Chapter 4
Interest Rates

Options, Futures, and Other Derivatives 10th Edition, Copyright © John C. Hull 2017

Types of Rates

- Treasury rate
- LIBOR
- Fed funds rate
- Repo rate

Copyright © John C. Hull 2017

Treasury Rate

- Rate on instrument issued by a government in its own currency

LIBOR

- LIBOR is the rate of interest at which a AA bank can borrow money on an unsecured basis from another bank
- For 5 currencies and 7 maturities ranging it is calculated daily by the from submissions from a number of major banks
- There have been some suggestions that banks manipulated LIBOR during certain periods. Why would they do this?

Copyright © John C. Hull 2017

The U.S. Fed Funds Rate

- Unsecured interbank overnight rate of interest
- Allows banks to adjust the cash (i.e., reserves) on deposit with the Federal Reserve at the end of each day
- The effective fed funds rate is the average rate on brokered transactions
- The central bank may intervene with its own transactions to raise or lower the rate
- Similar arrangements in other countries

Copyright © John C. Hull 2017

Repo Rate

- Repurchase agreement is an agreement where a financial institution that owns securities agrees to sell them for X and buy them bank in the future (usually the next day) for a slightly higher price, Y
- The financial institution obtains a loan.
- The rate of interest is calculated from the difference between X and Y and is known as the repo rate

Copyright © John C. Hull 2017

LIBOR swaps

- Most common swap is where LIBOR is exchanged for a fixed rate (discussed in Chapter 7)
- The swap rate where the 3 month LIBOR is exchanged for fixed has the same risk as a series of continually refreshed 3 month loans to AA-rated banks

OIS rate

- An overnight indexed swap is swap where a fixed rate for a period (e.g. 3 months) is exchanged for the geometric average of overnight rates.
- For maturities up to one year there is a single exchange
- For maturities beyond one year there are periodic exchanges, e.g. every quarter
- The OIS rate is a continually refreshed overnight rate

Copyright © John C. Hull 2016

The Risk-Free Rate

- The Treasury rate is considered to be artificially low because
  - Banks are not required to keep capital for Treasury instruments
  - Treasury instruments are given favorable tax treatment in the US
- OIS rates are now used as a proxy for risk-free rates in derivatives valuation

Copyright © John C. Hull 2016

Impact of Compounding

When we compound m times per year at rate R an amount A grows to $A(1+R/m)^m$ in one year

Compounding frequency	*Value of $100 in one year at 10%*
Annual (m=1)	110.00
Semiannual (m=2)	110.25
Quarterly (m=4)	110.38
Monthly (m=12)	110.47
Weekly (m=52)	110.51
Daily (m=365)	110.52

Copyright © John C. Hull 2017

Measuring Interest Rates

- The compounding frequency used for an interest rate is the unit of measurement
- The difference between quarterly and annual compounding is analogous to the difference between miles and kilometers

Copyright © John C. Hull 2017

Continuous Compounding

(Page 82-83)

- In the limit as we compound more and more frequently we obtain continuously compounded interest rates
- \$100 grows to \$100e^{RT} when invested at a continuously compounded rate R for time T
- \$100 received at time T discounts to \$100e^{-RT} at time zero when the continuously compounded discount rate is R

Copyright © John C. Hull 2017

Conversion Formulas *(Page 83)*

Define

R_c: continuously compounded rate

R_m: same rate with compounding *m* times per year

$$R_c = m \ln\left(1 + \frac{R_m}{m}\right)$$

$$R_m = m\left(e^{R_c/m} - 1\right)$$

Copyright © John C. Hull 2017

Examples

- 10% with semiannual compounding is equivalent to 2ln(1.05)=9.758% with continuous compounding
- 8% with continuous compounding is equivalent to $4(e^{0.08/4}-1)$=8.08% with quarterly compounding
- Rates used in option pricing are nearly always expressed with continuous compounding

Zero Rates

A zero rate (or spot rate), for maturity T is the rate of interest earned on an investment that provides a payoff only at time T

Options, Futures, and Other Derivatives 10th Edition, Copyright © John C. Hull 2017

Example *(Table 4.2, page 84)*

Maturity (years)	Zero rate (cont. comp.
0.5	5.0
1.0	5.8
1.5	6.4
2.0	6.8

Copyright © John C. Hull 2017

Bond Pricing

- To calculate the cash price of a bond we discount each cash flow at the appropriate zero rate
- In our example, the theoretical price of a two-year bond providing a 6% coupon semiannually is

$$3e^{-0.05\times 0.5} + 3e^{-0.058\times 1.0} + 3e^{-0.064\times 1.5}$$

$$+103e^{-0.068\times 2.0} = 98.39$$

Bond Yield

- The bond yield is the discount rate that makes the present value of the cash flows on the bond equal to the market price of the bond
- Suppose that the market price of the bond in our example equals its theoretical price of 98.39
- The bond yield (continuously compounded) is given by solving

$$3e^{-y\times 0.5}+3e^{-y\times 1.0}+3e^{-y\times 1.5}+103e^{-y\times 2.0}=98.39$$

to get y=0.0676 or 6.76%.

