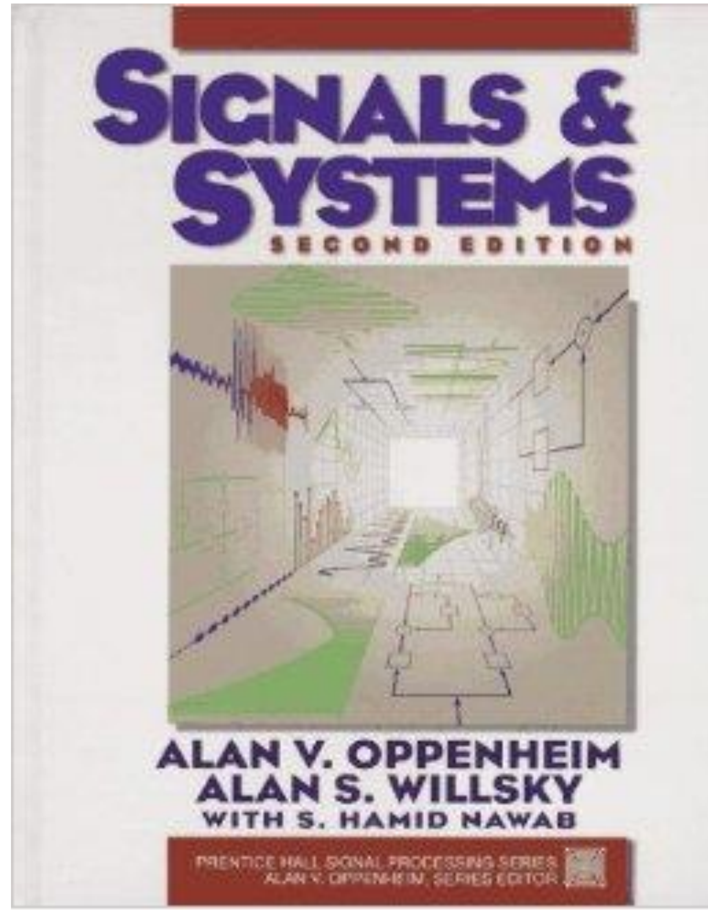
- Textbook:

A.V. Oppenheim and R.W. Schafer, Digital Signal Processing, Prentice-hall

- E-Learning:

http://nptel.ac.in/courses/index.php?subjectId=117104074

- General Reference: http://web.cecs.pdx.edu/~ece2xx/ECE222/Slides/



Outline of Chapter-1

Signals

1.0 Introduction

1.1 Continuous-Time & Discrete-Time Signals

- **1.1.l Examples and Mathematical Representation**
- **1.1.2 Signals Energy and Power**

1.2 Transformations of the Independent Variable

- **1.2.1 Examples of Transformations of the independent variable**
- **1.2.2 Periodic Signals**
- **1.2.3 Even and Odd signals**

1.3 Exponential and Sinusoidal Signals

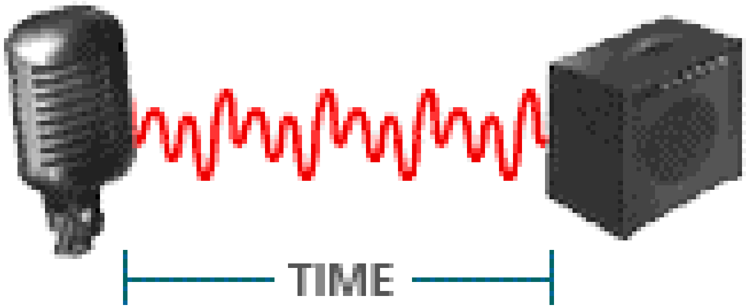
- **1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals**
- **1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals**

1.4 The Unit Impulse and Unit Step Functions

Introduction to Signals

Definition:

SIGNAL is a function or set of information representing a physical quantity that varies with time or any other independent variable.

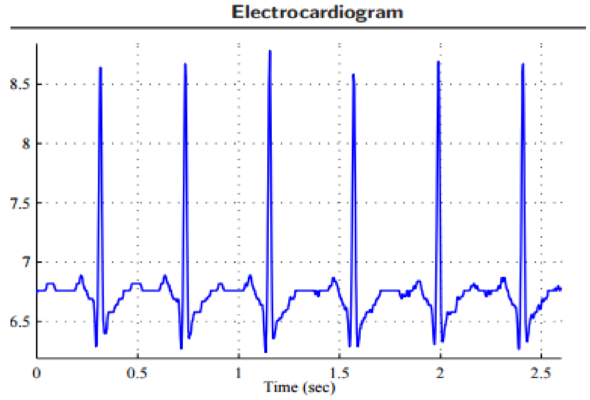


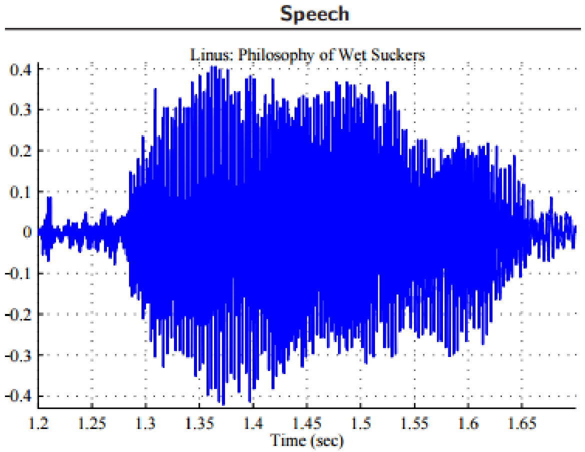
Examples:

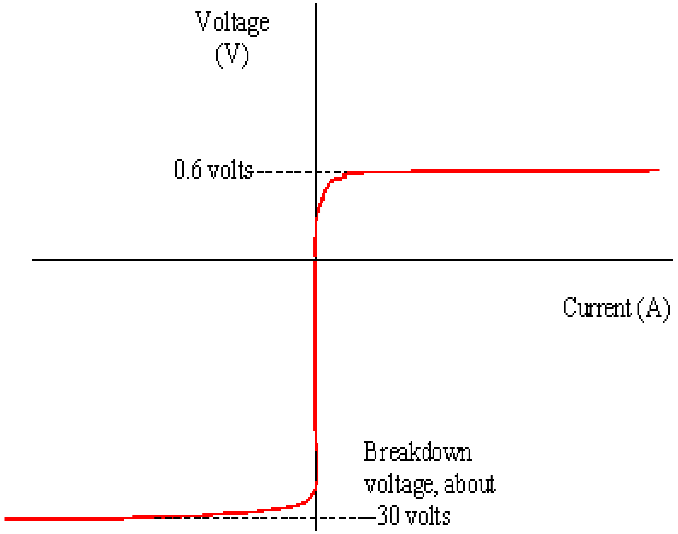
- Voltage/Current in a Circuit
- Speech/Music
- Any company's industrial average
- Force exerted on a shock absorber
- $A*\sin(\omega t+\phi)$

Introduction to Signals(Continued…..)

