

مادارات اشارات
ونظم

Ans Ques:- $\int_0^{\infty} 2 \cdot e^{at} \cdot e^{bt} dt$

Answer:- $\int_0^{\infty} 2 \cdot e^{(a+b)t} dt$

$$\Rightarrow 2 \cdot \left[\frac{e^{(a+b)t}}{(a+b)} \right]_0^{\infty}$$

$$\Rightarrow \frac{2}{(a+b)} \left[e^{(a+b)(\infty)} - e^{(a+b)(0)} \right]$$

$$\Rightarrow \frac{2}{(a+b)} \left[e^{\infty} - e^0 \right]$$

$$\Rightarrow \frac{2}{(a+b)} \left[\infty - 1 \right]$$

$$\Rightarrow \frac{2}{(a+b)} \left[\infty \right] = \infty \text{ Ans.}$$

$$\left\{ \begin{array}{l} e^{\infty} = \infty \\ e^0 = 1 \end{array} \right\}$$

Integration :- Some more practice :-

Ques 1 :- $\int_{-\infty}^{\infty} a \cdot e^{at} \cdot e^{bt} dt$

Answer :- $\int_{-\infty}^{\infty} a \cdot e^{at} \cdot e^{bt} dt$

$$\Rightarrow \int_{-\infty}^{\infty} a \cdot e^{(a+b)t} dt$$

$$\Rightarrow a \cdot \int_{-\infty}^{\infty} e^{(a+b)t} dt$$

$$\Rightarrow a \cdot \left[\frac{e^{(a+b)t}}{(a+b)} \right]_{-\infty}^{\infty}$$

$$\Rightarrow \frac{a}{(a+b)} \left[e^{(a+b)(\infty)} - e^{(a+b)(-\infty)} \right]$$

$$\Rightarrow \frac{a}{(a+b)} \left[e^{\infty} - e^{-\infty} \right] \quad \begin{cases} e^{\infty} = \infty \\ e^{-\infty} = 0 \end{cases}$$

$$\Rightarrow \frac{a}{(a+b)} [\infty - 0]$$

$$\Rightarrow \frac{a}{(a+b)} [\infty] = \infty \quad \underline{\underline{\text{Answer}}} \quad 2$$

Example.

← x → x

Solve the following

$$(i) \int_2^0 x^2 + 1 dx$$

$$(ii) \int_0^2 10x^2 + 10 dx$$

$$(iii) \int_0^2 (t^2 + 1) dt$$

Ans:-

$$(i) \int_2^0 x^2 + 1 dx \Rightarrow \left[\frac{x^3}{3} \right]_2^0 + [x]_2^0$$

$$\Rightarrow \frac{1}{3} [0 - 2^3] + [0 - 2] \Rightarrow -\frac{8}{3} - 2 \Rightarrow -\frac{14}{3} \underline{\underline{Ans.}}$$

$$(ii) \int_0^2 (10x^2 + 10) dx$$

$$\Rightarrow 10 \left[\frac{x^3}{3} \right]_0^2 + 10 [x]_0^2 \Rightarrow \frac{10}{3} [8 - 0] + 10 [2 - 0]$$

$$\Rightarrow \frac{80}{3} + 20 \Rightarrow \frac{140}{3} \underline{\underline{Ans.}}$$

$$(iii) \int_0^2 (t^2 + 1) dt$$

$$\Rightarrow \left[\frac{t^3}{3} \right]_0^2 + [t]_0^2$$

$$\Rightarrow \frac{1}{3} [8 - 0] + [2 - 0]$$

$$\Rightarrow \frac{8}{3} + 2 \Rightarrow \frac{14}{3} \underline{\underline{Ans.}}$$

(P-4) / 67

Integration is important
for chapter 13

Example → 1

(i) $\int 2x^2 dx$ (ii) $\int 2 \sin \theta d\theta$

(iii) $\int (6 + x^2 + \sin x) dx$ (iv) $\int 2 \cos x d\theta$

Ans:- (i) $\int 2x^2 dx \Rightarrow \frac{2}{3} x^3$ Ans.

(ii) $\int 2 \sin \theta d\theta \Rightarrow -2 \cos \theta$ Ans.

(iii) $\int (6 + x^2 + \sin x) dx$

$\Rightarrow \int 6 dx + \int x^2 dx + \int \sin x dx$

$\Rightarrow 6x + \frac{x^3}{3} - \cos x$ Ans.