Par Yield

- The par yield for a certain maturity is the coupon rate that causes the bond price to equal its face value.
- In our example we solve

$$\frac{c}{2}e^{-0.05\times0.5}+\frac{c}{2}e^{-0.058\times1.0}+\frac{c}{2}e^{-0.064\times1.5}$$

$$+\left(100+\frac{c}{2}\right)e^{-0.068\times2.0}=100$$

to get $c=6.87$

Copyright © John C. Hull 2017

Par Yield continued

In general if m is the number of coupon payments per year, d is the present value of \$1 received at maturity and A is the present value of an annuity of \$1 on each coupon date

$$c = \frac{(100 - 100d)m}{A}$$

(in our example, $m = 2$, $d = 0.87284$, and $A = 3.70027$)

Copyright © John C. Hull 2017

Data to Determine Zero Curve

(Table 4.3, page 86)

Bond Principal	Time to Maturity (yrs)	Coupon per year ($)*	Bond price ($)
100	0.25	0	99.6
100	0.50	0	99.0
100	1.00	0	97.8
100	1.50	4	102.5
100	2.00	5	105.0

* Half the stated coupon is paid each year

The Bootstrap Method

- An amount 0.4 can be earned on 99.6 during 3 months.
- Because $100=99.4e^{0.01603\times0.25}$ the 3-month rate is 1.603% with continuous compounding
- Similarly the 6 month and 1 year rates are 2.010% and 2.225% with continuous compounding

Copyright © John C. Hull 2017

The Bootstrap Method continued

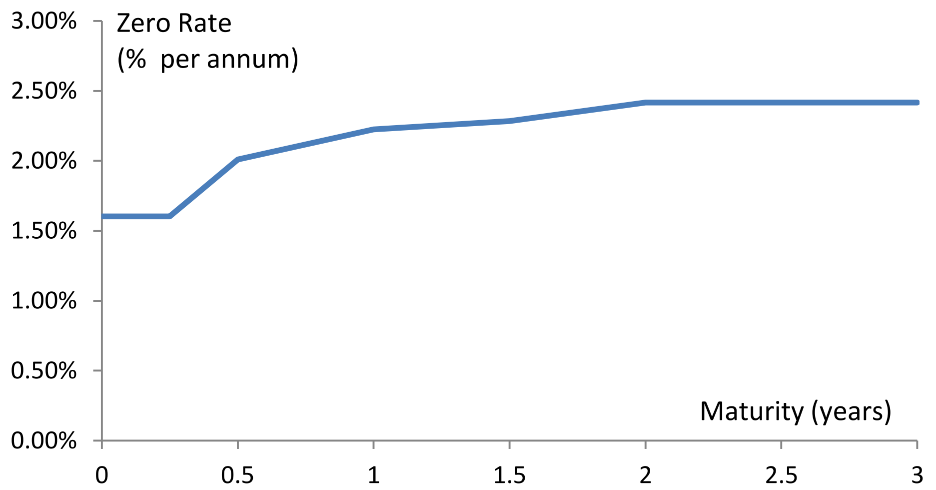
- To calculate the 1.5 year rate we solve

$$2e^{-0.02010\times0.5}+2e^{-0.02225\times1.0}+102e^{-R\times1.5}=102.5$$

to get R = 0.02284 or 2.284%

- Similarly the two-year rate is 2.416%

Zero Curve Calculated from the Data *(Figure 4.1, page 87)*



Application to OIS Rates

- OIS rates out to 1 year are zero rates
- OIS rates beyond one year are par yields,

Copyright © John C. Hull 2016

Forward Rates

- The forward rate is the future zero rate implied by today's term structure of interest rates

Formula for Forward Rates

- Suppose that the zero rates for time periods T_1 and T_2 are R_1 and R_2 with both rates continuously compounded.
- The forward rate for the period between times T_1 and T_2 is

$$\frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

- This formula is only approximately true when rates are not expressed with continuous compounding

Copyright © John C. Hull 2017

Application of the Formula

Year (n)	Zero rate for n-year investment (% per annum)	Forward rate for nth year (% per annum)
1	3.0	
2	4.0	5.0
3	4.6	5.8
4	5.0	6.2
5	5.5	6.5