Example of various signals(Graphs)

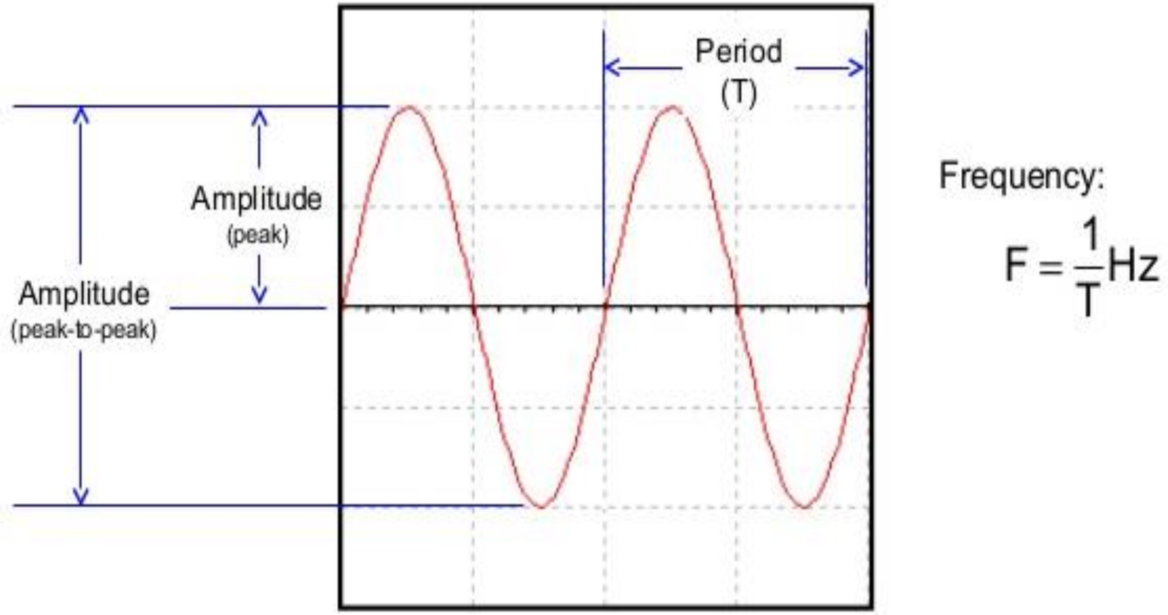






Introduction to Signals(Continued…..)

Measuring Signals



Classifications of Signals

Signals are classified into the following categories:

1. Continuous-Time Signals & Discrete-Time Signals
2. Even and Odd Signals
3. Periodic and Non-Periodic (aperiodic) Signals
4. Energy and Power Signals

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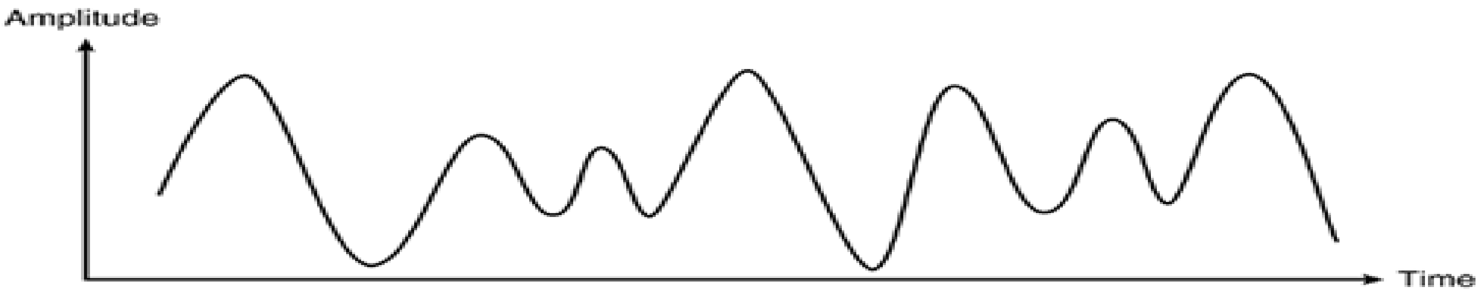
1. Continuous Time and Discrete Time Signals

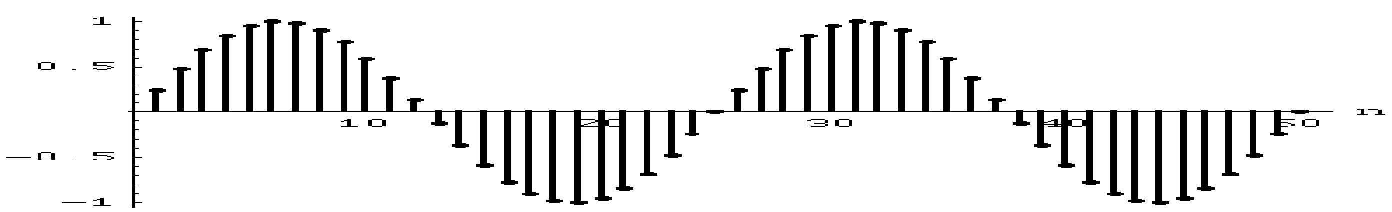
A signal is said to be **<u>Continuous</u>** when it is defined for all instants of time.

Example: Video, Audio



A signal is said to be **<u>discrete</u>** when it is defined at only discrete instants of time

Example: Stock Market, Temperature



Examples: CT vs. DT Signals

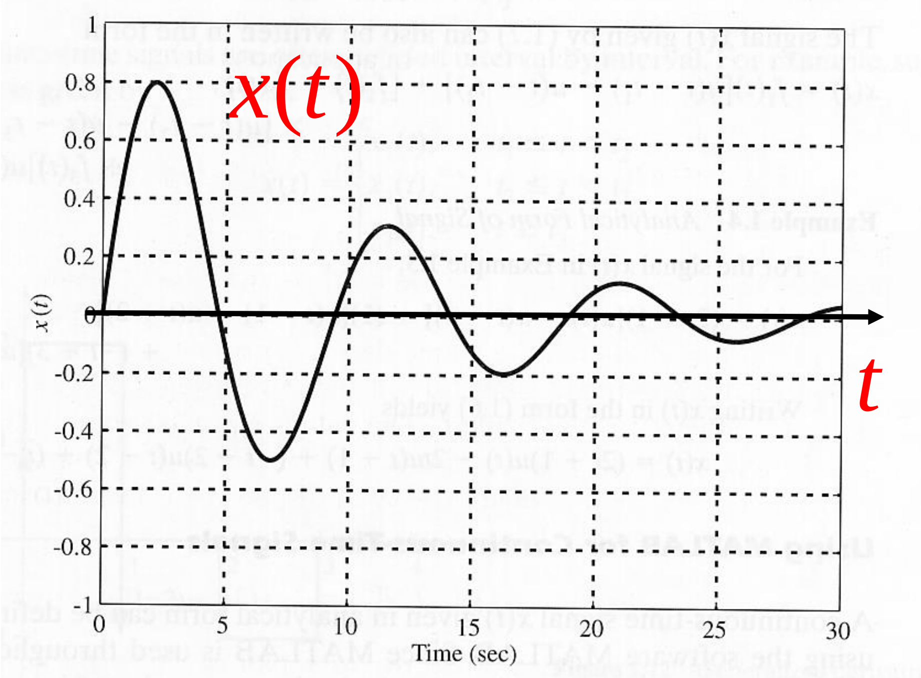


Figure 1.13 MATLAB plot of the signal $x(t) = e^{-0.1t} \sin \frac{2}{3} t$.

