

Linear Programming

B

MODULE

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Alaska Airlines



Alaska Airlines

LEARNING OBJECTIVES

- LO B.1** *Formulate* linear programming models, including an objective function and constraints 702
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The storm front closed in quickly on Boston's Logan Airport, shutting it down without warning. The heavy snowstorms and poor visibility sent airline passengers and ground crew scurrying. Because airlines use linear programming (LP) to schedule flights, hotels, crews, and refueling, LP has a direct impact on profitability. If an airline gets a major weather disruption at one of its hubs, a lot of flights may get canceled, which means a lot of crews and airplanes in the wrong places. LP is the tool that helps airlines unspool and cope with this weather mess.



Paul Italiano/Alamy

Why Use Linear Programming?

Many operations management decisions involve trying to make the most effective use of an organization's resources. Resources typically include machinery (such as planes, in the case of an airline), labor (such as pilots), money, time, and raw materials (such as jet fuel). These resources may be used to produce products (such as machines, furniture, food, or clothing) or services (such as airline schedules, advertising policies, or investment decisions). **Linear programming (LP)** is a widely used mathematical technique designed to help operations managers plan and make the decisions necessary to allocate resources.

A few examples of problems in which LP has been successfully applied in operations management are:

1. Scheduling school buses to *minimize* the total distance traveled when carrying students
2. Allocating police patrol units to high crime areas to *minimize* response time to 911 calls
3. Scheduling tellers at banks so that needs are met during each hour of the day while *minimizing* the total cost of labor
4. Selecting the product mix in a factory to make best use of machine- and labor-hours available while *maximizing* the firm's profit
5. Picking blends of raw materials in feed mills to produce finished feed combinations at *minimum* cost
6. Determining the distribution system that will *minimize* total shipping cost from several warehouses to various market locations
7. Developing a production schedule that will satisfy future demands for a firm's product and at the same time *minimize* total production and inventory costs
8. Allocating space for a tenant mix in a new shopping mall to *maximize* revenues to the leasing company

Linear programming (LP)

A mathematical technique designed to help operations managers plan and make decisions necessary to allocate resources.

VIDEO B.1

Scheduling Challenges at Alaska Airlines

Requirements of a Linear Programming Problem

All LP problems have four requirements: an objective, constraints, alternatives, and linearity:

1. LP problems seek to *maximize* or *minimize* some quantity (usually profit or cost). We refer to this property as the **objective function** of an LP problem. The major objective of a typical firm is to maximize dollar profits in the long run. In the case of a trucking or airline distribution system, the objective might be to minimize shipping costs.
2. The presence of restrictions, or **constraints**, limits the degree to which we can pursue our objective. For example, deciding how many units of each product in a firm’s product line to manufacture is restricted by available labor and machinery. We want, therefore, to maximize or minimize a quantity (the objective function) subject to limited resources (the constraints).
3. There must be *alternative courses of action* to choose from. For example, if a company produces three different products, management may use LP to decide how to allocate among them its limited production resources (of labor, machinery, and so on). If there were no alternatives to select from, we would not need LP.
4. The objective and constraints in linear programming problems must be expressed in terms of *linear equations* or *inequalities*. Linearity implies proportionality and additivity. If x_1 and x_2 are decision variables, there can be no products (e.g., x_1x_2) or powers (e.g., x_1^3) in the objective or constraints. For example, the expression $5x_1 + 8x_2 \leq 250$ is linear; however, the expression $5x_1 + 8x_2 - 2x_1x_2 \leq 300$ is not linear.

Objective function

A mathematical expression in linear programming that maximizes or minimizes some quantity (often profit or cost, but any goal may be used).

Constraints

Restrictions that limit the degree to which a manager can pursue an objective.

Formulating Linear Programming Problems

One of the most common linear programming applications is the *product-mix problem*. Two or more products are usually produced using limited resources. The company would like to determine how many units of each product it should produce to maximize overall profit given its limited resources. Let’s look at an example.

STUDENT TIP

Here we set up an LP example that we will follow for most of this module.