(iv) $\int 2 \cos x d\theta$

$\Rightarrow 2 \cos x \int 1 d\theta$ } -7

$\Rightarrow 2 \cos x \cdot \theta$

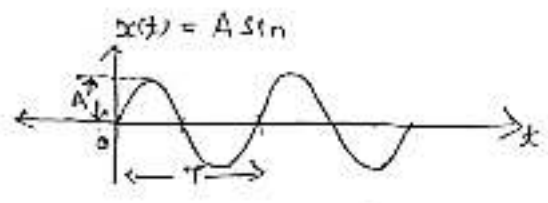
$\Rightarrow \frac{2 \sin x}{x}$

[(P-3)/G]

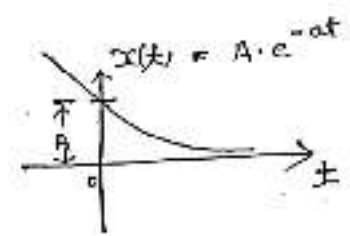
Continuous - Graph

1. $x(t) = A \sin(\omega t + \phi)$

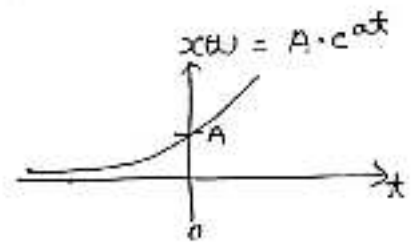
Angular frequency $\omega = \frac{2\pi}{T}$ "or" $2\pi f$
 { Time period of one cycle } frequency



2. $x(t) = A \cdot e^{-at}$

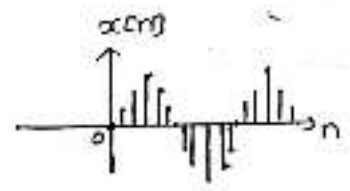


3. $x(t) = A \cdot e^{at}$

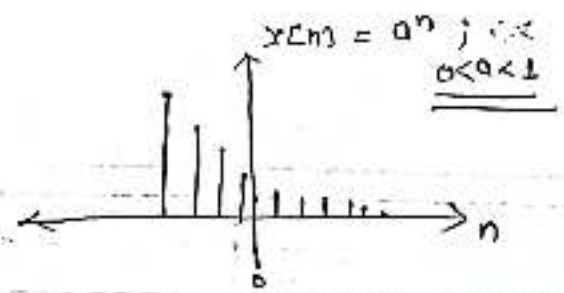
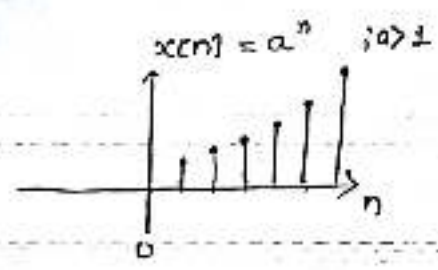


Discrete Graph

1. $x[n] = A \cdot \sin(\omega n + \phi)$



2. $x[n] = a^n$



Formulas "Signals & Systems"

{
• $i^2 = -1$
• i or j , use any one.

1. $e^{i\theta} = \cos\theta + i\sin\theta$
2. $e^{-i\theta} = \cos\theta - i\sin\theta$
3. $\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta$
4. $\frac{e^{i\theta} - e^{-i\theta}}{2i} = \sin\theta$

Important OS.

05. $e^a \times e^b = e^{(a+b)}$ ✓
06. $\frac{e^a}{e^b} = e^{a-b}$
07. $e^0 = 1$; $e^\infty = \infty$; $e^{-\infty} = 0$
08. $\int 1 \cdot dt = t$
09. $\int t^n dt = \frac{t^{n+1}}{n+1}$ or $\int t^n dt = \frac{t^{n+1}}{n+1}$
10. $\int e^t dt = e^t$
11. $\int e^{at} dt = \frac{e^{at}}{a}$
12. $\int \sin(x) dt = -\cos(x)$
13. $\int \cos(x) dt = +\sin(x)$
14. $\int_a^b t dt = \left[\frac{t^2}{2} \right]_a^b \Rightarrow \frac{1}{2} [(b)^2 - (a)^2]$