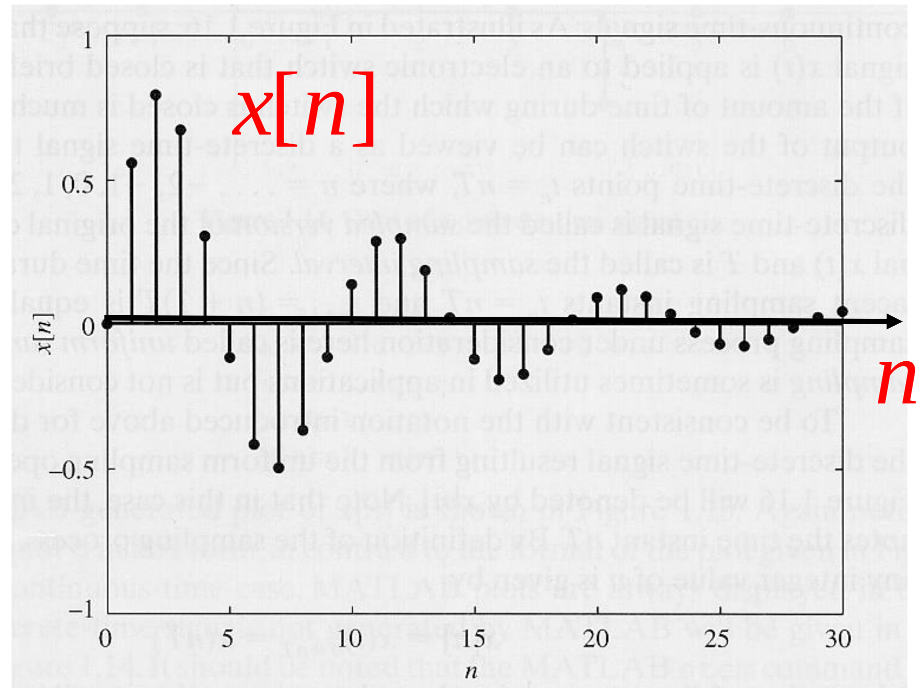
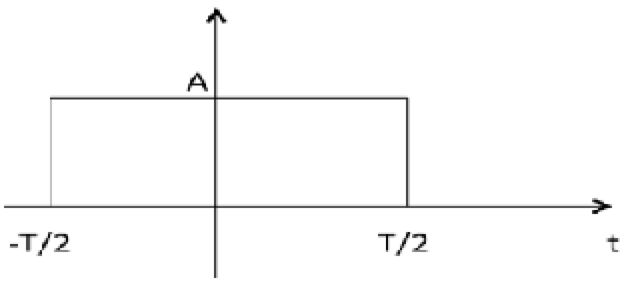


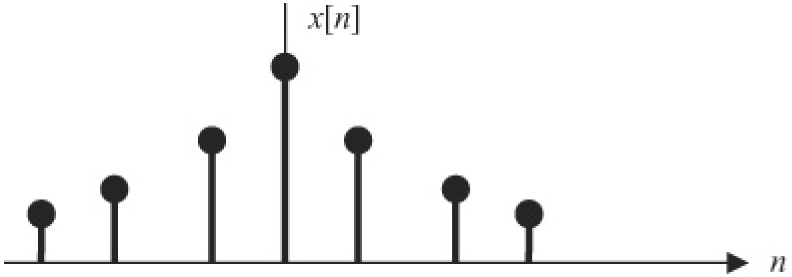
Figure 1.17 Sampled continuous-time signal.

2. Even and Odd Signals

A signal is said to be even when it satisfies the condition

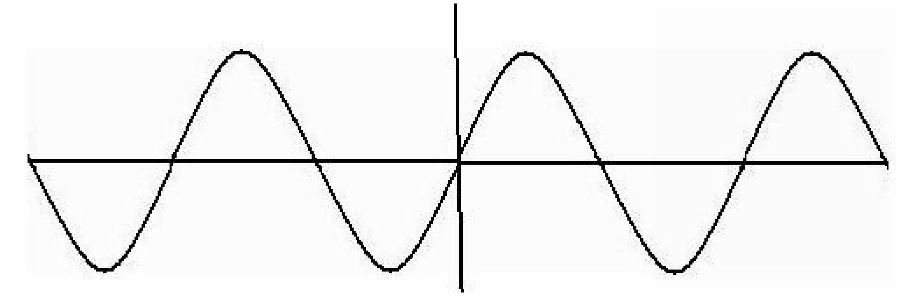
X(t) = X(-t) ; X[n] = X[-n]

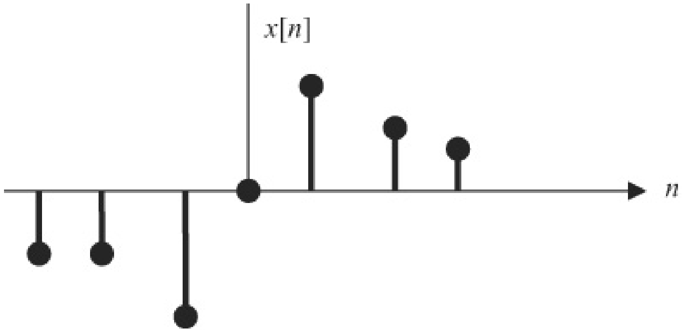




A signal is said to be odd when it satisfies the condition

X(-t) = -X(t) ; X[-n] = -X[n]



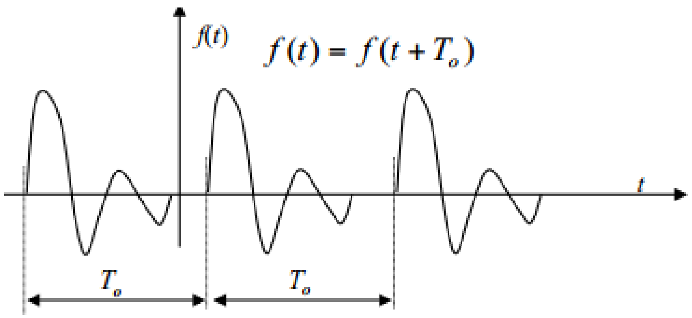


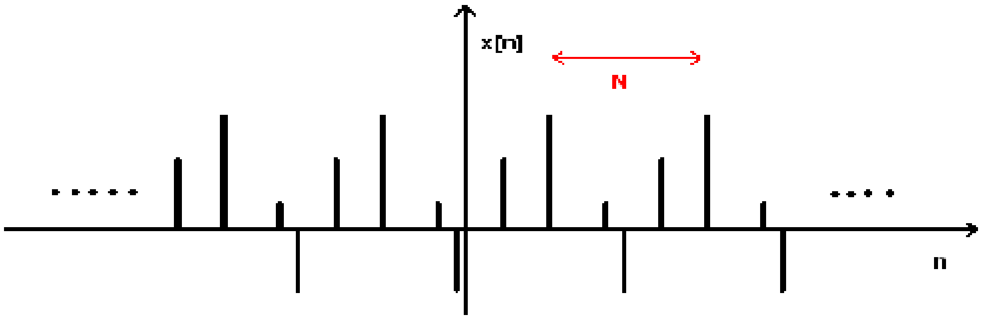
3. Periodic and Non-periodic (Aperiodic) Signals

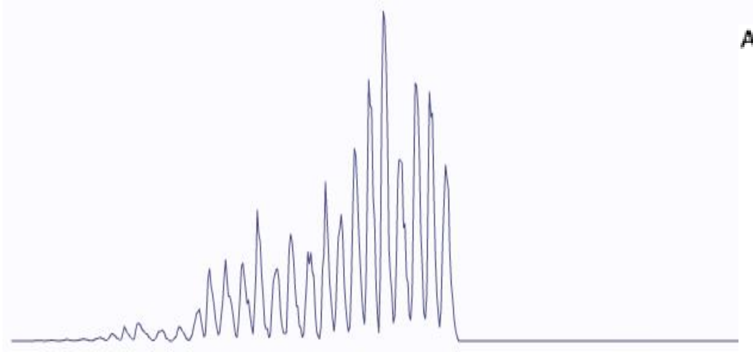
A signal is said to be periodic if it satisfies the condition