Copyright © John C. Hull 2017

Instantaneous Forward Rate

- The instantaneous forward rate for a maturity T is the forward rate that applies for a very short time period starting at T. It is

$$R + T\frac{\partial R}{\partial T}$$

where R is the T-year rate

Upward vs Downward Sloping Yield Curve

- For an upward sloping yield curve:
  Fwd Rate > Zero Rate > Par Yield

- For a downward sloping yield curve
  Par Yield > Zero Rate > Fwd Rate

Copyright © John C. Hull 2017

Forward Rate Agreement

- A forward rate agreement (FRA) is an OTC agreement that a certain rate will apply to a certain principal during a certain future time period

Copyright © John C. Hull 2017

Forward Rate Agreement: Key Results

- An FRA is equivalent to an agreement where interest at a predetermined rate, R_K is exchanged for interest at the market rate
- An FRA can be valued by assuming that the forward LIBOR interest rate, R_F, is certain to be realized
- This means that the value of an FRA is the present value of the difference between the interest that would be paid at interest at rate R_F and the interest that would be paid at rate R_K

Valuation Formulas

- If the period to which an FRA applies lasts from T_1 to T_2, we assume that R_F and R_K are expressed with a compounding frequency corresponding to the length of the period between T_1 and T_2
- With an interest rate of R_K, the interest cash flow is $R_K(T_2 - T_1)$ at time T_2
- With an interest rate of R_F, the interest cash flow is $R_F(T_2 - T_1)$ at time T_2

Valuation Formulas *continued*

- When the rate R_K will be received on a principal of L the value of the FRA is the present value of

$$(R_K - R_F)(T_2 - T_1)$$

received at time T_2

- When the rate R_K will be received on a principal of L the value of the FRA is the present value of

$$(R_F - R_K)(T_2 - T_1)$$

received at time T_2

Copyright © John C. Hull 2017

Example

- An FRA entered into some time ago ensures that a company will receive 4% (s.a.) on $100 million for six months starting in 1 year
- Forward LIBOR for the period is 5% (s.a.)
- The 1.5 year risk-free rate is 4.5% with continuous compounding
- The value of the FRA (in $ millions) is

$$100 \times (0.04 - 0.05) \times 0.5 \times e^{-0.045 \times 1.5} = -0.467$$

Example continued

- If the six-month LIBOR interest rate in one year turns out to be 5.5% (s.a.) there will be a payoff (in $ millions) of

$$100 \times (0.04 - 0.055) \times 0.5 = -0.75$$

in 1.5 years

- The transaction might be settled at the one-year point for the present value of this

Copyright © John C. Hull 2017

Duration *(page 94-97)*

- Duration of a bond that provides cash flow c_i at time t_i is

$$D = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-y t_i}}{B} \right]$$

where B is its price and y is its yield (continuously compounded)

Copyright © John C. Hull 2017

Key Duration Relationship

- Duration is important because it leads to the following key relationship between the change in the yield on the bond and the change in its price

$$\frac{\Delta B}{B} = -D\Delta y$$

Copyright © John C. Hull 2017

Key Duration Relationship
continued

- When the yield y is expressed with compounding m times per year

$$\Delta B = -\frac{BD\Delta y}{1 + y/m}$$

- The expression

$$\frac{D}{1 + y/m}$$

is referred to as the "modified duration"

Bond Portfolios

- The duration for a bond portfolio is the weighted average duration of the bonds in the portfolio with weights proportional to prices
- The key duration relationship for a bond portfolio describes the effect of small parallel shifts in the yield curve
- What exposures remain if duration of a portfolio of assets equals the duration of a portfolio of liabilities?

Convexity

The convexity, C, of a bond is defined as

$$C = \frac{1}{B}\frac{\partial^2 B}{\partial y^2} = \frac{\sum_{i=1}^{n} c_i t_i^2 e^{-yt_i}}{B}$$

This leads to a more accurate relationship

$$\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

When used for bond portfolios it allows larger shifts in the yield curve to be considered, but the shifts still have to be parallel

Theories of the Term Structure
Page 99-101

- Expectations Theory: forward rates equal expected future zero rates
- Market Segmentation: short, medium and long rates determined independently of each other
- Liquidity Preference Theory: forward rates higher than expected future zero rates

Copyright © John C. Hull 2017

Liquidity Preference Theory

- Suppose that the outlook for rates is flat and you have been offered the following choices

Maturity	Deposit rate	Mortgage rate
1 year	3%	6%
5 year	3%	6%

- Which would you choose as a depositor? Which for your mortgage?

Liquidity Preference Theory
cont

- To match the maturities of borrowers and lenders a bank has to increase long rates above expected future short rates
- In our example the bank might offer

Maturity	Deposit rate	Mortgage rate
1 year	3%	6%
5 year	4%	7%

Copyright © John C. Hull 2017