(CP-1) / G1

Ques 3:- $\int_{-\infty}^0 2 \cdot e^{at} \cdot e^{bt} dt$

Ans:- $\int_{-\infty}^0 2 \cdot e^{(a+b)t} dt$

$$\Rightarrow 2 \cdot \left[\frac{e^{(a+b)t}}{(a+b)} \right]_{-\infty}^0$$

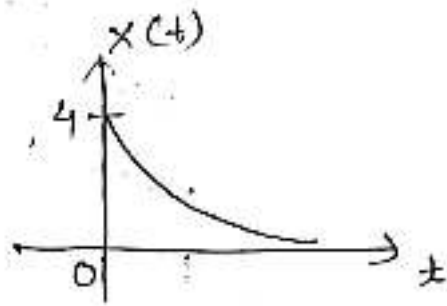
$$\Rightarrow \frac{2}{(a+b)} \left[e^{(a+b)(0)} - e^{(a+b)(-\infty)} \right]$$

$$\Rightarrow \frac{2}{(a+b)} \left[e^0 - e^{-\infty} \right] \quad \left\{ \begin{array}{l} e^0 = 1 \\ e^{-\infty} = 0 \end{array} \right.$$

$$\Rightarrow \frac{2}{(a+b)} [1 - 0]$$

$$= \frac{2}{a+b} \quad \underline{\underline{\text{Answer}}}$$

Question:- Find the energy of signal $x(t)$.



Ans:-

$$\text{Energy of } x(t) = \int_{-\infty}^{\infty} [x(t)]^2 dt$$

$$\text{Energy of given signal} \Rightarrow \int_0^{\infty} (4 \cdot e^{-t})^2 dt$$

$$E \Rightarrow \int_0^{\infty} (16 \cdot e^{-2t}) dt$$

$$E \Rightarrow 16 \int_0^{\infty} e^{-2t} dt$$

$$E \Rightarrow 16 \cdot \left[\frac{e^{-2t}}{-2} \right]_0^{\infty}$$

$$E \Rightarrow \frac{16}{(-2)} [e^{-2(\infty)} - e^{-2(0)}]$$

$$E \Rightarrow -8 [e^{-\infty} - e^0]$$

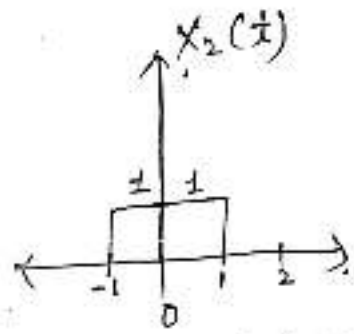
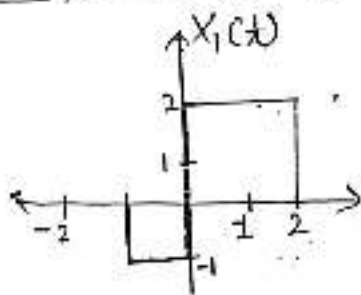
$$E \Rightarrow -8 [0 - 1]$$

$$E \Rightarrow 8 \text{ J } \underline{\underline{\text{Ans}}}$$

(1/2/2021)