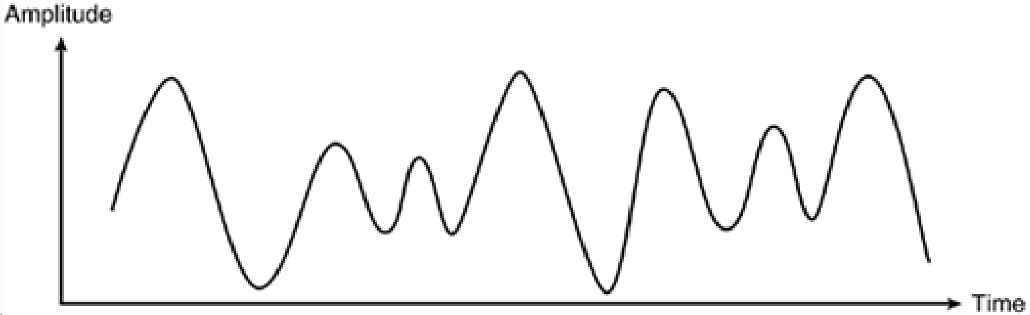
$x(t) = x(t + T)$ or $x(n) = x(n + N)$ for all t or n

Where T = fundamental time period, $1/T = f$ = fundamental frequency.









4. Energy and Power Signals

A signal is said to be energy signal when it has finite energy.

$$\text{Energy } E = \int_{-\infty}^{\infty} x^2(t)\,dt \qquad E = \sum_{n=-\infty}^{\infty} \{x[n]\}^2$$

A signal is said to be power signal when it has finite power.

$$\text{Power } P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt \qquad P = \lim_{N\to\infty} \left(1/(2N+1)\right) \sum_{n=-N}^{N} \{x[n]\}^2$$

NOTE:

A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal. Power of energy signal = 0 & Energy of power signal = ∞

:

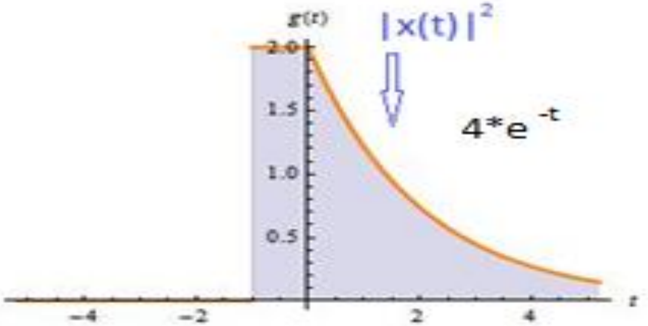
Energy and Power Signals

Example 1

- Determine the energy of the given signal as in figure

Solution:

- $E = x^2\,dt$
- $E = x^2\,dt + e^{-t}dt$
- $E = 4[t] + 4[-e^{-t}] +$
- $E = 4[0-(-1)] + 4[-(e^{\infty} - e^{0})]$
- $E = 4+4$
- E= 8 joules

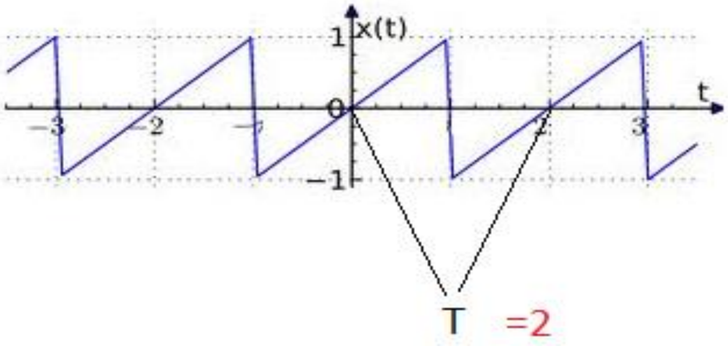


Example 2

- Determine the power of the given signal as in figure

Solution:

- $P = x^2\,dt$
- $P = x^2\,dt$
- $P = 1/2\,[t^{2+1}/2+1]$
- $P = 1/2\,[t^3/3]$
- $P = ((1/2) * 3)\,[1^3 - (-1)^3] = 1/3$ watts



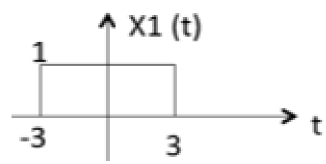
Basic Mathematical Operations on Signals

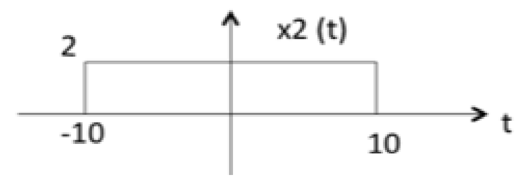
1. Addition
2. Subtraction
3. Multiplication

1. Addition

Example:- For the given signals $x_1(t)$ and $x_2(t)$ plot the Signal

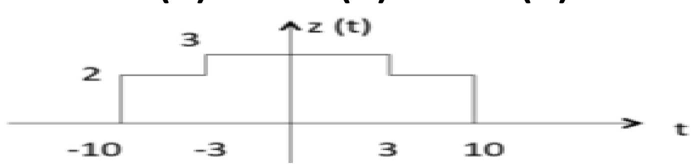
$$z_1(t) = x_1(t) + x_2(t)$$





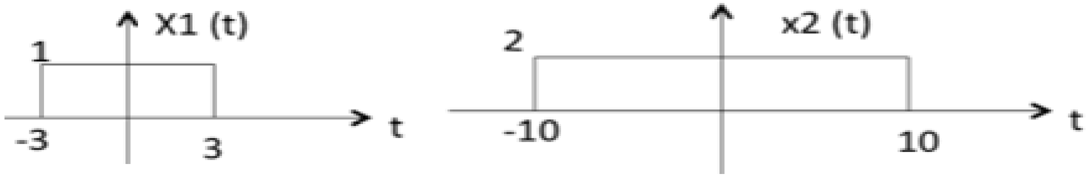
Answer: As seen from the diagram above,

- $-10 < t < -3$ amplitude of $z(t) = x_1(t) + x_2(t) = 0 + 2 = 2$
- $-3 < t < 3$ amplitude of $z(t) = x_1(t) + x_2(t) = 1 + 2 = 3$
- $3 < t < 10$ amplitude of $z(t) = x1(t) + x2(t) = 0 + 2 = 2$



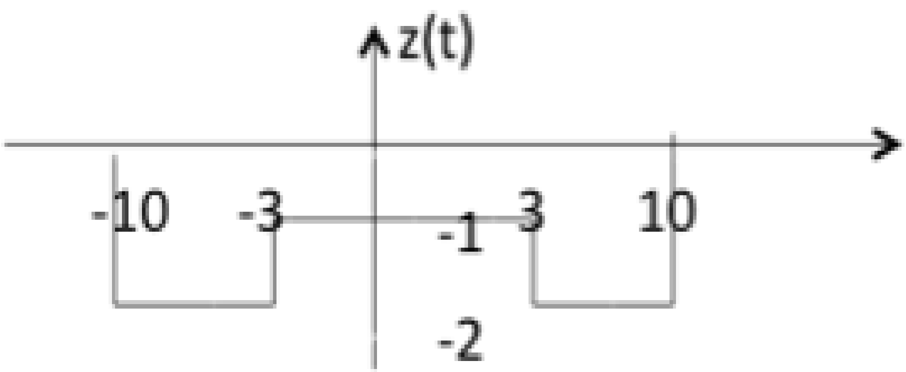
2. Subtraction

Example-2: For the given signals x**1**(t) and x**2**(t) plot the Signal z**1**(t) = x**1**(t) - x**2**(t)