Glickman Electronics Example

The Glickman Electronics Company in Washington, DC, produces two products: (1) the Glickman x-pod and (2) the Glickman BlueBerry. The production process for each product is similar in that both require a certain number of hours of electronic work and a certain number of labor-hours in the assembly department. Each x-pod takes 4 hours of electronic work and 2 hours in the assembly shop. Each BlueBerry requires 3 hours in electronics and 1 hour in assembly. During the current production period, 240 hours of electronic time are available, and 100 hours of assembly department time are available. Each x-pod sold yields a profit of \$7; each BlueBerry produced may be sold for a \$5 profit.

Glickman’s problem is to determine the best possible combination of x-pods and BlueBerrys to manufacture to reach the maximum profit. This product-mix situation can be formulated as a linear programming problem.

We begin by summarizing the information needed to formulate and solve this problem (see Table B.1). Further, let’s introduce some simple notation for use in the objective function and constraints. Let:

$$X_1 = \text{number of x-pods to be produced}$$

$$X_2 = \text{number of BlueBerrys to be produced}$$

ACTIVE MODEL B.1

This example is further illustrated in Active Model B.1 in MyOMLab.

TABLE B.1

Glickman Electronics Company Problem Data

DEPARTMENT	HOURS REQUIRED TO PRODUCE ONE UNIT		AVAILABLE HOURS THIS WEEK
	X-PODS (X_1)	BLUEBERRYS (X_2)	
Electronic	4	3	240
Assembly	2	1	100
Profit per unit	\$7	\$5	

Now we can create the LP *objective function* in terms of X_1 and X_2 :

$$\text{Maximize profit} = \$7X_1 + \$5X_2$$

Our next step is to develop mathematical relationships to describe the two constraints in this problem. One general relationship is that the amount of a resource used is to be less than or equal to (\leq) the amount of resource *available*.

LO B.1 Formulate linear programming models, including an objective function and constraints

First constraint: Electronic time used is \leq Electronic time available.

$$4X_1 + 3X_2 \leq 240 \text{ (hours of electronic time)}$$

Second constraint: Assembly time used is \leq Assembly time available.

$$2X_1 + 1X_2 \leq 100 \text{ (hours of assembly time)}$$

Both these constraints represent production capacity restrictions and, of course, affect the total profit. For example, Glickman Electronics cannot produce 70 x-pods during the production period because if $X_1 = 70$, both constraints will be violated. It also cannot make $X_1 = 50$ x-pods and $X_2 = 10$ BlueBerrys. This constraint brings out another important aspect of linear programming; that is, certain interactions will exist between variables. The more units of one product that a firm produces, the fewer it can make of other products.

Graphical Solution to a Linear Programming Problem

The easiest way to solve a small LP problem such as that of the Glickman Electronics Company is the **graphical solution approach**. The graphical procedure can be used only when there are two **decision variables** (such as number of x-pods to produce, X_1 , and number of BlueBerrys to produce, X_2). When there are more than two variables, it is *not* possible to plot the solution on a two-dimensional graph; we then must turn to more complex approaches described later in this module.

Graphical solution approach

A means of plotting a solution to a two-variable problem on a graph.

Decision variables

Choices available to a decision maker.

Graphical Representation of Constraints

To find the optimal solution to a linear programming problem, we must first identify a set, or region, of feasible solutions. The first step in doing so is to plot the problem's constraints on a graph.

The variable X_1 (x-pods, in our example) is usually plotted as the horizontal axis of the graph, and the variable X_2 (BlueBerrys) is plotted as the vertical axis. The complete problem may be restated as:

$$\text{Maximize profit} = \$7X_1 + \$5X_2$$

Subject to the constraints:

$$4X_1 + 3X_2 \leq 240 \text{ (electronics constraint)}$$

$$2X_1 + 1X_2 \leq 100 \text{ (assembly constraint)}$$

$$X_1 \geq 0 \text{ (number of x-pods produced is greater than or equal to 0)}$$

$$X_2 \geq 0 \text{ (number of BlueBerrys produced is greater than or equal to 0)}$$

(These last two constraints are also called *nonnegativity constraints*.)

The first step in graphing the constraints of the problem is to convert the constraint *inequalities* into *equalities* (or equations):

$$\text{Constraint A: } 4X_1 + 3X_2 = 240$$

$$\text{Constraint B: } 2X_1 + 1X_2 = 100$$

The equation for constraint A is plotted in Figure B.1 and for constraint B in Figure B.2.