Basic Mathematic operation

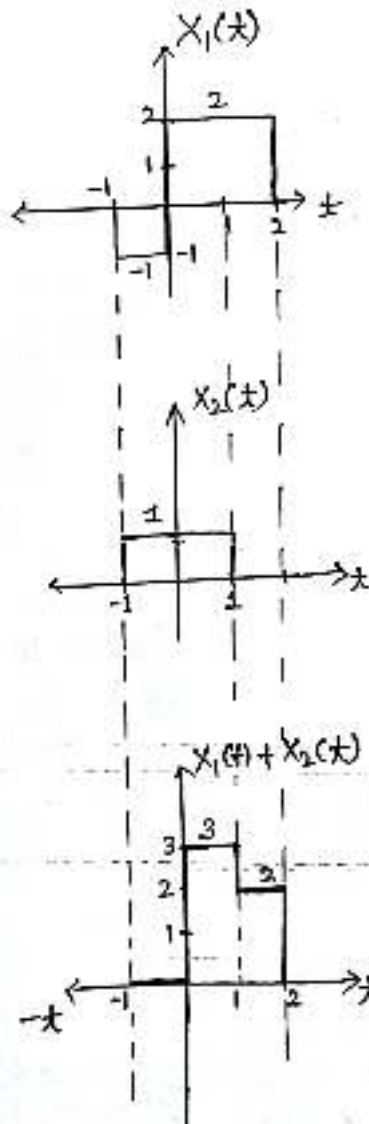
Ans:- Solve the followings:-



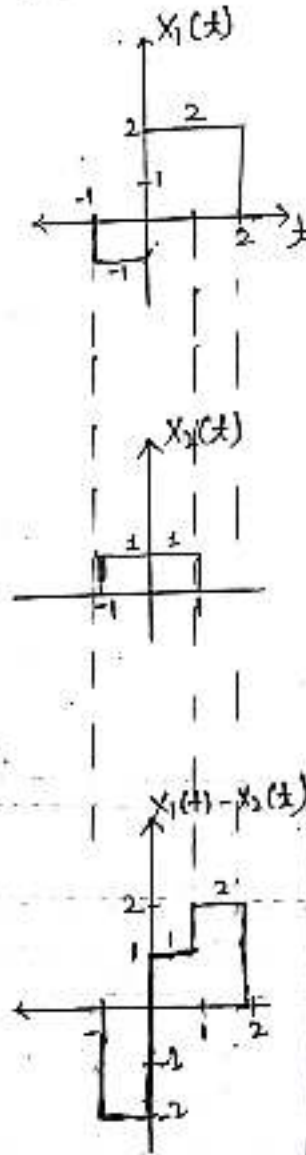
- (i) $X_1(t) + X_2(t)$ (ii) $X_1(t) - X_2(t)$ (iii) $X_1(t) \cdot X_2(t)$