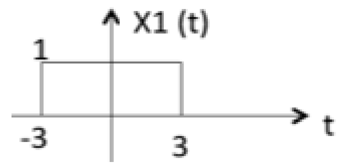
Answer: As seen from the diagram above,

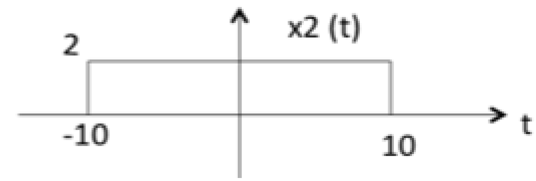
- -10 < t < -3 amplitude of z (t) = x1(t) - x2(t) = 0 - 2 = -2
- -3 < t < 3 amplitude of z (t) = x1(t) - x2(t) = 1 - 2 = -1
- 3 < t < 10 amplitude of z (t) = x1(t) + x2(t) = 0 - 2 = -2



3. Multiplication

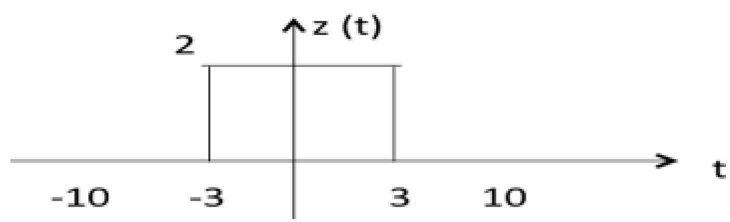
Example-3: For the given signals x**1**(t) and x**2**(t) plot the Signal z**1**(t) = x**1**(t) * x**2**(t)





Answer:

- As seen from the diagram above,
- -10 < t < -3 amplitude of z (t) = x1(t) ×x2(t) = 0 ×2 = 0
- -3 < t < 3 amplitude of z (t) = x1(t) ×x2(t) = 1 ×2 = 2
- 3 < t < 10 amplitude of z (t) = x1(t) × x2(t) = 0 × 2 = 0



Transformations of the Independent Variable

1.Time Shifting

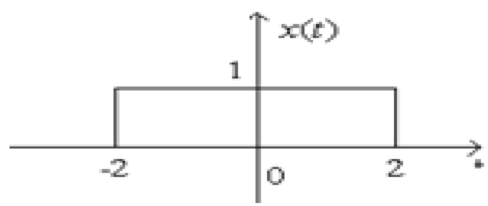
2. Time Scaling

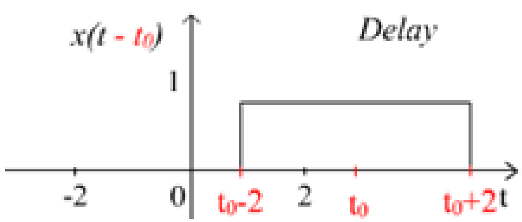
3. Time Reversal

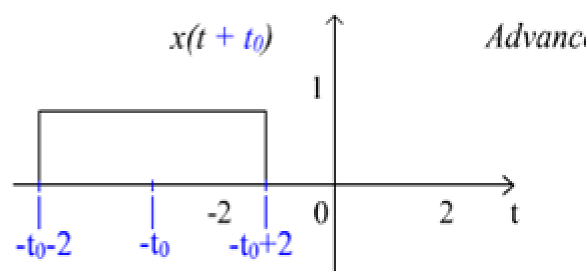
1. Time Shifting:

$x(t \pm t_0)$ is time shifted version of the signal $x(t)$.

- $x(t + t_0) \longrightarrow\longrightarrow$ negative shift
- $x(t - t_0) \longrightarrow\longrightarrow$ positive shift





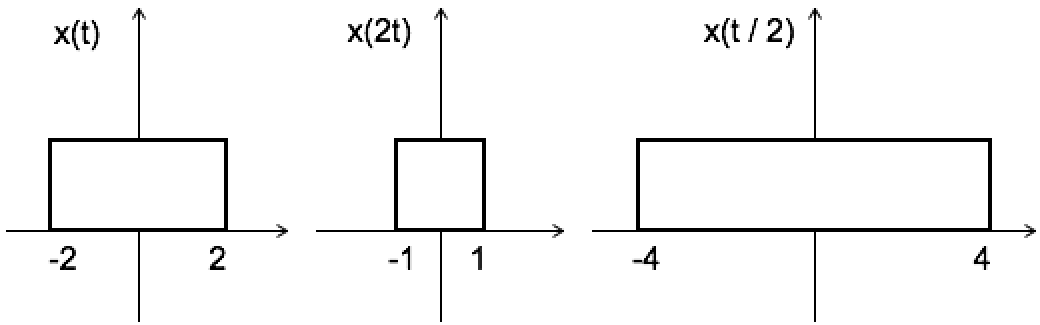


2. Time Scaling:

x(At) is time scaled version of the signal x(t). where A is always positive.

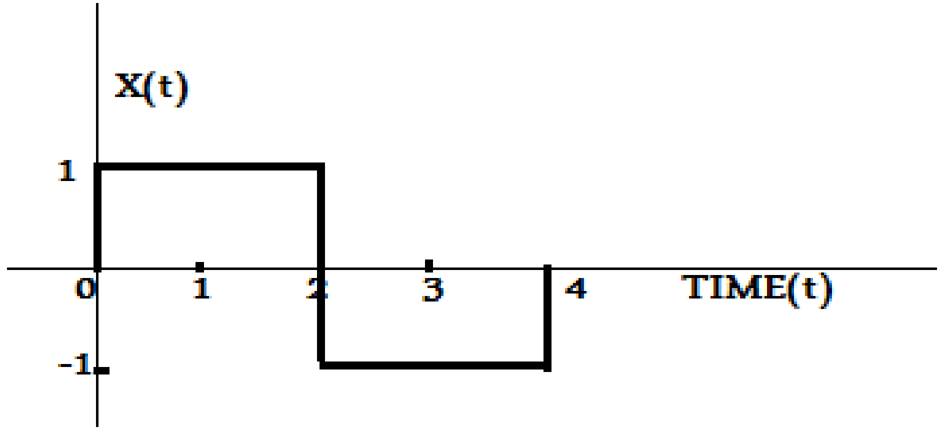
$|A| > 1 \longrightarrow\longrightarrow$ Compression of the signal