To plot the line in Figure B.1, all we need to do is to find the points at which the line $4X_1 + 3X_2 = 240$ intersects the X_1 and X_2 axes. When $X_1 = 0$ (the location where the line touches the X_2 axis), it implies that $3X_2 = 240$ and that $X_2 = 80$. Likewise, when $X_2 = 0$, we see that $4X_1 = 240$ and that $X_1 = 60$. Thus, constraint A is bounded by the line running from $(X_1 = 0, X_2 = 80)$ to $(X_1 = 60, X_2 = 0)$. The shaded area represents all points that satisfy the original *inequality*.

Constraint B is illustrated similarly in Figure B.2. When $X_1 = 0$, then $X_2 = 100$; and when $X_2 = 0$, then $X_1 = 50$. Constraint B, then, is bounded by the line between

STUDENT TIP

We named the decision variables X_1 and X_2 here, but any notations (e.g., x - p and B or X and Y) would do as well.

and then add it to the first equation:

$$\begin{array}{r} +4X_1 + 3X_2 = 240 \\ -4X_1 - 2X_2 = -200 \\ \hline + 1X_2 = 40 \end{array}$$

or:

$$X_2 = 40$$

Doing this has enabled us to eliminate one variable, X_1 , and to solve for X_2 . We can now substitute 40 for X_2 in either of the original constraint equations and solve for X_1 . Let us use the first equation. When $X_2 = 40$, then:

$$\begin{aligned} 4X_1 + 3(40) &= 240 \\ 4X_1 + 120 &= 240 \\ 4X_1 &= 120 \\ X_1 &= 30 \end{aligned}$$

Thus, the optimal solution has the coordinates ($X_1 = 30, X_2 = 40$). The profit at this point is $\$7(30) + \$5(40) = \$410$.

Corner-Point Solution Method

A second approach to solving linear programming problems employs the **corner-point method**. This technique is simpler in concept than the iso-profit line approach, but it involves looking at the profit at every corner point of the feasible region.

Corner-point method
A method for solving graphical linear programming problems.

The mathematical theory behind linear programming states that an optimal solution to any problem (that is, the values of X_1, X_2 that yield the maximum profit) will lie at a *corner point*, or *extreme point*, of the feasible region. Hence, it is necessary to find only the values of the variables at each corner; the maximum profit or optimal solution will lie at one (or more) of them.

Once again we can see (in Figure B.7) that the feasible region for the Glickman Electronics Company problem is a four-sided polygon with four corner, or extreme, points. These points are labeled ①, ②, ③, and ④ on the graph. To find the (X_1, X_2) values producing the maximum profit, we find out what the coordinates of each corner point are, then determine and compare their profit levels. (We showed how to find the coordinates for point ③ in the previous section describing the iso-profit line solution method.)

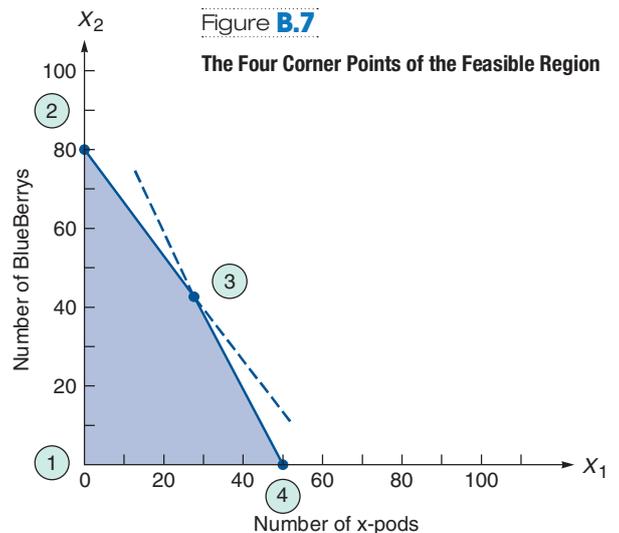


Figure B.7

The Four Corner Points of the Feasible Region

- Point ①: ($X_1 = 0, X_2 = 0$) Profit $\$7(0) + \$5(0) = \$0$
- Point ②: ($X_1 = 0, X_2 = 80$) Profit $\$7(0) + \$5(80) = \$400$
- Point ③: ($X_1 = 30, X_2 = 40$) Profit $\$7(30) + \$5(40) = \$410$
- Point ④: ($X_1 = 50, X_2 = 0$) Profit $\$7(50) + \$5(0) = \$350$

Because point ③ produces the highest profit of any corner point, the product mix of $X_1 = 30$ x-pods and $X_2 = 40$ BlueBerrys is the optimal solution to the Glickman Electronics problem. This solution will yield a profit of \$410 per production period; it is the same solution we obtained using the iso-profit line method.