Ans:-

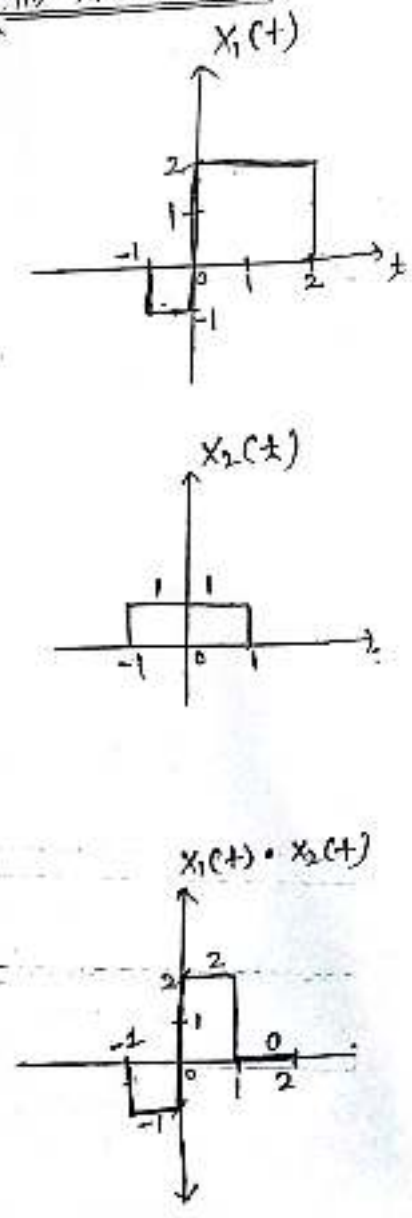
(i) $X_1(t) + X_2(t)$



(ii) $X_1(t) - X_2(t)$



(iii) $X_1(t) \cdot X_2(t)$

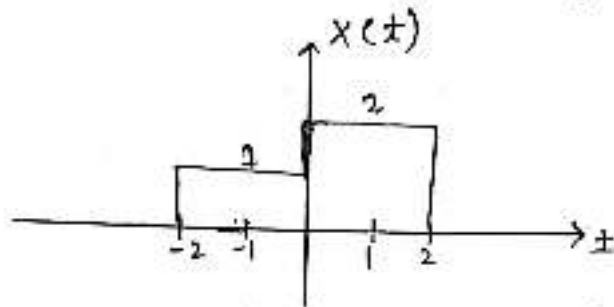


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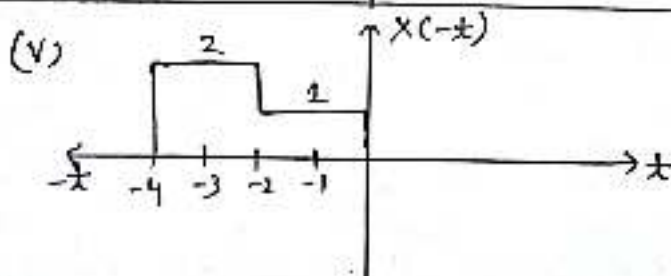
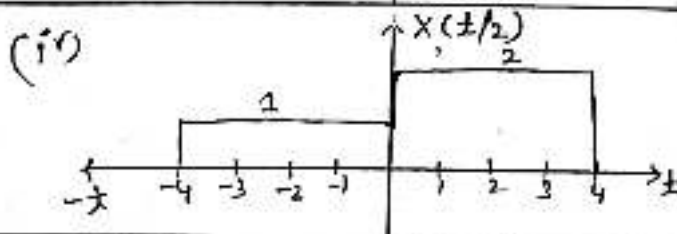
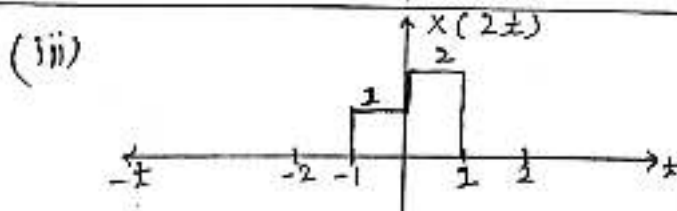
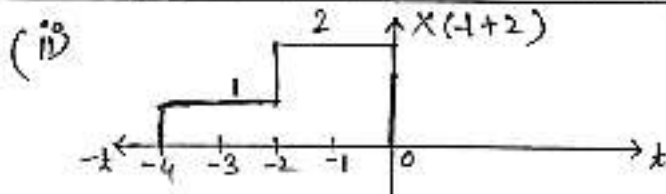
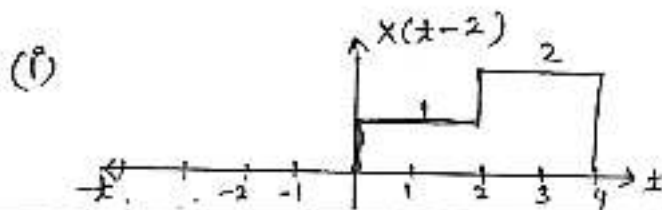
Transformations of $x(t)$

Question:- Solve the followings for the given - signal $x(t)$.



- (i) $x(t-2)$ (ii) $x(2t)$ (iv) $x(-t)$
 (iii) $x(t+2)$ (v) $x(t/2)$

Ans:-



Chapter → 2

Question 1:- Find the followings are with memory or without memory system:

(i) $y(t) = 2x(t)$

(ii) $y(t) = x(t) + 4x(t)$

(iii) $y(t) = 2x(t-5)$

Answer:-

(i) $y(t) = 2x(t)$

Step-1 \Rightarrow put, $t = 0$

$$y(0) = 2x(0)$$

Step-2 \Rightarrow put, $t = -2$

$$y(-2) = 2x(-2)$$

output $\{y(t)\}$ depends upon present input $\{x(t)\}$

So, this is without memory system.

(ii) $y(t) = x(t) + 4x(t)$

Step-1: put, $t = 0$

$$y(0) = x(0) + 4x(0)$$

Step-2 \Rightarrow put $t = -2$

$$y(-2) = x(-2) + 4x(-2)$$

Output $\{y(t)\}$ depends upon present I/p.

So, without memory system

(0.4/2021)

$$(iv) \quad y(t) = 2x(t-5)$$

Step 2 $t = -2$

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Step-1 Put $t = 0$

$$y(0) = 2x(0-5)$$

$$y(0) = 2x(-5)$$

$$y(-2) = 2x(-2-5)$$

output depends upon past input.

So,

System is with memory.

Question- Find the given system is causal or not?

// (i) $y(t) = 2x(t) + x(t-2)$

(ii) $y[n] = 4x^2[n] - 2x[n+2]$

Answer (i) $y(t) = 2x(t) + x(t-2)$

Step 1:- Put $t = 0$

$$y(0) = 2x(0) + x(0-2)$$

\Rightarrow output depends upon "present input" and "past input".

So, system is causal.

(ii) $y[n] = 4x^2[n] - 2x[n+2]$

Step 1:- Put $t = 0$

$$y[0] = 4x^2[0] - 2x[0+2]$$

\Rightarrow output depends upon "present input" and "past future".

So, system is non-causal.

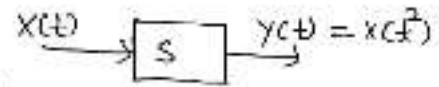
14 (p-2 / Ch-2)

Chapter - 2

Example 1 :- For the given system, $y(t) = x(t^2)$ is given or not?

Answer :-

Step 1 :- Take two individual inputs, and then check the output



$$x_1(t) \rightarrow y_1(t) \Rightarrow x_1(t^2)$$

$$x_2(t) \rightarrow y_2(t) \Rightarrow x_2(t^2)$$

add both outputs.

$$\underbrace{y_1(t) + y_2(t)}_{y'(t)} = x_1(t^2) + x_2(t^2) \quad \text{--- (1)}$$

to the system

Step 2 :- Two inputs are going at the same time, and then check the output.

$$\underbrace{\{x_1(t) + x_2(t)\}}_{x''(t)} \rightarrow \boxed{S} \rightarrow y''(t) = x''(t^2)$$

$$y''(t) = x_1(t^2) + x_2(t^2) \quad \text{--- (2)}$$

Now,

$y'(t) = y''(t)$; means both the output are same. So system is Linear.