$|A| < 1 \longrightarrow\longrightarrow$ Expansion of the signal

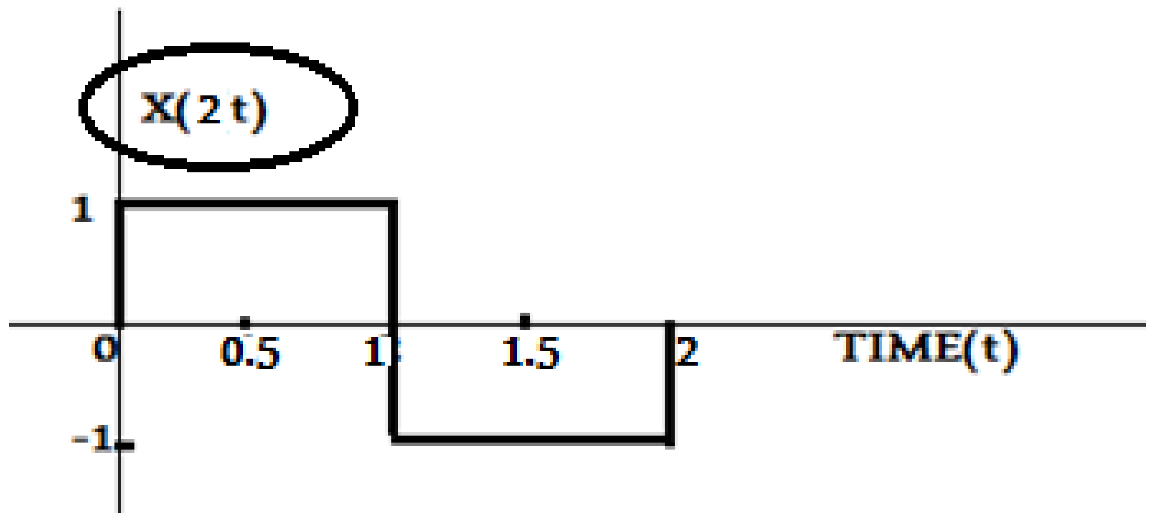


Note: u(at) = u(t) time scaling is not applicable for unit step function.

Example:- For the given signal X(t) shown in figure below, sketch the Time scaled signal X(2t).



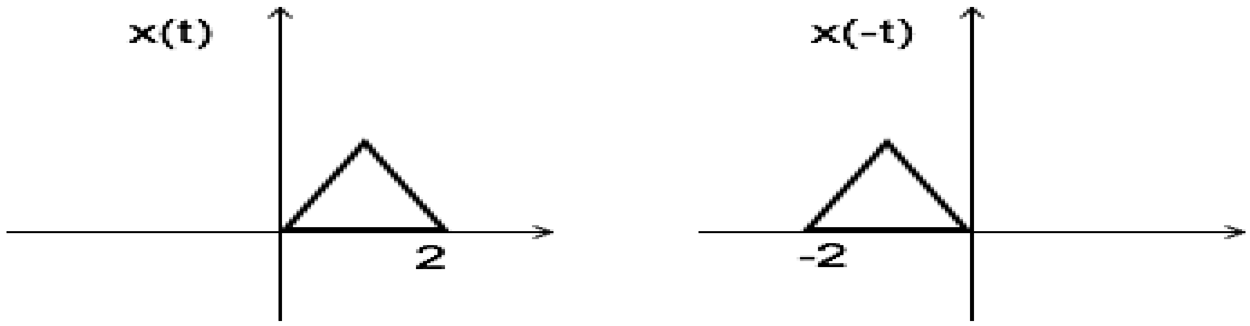
Solution:



3. Time Reversal:

The Time reversal of a signal x(t) can be obtained by folding the signal about t=0, its denoted by x(-t).

Example:-



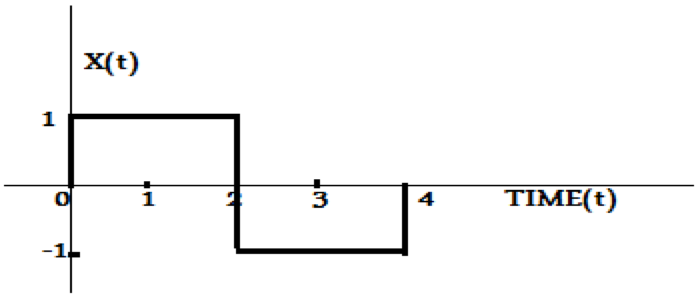
Example:- For the given signal X(t) shown in figure below, sketch each of the following signals.

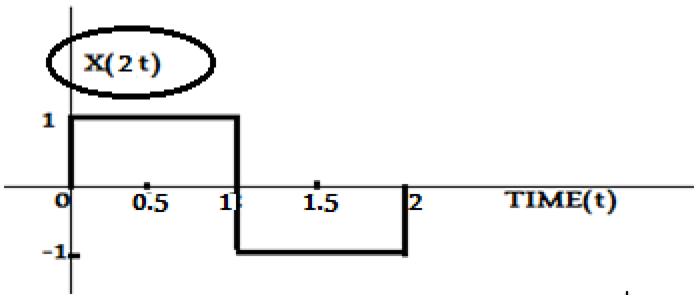
(a) Time scaled signal X(2t).

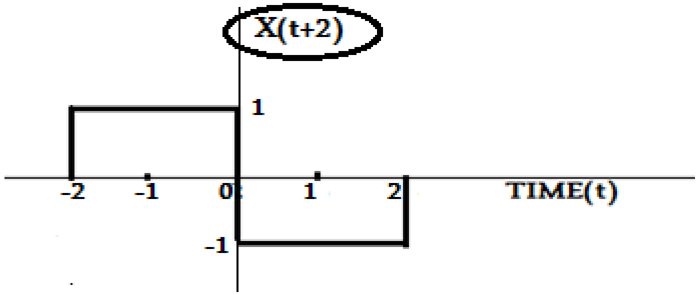
(b) Time shifted signal X(t+2).

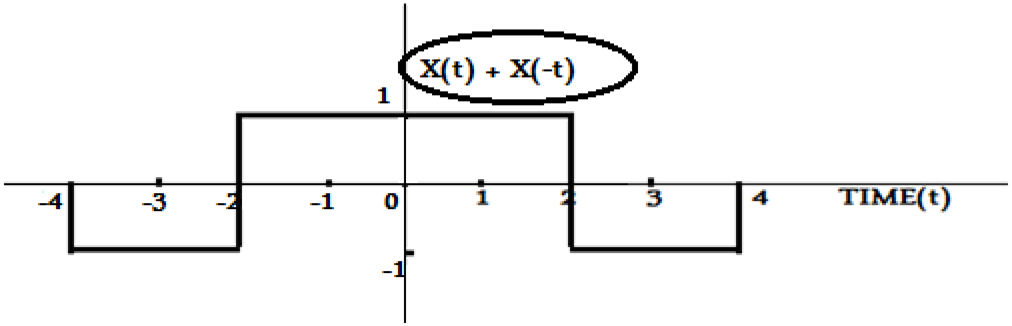
(c) X(t) + X(-t).

Solution:









Example:- For the given signal X(t) shown in Figure-6, **sketch** each of the following signals.

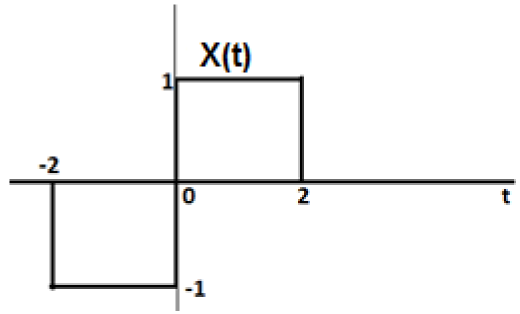
a. Time reversed signal X(-t) .

b. Time scaled signal X(2t).

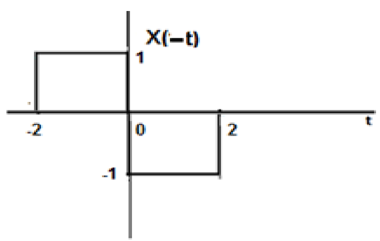
c. Time shifted signal X(t+2).