LO B.3 Graphically solve an LP problem with the corner-point method

Sensitivity Analysis

Operations managers are usually interested in more than the optimal solution to an LP problem. In addition to knowing the value of each decision variable (the X_i s) and the value of the objective function, they want to know how sensitive these answers are to input **parameter** changes. For example, what happens if the coefficients of the objective function are not exact, or if they change by 10% or 15%? What happens if the right-hand-side values of the constraints

Parameter
Numerical value that is given in a model.

Allowable Ranges for Objective Function Coefficients As the unit profit contribution of either product changes, the slope of the iso-profit lines we saw earlier in Figure B.5 changes. The size of the feasible region, however, remains the same. That is, the locations of the corner points do not change.

The limits to which the profit coefficient of x-pods or BlueBerrys can be changed without affecting the optimality of the current solution is revealed by the values in the **Allowable Increase** and **Allowable Decrease** columns of the *Sensitivity Report* in Program B.1. The allowable increase in the objective function coefficient for BlueBerrys is only \$0.25. In contrast, the allowable decrease is \$1.50. Hence, if the unit profit of BlueBerrys drops to \$4 (i.e., a decrease of \$1 from the current value of \$5), it is still optimal to produce 30 x-pods and 40 BlueBerrys. The total profit will drop to \$370 (from \$410) because each BlueBerry now yields less profit (of \$1 per unit). However, if the unit profit drops below \$3.50 per BlueBerry (i.e., a decrease of more than \$1.50 from the current \$5 profit), the current solution is no longer optimal. The LP problem will then have to be resolved using Solver, or other software, to find the new optimal corner point.

STUDENT TIP ◆

LP problems can be structured to minimize costs as well as maximize profits.

Solving Minimization Problems

Many linear programming problems involve *minimizing* an objective such as cost instead of maximizing a profit function. A restaurant, for example, may wish to develop a work schedule to meet staffing needs while minimizing the total number of employees. Also, a manufacturer may seek to distribute its products from several factories to its many regional warehouses in a way that minimizes total shipping costs.

Minimization problems can be solved graphically by first setting up the feasible solution region and then using either the corner-point method or an **iso-cost** line approach (which is analogous to the iso-profit approach in maximization problems) to find the values of X_1 and X_2 that yield the minimum cost.

Example B1 shows how to solve a minimization problem.

iso-cost

An approach to solving a linear programming minimization problem graphically.

Example B1

A MINIMIZATION PROBLEM WITH TWO VARIABLES

Cohen Chemicals, Inc., produces two types of photo-developing fluids. The first, a black-and-white picture chemical, costs Cohen \$2,500 per ton to produce. The second, a color photo chemical, costs \$3,000 per ton.

Based on an analysis of current inventory levels and outstanding orders, Cohen's production manager has specified that at least 30 tons of the black-and-white chemical and at least 20 tons of the color chemical must be produced during the next month. In addition, the manager notes that an existing inventory of a highly perishable raw material needed in both chemicals must be used within 30 days. To avoid wasting the expensive raw material, Cohen must produce a total of at least 60 tons of the photo chemicals in the next month.

APPROACH ► Formulate this information as a minimization LP problem.

Let:

$$\begin{aligned} X_1 &= \text{number of tons of black-and-white photo chemical produced} \\ X_2 &= \text{number of tons of color photo chemical produced} \\ \text{Objective: Minimize cost} &= \$2,500X_1 + \$3,000X_2 \end{aligned}$$

Subject to:

$$\begin{aligned} X_1 &\geq 30 \text{ tons of black-and-white chemical} \\ X_2 &\geq 20 \text{ tons of color chemical} \\ X_1 + X_2 &\geq 60 \text{ tons total} \\ X_1, X_2 &\geq 0 \text{ nonnegativity requirements} \end{aligned}$$

SOLUTION ► To solve the Cohen Chemicals problem graphically, we construct the problem's feasible region, shown in Figure B.9.