Example 2:- For the given system, $y(t) = x^2(t)$ is linear or not

Answer:-

Step 1:- Take two individual inputs and then check the output.

$$x_1(t) \rightarrow y_1(t) = x_1^2(t)$$

$$x_2(t) \rightarrow y_2(t) = x_2^2(t)$$

Add $y_1(t) + y_2(t)$

$$\underbrace{y_1(t) + y_2(t)}_{y'(t)} = x_1^2(t) + x_2^2(t) \quad \text{--- (1)}$$

Step 2:- Two inputs are going ^{to the system} at the same time and then check the output

$$\underbrace{x_1(t) + x_2(t)}_{x''(t)} \rightarrow y''(t) = [x''(t)]^2$$

$$y''(t) = [x''(t)]^2$$

$$y''(t) = [x_1(t) + x_2(t)]^2$$

$$y''(t) = x_1^2(t) + x_2^2(t) + 2x_1(t) \cdot x_2(t) \quad \text{--- (2)}$$

Now,

Equation (1) \neq Equation (2)

So the given system is not linear.

Example 3:-

for the given system, $y(n) = n^2 x(n)$
is linear or not?

Answer:-

Step 1:- Take two individual inputs and then check the output.

• $x_1(n) \rightarrow y_1(n) = n^2 x_1(n)$

• $x_2(n) \rightarrow y_2(n) = n^2 x_2(n)$

Add $y_1(n)$ and $y_2(n)$

$$\underbrace{y_1(n) + y_2(n)}_{y'(n)} = n^2 x_1(n) + n^2 x_2(n) \quad \text{--- (1)}$$

Step 2:-

$$\underbrace{x_1(n) + x_2(n)}_{x''(n)} \rightarrow y''(n) \rightarrow n^2 x''(n)$$

Now, $y''(n) = n^2 [x_1(n) + x_2(n)]$
 $y''(n) = n^2 x_1(n) + n^2 x_2(n) \quad \text{--- (2)}$

Equation (1) = Equation (2)

Hence,

System is Linear.

Example 4:- For the given system, $y[n] = x^2[n]$ is
linear or not?

↑
Ans:- Not linear.

Example 5:- For the given system, $y[n] = 2x[n-5]$
is linear or not?

↑
Ans:- Linear

Example 6:- For the given system, $y(t) = \frac{x(t)}{t}$
is linear or not?

Ans:- Linear

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(P-6 / Ch-2)

Procedure

Time-Variant Property

Step (1) :- Delay the time by k ; only \pm with $x(t)$. (1)

Step (2) :- Delay the time by k ; in all \pm . (2)

Step (3) :- If eq(1) = eq(2)

then

System is Time-Invariant.

• Examples for the given system,

$y[n] = x[n]/n$ is time-invariant or not?

Answer:- Step (1):- $y[n-k] = x[n-k]/n$ (1)

Step (2):- $y[n-k] = x[n-k]/(n-k)$ (2)

Step (3) $Eq(1) \neq Eq(2)$

System is not Time-Invariant.

Example for the given system,

$y[n] = 5x[n-10]$ is time-invariant or not?

Answer:- Step (1):- $y[n-k] = 5x[n-k-10]$ (1)

Step (2):- $y[n-k] = 5x[n-10-k]$ (2)

$Eq(1) = Eq(2)$

System is Time-Invariant.

Chapter 3

$h(t)$ = Impulse Response.

$x(t)$ = Input signal.

$y(t)$ = output signal of a continuous-time -
- LTI system

$$h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau) d\tau$$

$y(t) \Rightarrow$ "or"

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau$$

Question :- Find the output of a continuous-time LTI system from range "0 to t ", if impulse response is $h(t) = e^{-at}$ and input is $x(t) = u(t)$.