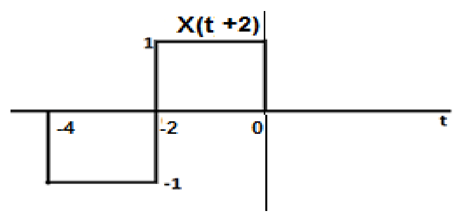
d. { X(t) + X(-t) }

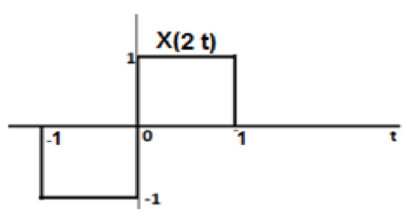
e. { X(t) - X(-t) }

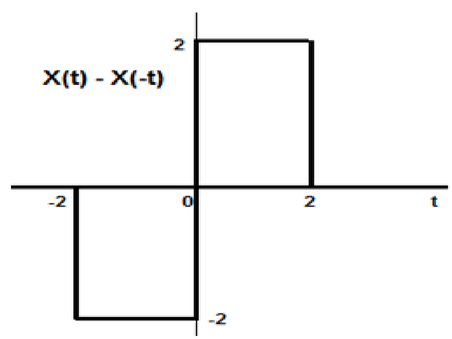


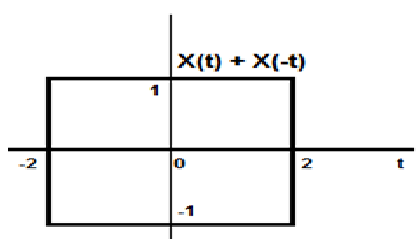
Solution:











Continuous-Time Sinusoidal And Exponential Signals

1. Continuous-Time Sinusoidal Signal:-

- Sinusoids and exponentials are important in signal and system analysis because they arise naturally in the solutions of the differential equations.
- Sinusoidal Signals can expressed in either of two ways :

Cyclic frequency *form*-

$$A \sin 2\Pi f_o t = A \sin(2\Pi/T_o)t$$

Radian frequency form-

$$A \sin \omega_o t$$

$\omega_o = 2\Pi f_o = 2\Pi/T_o$

T_o = Time Period of the Sinusoidal Wave

$x(t) = A \sin(2\Pi f_o t + \theta)$

$= A \sin(\omega_o t + \theta)$

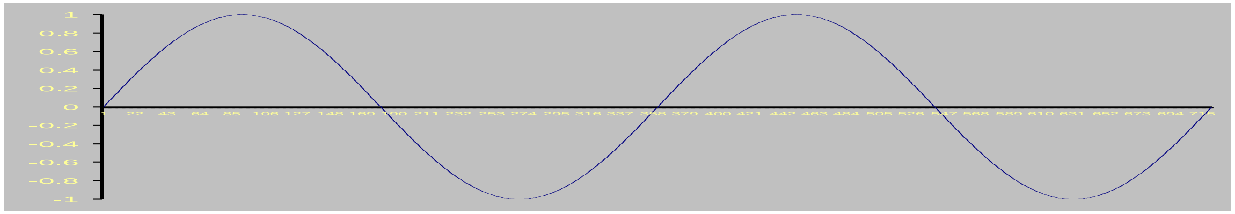
Sinusoidal signal as shown in below figure

θ = Phase of sinusoidal wave

A = amplitude of a sinusoidal or exponential signal

f_o = fundamental cyclic frequency of sinusoidal signal

ω_o = radian frequency



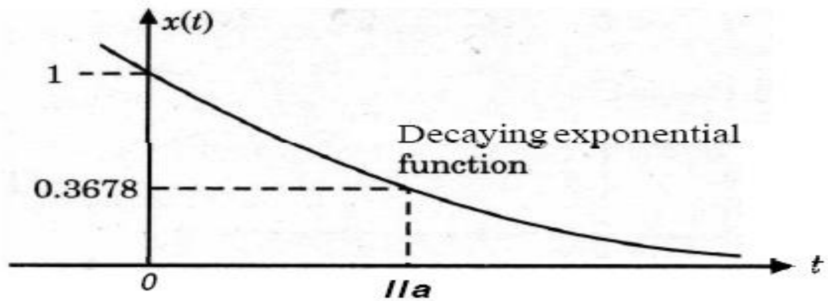
2. Continuous-Time Exponential Signal:-

$x(t) = Ae^{at}$ Real Exponential

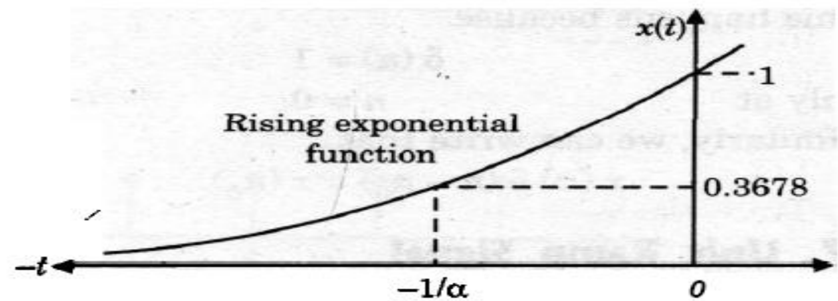
$= Ae^{j\omega t} = A[\cos(\omega_o t) + j \sin(\omega_o t)]$ Complex Exponential

Real Exponential Signals and damped Sinusoidal

$x(t) = e^{-at}$

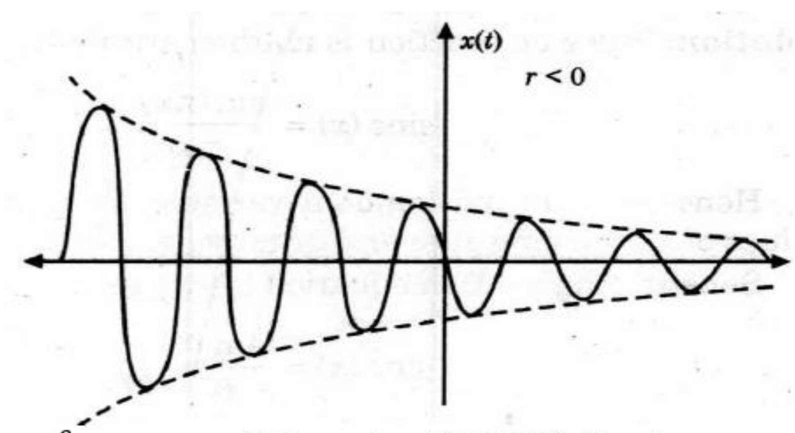


$x(t) = e^{\alpha t}$

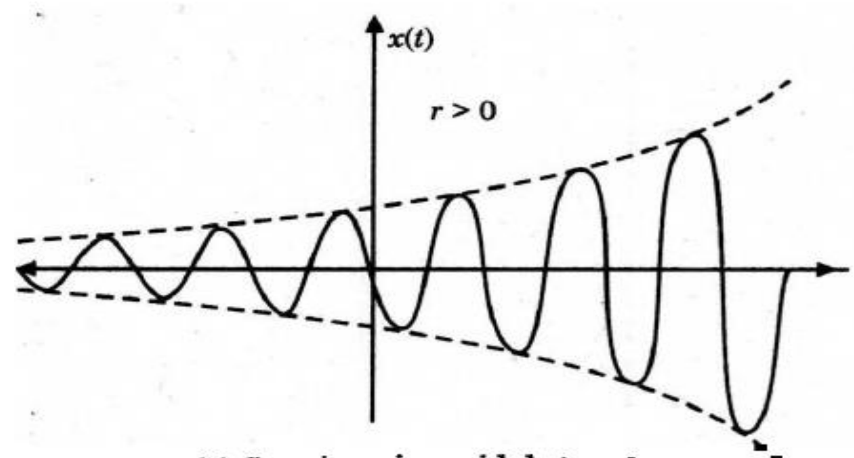


A discrete time exponential signal is expressed as

$$x(n) = a^n$$



(b) Decaying sinusoidal signal



(a) Growing sinusoidal signal

$$e^{rt} \cos(\omega_0 t + \theta)$$

Discrete Time Sinusoidal and Exponential Signals

- DT signals can be defined in a manner analogous to their continuous-time counter part