STUDENT TIP ◆

Now we look at three larger problems—ones that have more than two decision variables each and therefore are not graphed.

Linear Programming Applications

The foregoing examples each contained just two variables (X_1 and X_2). Most real-world problems (as we saw in the UPS *OM in Action* box) contain many more variables, however. Let's use the principles already developed to formulate a few more-complex problems. The practice you will get by “paraphrasing” the following LP situations should help develop your skills for applying linear programming to other common operations situations.

LO B.6 Formulate production-mix, diet, and labor scheduling problems

Production-Mix Example

Example B2 involves another *production-mix* decision. Limited resources must be allocated among various products that a firm produces. The firm's overall objective is to manufacture the selected products in such quantities as to maximize total profits.

Example B2

A PRODUCTION-MIX PROBLEM

Failsafe Electronics Corporation primarily manufactures four highly technical products, which it supplies to aerospace firms that hold NASA contracts. Each of the products must pass through the following departments before they are shipped: wiring, drilling, assembly, and inspection. The time requirements in each department (in hours) for each unit produced and its corresponding profit value are summarized in this table:

PRODUCT	DEPARTMENT				UNIT PROFIT
	WIRING	DRILLING	ASSEMBLY	INSPECTION	
XJ201	.5	3	2	.5	\$ 9
XM897	1.5	1	4	1.0	\$12
TR29	1.5	2	1	.5	\$15
BR788	1.0	3	2	.5	\$11

The production time available in each department each month and the minimum monthly production requirement to fulfill contracts are as follows:

DEPARTMENT	CAPACITY (HOURS)	PRODUCT	MINIMUM PRODUCTION LEVEL
Wiring	1,500	XJ201	150
Drilling	2,350	XM897	100
Assembly	2,600	TR29	200
Inspection	1,200	BR788	400

APPROACH ► Formulate this production-mix situation as an LP problem. The production manager first specifies production levels for each product for the coming month. He lets:

- X_1 = number of units of XJ201 produced
- X_2 = number of units of XM897 produced
- X_3 = number of units of TR29 produced
- X_4 = number of units of BR788 produced

SOLUTION ► The LP formulation is:

Objective: Maximize profit = $9X_1 + 12X_2 + 15X_3 + 11X_4$

subject to:

$$.5X_1 + 1.5X_2 + 1.5X_3 + 1X_4 \leq 1,500 \text{ hours of wiring available}$$

$$3X_1 + 1X_2 + 2X_3 + 3X_4 \leq 2,350 \text{ hours of drilling available}$$

$$2X_1 + 4X_2 + 1X_3 + 2X_4 \leq 2,600 \text{ hours of assembly available}$$

$$.5X_1 + 1X_2 + .5X_3 + .5X_4 \leq 1,200 \text{ hours of inspection}$$

$$X_1 \geq 150 \text{ units of XJ201}$$

$$X_2 \geq 100 \text{ units of XM897}$$

$$X_3 \geq 200 \text{ units of TR29}$$

$$X_4 \geq 400 \text{ units of BR788}$$

$$X_1, X_2, X_3, X_4 \geq 0$$

or:

$$4P_1 + 4P_2 + 4P_3 + 4P_4 + 4P_5 \leq 0.50(112)$$

$$F, P_1, P_2, P_3, P_4, P_5 \geq 0$$

There are two alternative optimal schedules that Mexico City Bank can follow. The first is to employ only 10 full-time tellers ($F = 10$) and to start 7 part-timers at 10 A.M. ($P_2 = 7$), 2 part-timers at 11 A.M. and noon ($P_3 = 2$ and $P_4 = 2$), and 3 part-timers at 1 P.M. ($P_5 = 3$). No part-timers would begin at 9 A.M.

The second solution also employs 10 full-time tellers, but starts 6 part-timers at 9 A.M. ($P_1 = 6$), 1 part-timer at 10 A.M. ($P_2 = 1$), 2 part-timers at 11 A.M. and noon ($P_3 = 2$ and $P_4 = 2$), and 3 part-timers at 1 P.M. ($P_5 = 3$). The cost of either of these two policies is \$1,086 per day.