Answer :-

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau) d\tau$$

$$y(t) = \int_0^t e^{-a\tau} \cdot u(t-\tau) d\tau$$

$$y(t) = \int_0^t e^{-a\tau} d\tau$$

$$y(t) = \left[\frac{e^{-a\tau}}{-a} \right]_0^t$$

$$y(t) = \frac{1}{-a} [e^{-at} - e^{-a \cdot 0}]$$

$$y(t) = \frac{e^{-at} - 1}{-a} \Rightarrow \frac{1 - e^{-at}}{a}$$

Question 2:-

consider a LTI - system. Assume the impulse response of the system is $h(t) = 2 \cdot e^{-at}$ and input $x(t) = u(t)$. Find the output for range 0 to t ?

Answer:-

$$y(t) = u(t) * x(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau) d\tau$$

$$y(t) = \int_0^t 2 \cdot e^{-a\tau} \cdot u(t-\tau) d\tau$$

$$y(t) = \int_0^t 2 \cdot e^{-a\tau} d\tau$$

$$y(t) = 2 \cdot \left[\frac{e^{-a\tau}}{-a} \right]_0^t$$

$$y(t) = \frac{2}{-a} [e^{-at} - e^{-a \cdot 0}]$$

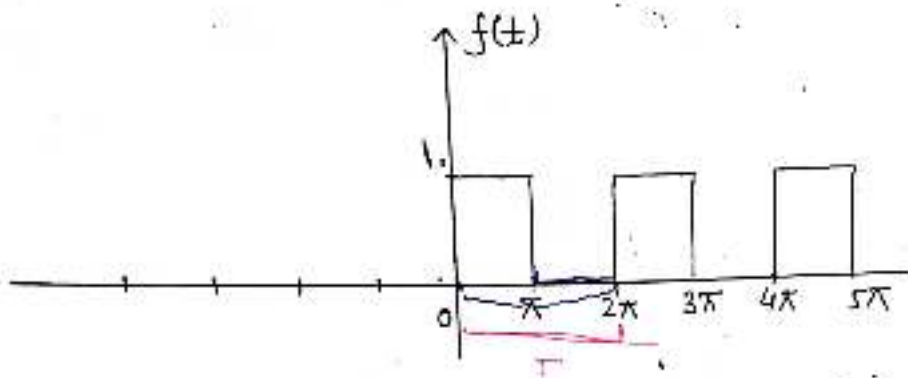
$$y(t) = \frac{2}{-a} [e^{-at} - e^0]$$

$$y(t) = \frac{2}{-a} [e^{-at} - 1]$$

$$y(t) = \frac{2 [1 - e^{-at}]}{a} \quad \underline{\text{Ans.}}$$

Chapter → 4

Question → find the fourier series of $f(t)$ given below:-



Answer:-

fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

or $x(t)$

(i) $a_0 = \frac{2}{T} \int_{\text{cycle lower limit}}^{\text{cycle upper limit}} f(t) dt$ $\Rightarrow \frac{2}{2\pi} \int_0^{2\pi} 1 \cdot dt$

Time-period of one cycle.

Signal Amplitude

$$\Rightarrow \frac{1}{\pi} \left[\int_0^{\pi} 1 dt + \int_{\pi}^{2\pi} 0 dt \right]$$

$T =$ الدائرة كاملة

$f(t) =$ الإغلاق

$$\Rightarrow \frac{1}{\pi} \int_0^{\pi} 1 dt$$

$$\Rightarrow \frac{1}{\pi} [t]_0^{\pi}$$

$$\Rightarrow \frac{1}{\pi} [\pi - 0] \Rightarrow \frac{\pi}{\pi} \Rightarrow 1$$

$$a_n = \frac{2}{T} \int_0^{2\pi} f(t) \cdot \cos(nt) dt$$

$$a_n \Rightarrow \frac{2}{2\pi} \left[\int_0^{\pi} 1 \cdot \cos(nt) dt + \int_{\pi}^{2\pi} 0 dt \right]$$

$$a_n \Rightarrow \frac{1}{\pi} \left[\frac{\sin(nt)}{n} \right]_0^{\pi}$$

$$a_n \Rightarrow \frac{1}{n\pi} \left[\sin(n\pi) - \sin(n \cdot 0) \right]$$

$$a_n \Rightarrow \frac{1}{n\pi} \left[\sin n\pi \right] \#$$

$$b_n = \frac{2}{T} \int_0^{2\pi} f(t) \cdot \sin(nt) dt$$

$$b_n \Rightarrow \frac{2}{2\pi} \left[\int_0^{\pi} 1 \cdot \sin(nt) dt + \int_{\pi}^{2\pi} 0 dt \right]$$