  $x[n] = A \sin(2\Pi n/N_o+\theta)$

  $= A \sin(2\Pi F_o n+ \theta)$ Discrete Time Sinusoidal Signal

$x[n] = a^n$ Discrete Time Exponential Signal

n = the discrete time

A = amplitude

θ = phase shifting radians,

N_o = Discrete Period of the wave

$1/N_0 = F_o = \Omega_o/2\Pi$ = Discrete Frequency

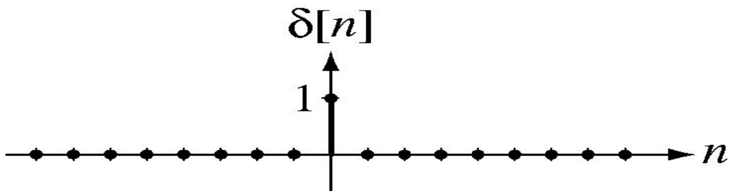
Unit Impulse and Unit Step functions

1. **Discrete-Time Unit Impulse and Unit Step Sequences**
2. **Continuous-Time Unit Step and Unit Impulse Functions**

1. Discrete-Time Unit Impulse and Unit Step Sequences

1(a) Discrete Time Unit Impulse Function

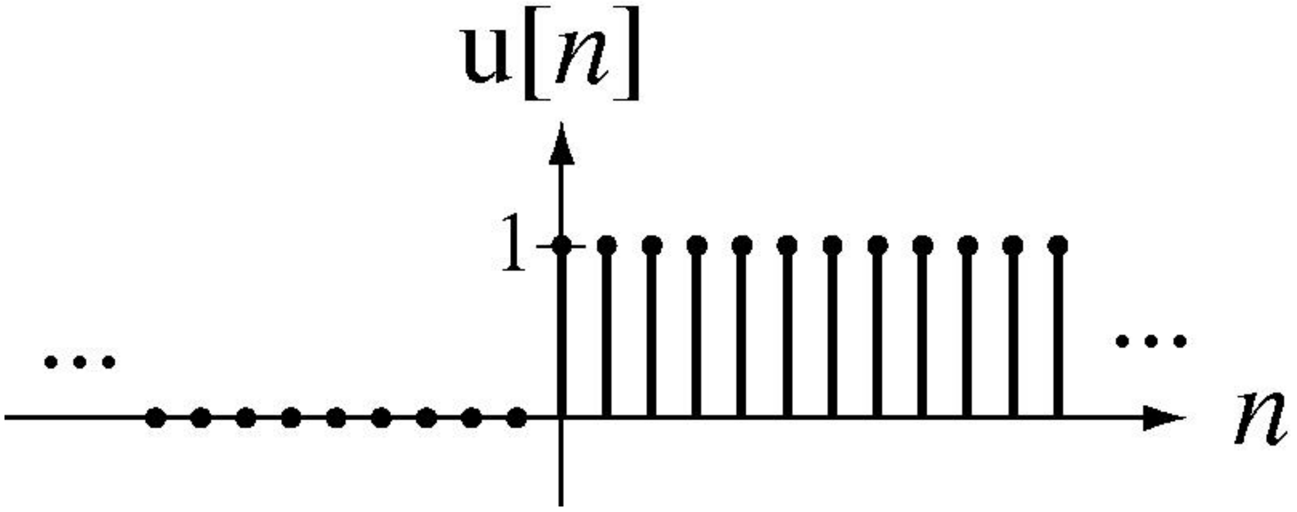
$$\delta[n] = \begin{cases} 1 \ , & n = 0 \\ 0 \ , & n \neq 0 \end{cases}$$



$\delta[n] = \delta[an]$ for any non-zero, finite integer a.

1(b) Unit Step Sequence functions

$$u[n] = \begin{cases} 1 \ , & n \geq 0 \\ 0 \ , & n < 0 \end{cases}$$



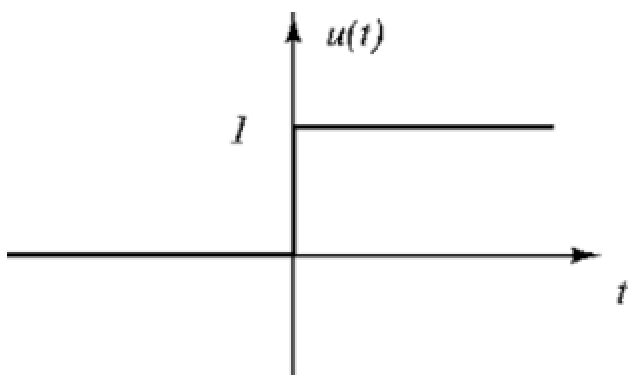
2. Continuous-Time Unit Step and Unit Impulse Functions

2(a) Continuous-Time Unit Step Function

Definition: The unit step function, u(t), is defined as

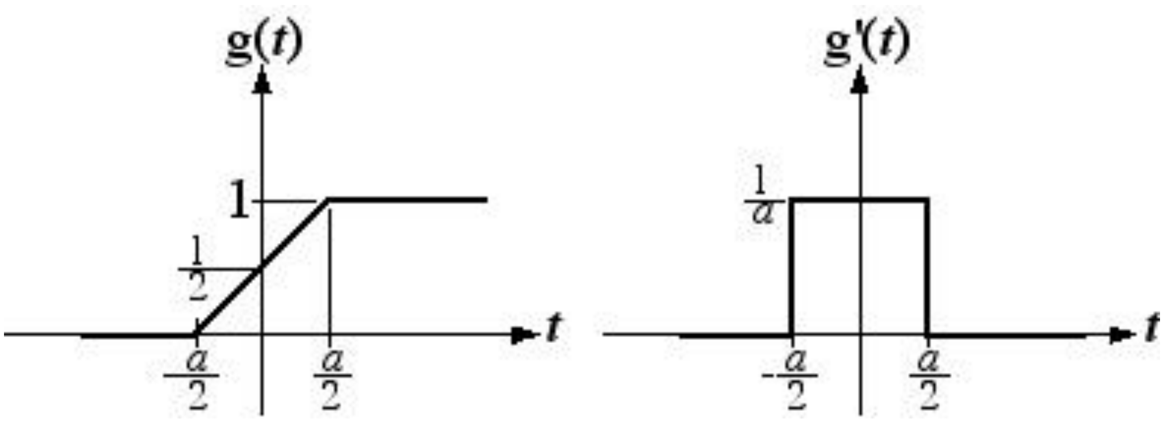
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

That is, u is a function of time t, and u has value zero when time is negative (before we flip the switch); and value one when time is positive (from when we flip the switch).



2(b) Unit Impulse Function

As a approaches zero, $g(t)$ approaches a unit step and $g'(t)$ approaches a unit impulse



So unit impulse function is the **derivative** of the unit step function or unit step is the integral of the unit impulse function

Videos

1. **https://www.youtube.com/watch?v=7Z3LE5uM-6Y&list=PLbMVogVj5nJQQZbah2uRZIRZ_9kfoqZyx**

2. Signals & Systems Tutorial

https://www.youtube.com/watch?v=yLezP5ziz0U&list=PL56ED47DCECCD69B2