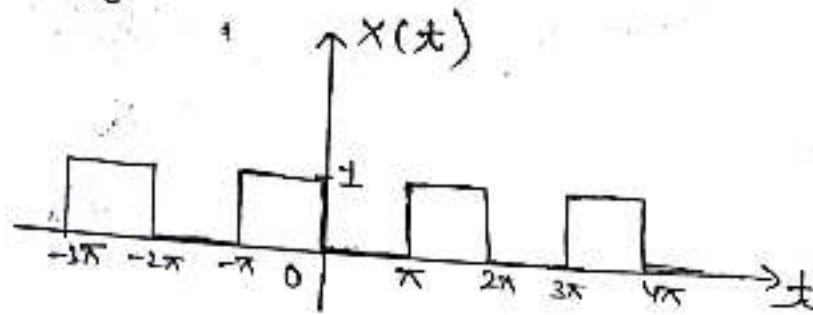
$$b_n = \frac{1}{\pi} \int_0^{\pi} 1 \cdot \sin(nt) dt$$

$$b_n = \frac{1}{\pi} \left[-\frac{\cos(nt)}{n} \right]_0^{\pi}$$

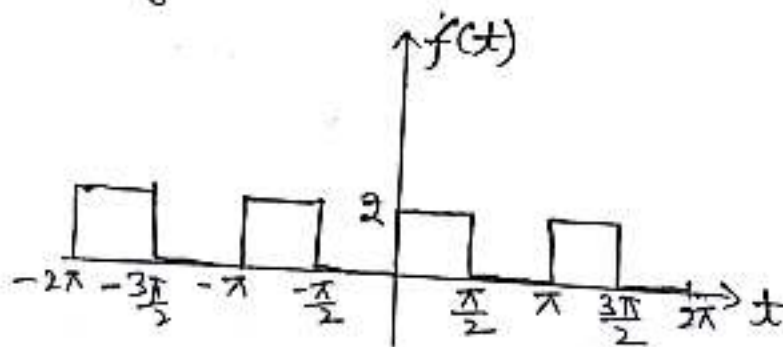
$$b_n = \frac{-1}{n\pi} \left[\cos(n\pi) - \cos(n \cdot 0) \right]$$

$$b_n = \frac{-1}{n\pi} \left[\cos(n\pi) - 1 \right]$$

Question 2 → find the fourier series of $x(t)$ given below.



Question 3 → find the fourier series of $f(t)$ given below.



Chapter 5

Question: 1 → Find the Fourier transform of signal $x(t)$

$$x(t) = e^{-at} u(t)$$

Soln:-

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$X(j\omega) = \int_0^{\infty} e^{-at} \cdot e^{-j\omega t} dt$$

$$X(j\omega) \Rightarrow \int_0^{\infty} e^{-t(a+j\omega)} dt$$

$$X(j\omega) \Rightarrow \left[\frac{e^{-t(a+j\omega)}}{-(a+j\omega)} \right]_0^{\infty}$$

$$X(j\omega) \Rightarrow \frac{e^{-\infty(a+j\omega)} - e^{-0(a+j\omega)}}{-(a+j\omega)}$$

$$X(j\omega) \Rightarrow \frac{e^{-\infty} - e^0}{-(a+j\omega)}$$

$$X(j\omega) = \frac{0 - 1}{-(a+j\omega)}$$

$$X(j\omega) = \frac{1}{(a+j\omega)} \quad (P_{-1} / (C_{-1} - r))$$

Question-2 :- Find the fourier transform of $f(t)$
is given as :-

$$f(t) = 2 \cdot e^{-at} u(t)$$

Answer :-

$$f(j\omega) \Rightarrow \int_0^{\infty} 2 \cdot e^{-at} \cdot e^{-j\omega t} dt$$

$$f(j\omega) \Rightarrow 2 \int_0^{\infty} e^{-t(a+j\omega)} dt$$

$$f(j\omega) \Rightarrow 2 \left[\frac{e^{-t(a+j\omega)}}{-(a+j\omega)} \right]_0^{\infty}$$

$$f(j\omega) \Rightarrow 2 \left[\frac{e^{-\infty(a+j\omega)} - e^{-0(a+j\omega)}}{-(a+j\omega)} \right]$$

$$f(j\omega) \Rightarrow 2 \left[\frac{e^{-\infty} - e^0}{-(a+j\omega)} \right]$$

$$f(j\omega) \Rightarrow \frac{2 [0 - 1]}{-(a+j\omega)}$$

$$f(j\omega) \Rightarrow \frac{2}{(a+j\omega)}$$

$$f(j\omega) = \frac{2}{(a+j\omega)} \underline{\underline{\text{Ans}}}$$