INSIGHT ► It is not unusual for multiple optimal solutions to exist in large LP problems. In this case, it gives management the option of selecting, at the same cost, between schedules. To find an alternate optimal solution, you may have to enter the constraints in a different sequence.

LEARNING EXERCISE ► The bank decides to give part-time employees a raise to \$7 per hour. Does the solution change? [Answer: Yes, cost = \$1,142, $F = 10$, $P_1 = 6$, $P_2 = 1$, $P_3 = 2$, $P_4 = 5$, $P_5 = 0$.]

RELATED PROBLEMS ► B.36

The Simplex Method of LP

Most real-world linear programming problems have more than two variables and thus are too complex for graphical solution. A procedure called the **simplex method** may be used to find the optimal solution to such problems. The simplex method is actually an algorithm (or a set of instructions) with which we examine corner points in a methodical fashion until we arrive at the best solution—highest profit or lowest cost. Computer programs (such as Excel OM and POM for Windows) and Excel spreadsheets are available to solve linear programming problems via the simplex method.

For details regarding the algebraic steps of the simplex algorithm, see [Tutorial 3 at our text student download site](#) or in [MyOMLab](#), or refer to a management science textbook.²

Integer and Binary Variables

All the examples we have seen in this module so far have produced integer solutions. But it is very common to see LP solutions where the decision variables are not whole numbers. Computer software provides a simple way to guarantee only integer solutions. In addition, computers allow us to create special decision variables called **binary variables** that can only take on the values of 0 or 1. Binary variables allow us to introduce “yes-or-no” decisions into our linear programs and to introduce special logical conditions.

Creating Integer and Binary Variables

If we wish to ensure that decision variable values are integers rather than fractions, it is generally *not* good practice to simply round the solutions to the nearest integer values. The rounded solutions may not be optimal and, in fact, may not even be feasible. Fortunately, all LP software programs have simple ways to add constraints that enforce some or all of the decision variables to be either integer or binary. The main disadvantage of introducing such constraints is that larger programs may take longer to solve. The same LP that may take 3 seconds to solve on a computer could take several hours or more to solve if many of its variables are forced to be integer or binary. For relatively small programs, though, the difference may be unnoticeable.

Using Excel’s Solver (see *Using Software to Solve LP Problems* later in this module), integer and binary constraints can be added by clicking **Add** from the main Solver dialog box. Using the **Add Constraint** dialog box (see Program B.2), highlight the decision variables themselves under **Cell Reference**:. Then select **int** or **bin** to ensure that those variables are integer or binary, respectively, in the optimal solution.

Simplex method

An algorithm for solving linear programming problems of all sizes.

Binary variables

Decision variables that can only take on the value of 0 or 1.

Summary

This module introduces a special kind of model, linear programming. LP has proven to be especially useful when trying to make the most effective use of an organization's resources.

The first step in dealing with LP models is problem formulation, which involves identifying and creating an objective function and constraints. The second step is to solve

the problem. If there are only two decision variables, the problem can be solved graphically, using the corner-point method or the iso-profit/iso-cost line method. With either approach, we first identify the feasible region, then find the corner point yielding the greatest profit or least cost. LP is used in a wide variety of business applications, as the examples and homework problems in this module reveal.

Key Terms

Linear programming (LP) (p. 700)

Objective function (p. 701)

Constraints (p. 701)

Graphical solution approach (p. 702)

Decision variables (p. 702)

Feasible region (p. 703)

Iso-profit line method (p. 703)

Corner-point method (p. 705)

Parameter (p. 705)

Sensitivity analysis (p. 706)

Shadow price (or dual value) (p. 707)

Iso-cost (p. 708)

Simplex method (p. 713)

Binary variables (p. 713)

Discussion Questions

- List at least four applications of linear programming problems.
- What is a "corner point"? Explain why solutions to linear programming problems focus on corner points.
- Define the feasible region of a graphical LP problem. What is a feasible solution?
- Each linear programming problem that has a feasible region has an infinite number of solutions. Explain.
- Under what circumstances is the objective function more important than the constraints in a linear programming model?
- Under what circumstances are the constraints more important than the objective function in a linear programming model?
- Why is the diet problem, in practice, applicable for animals but not particularly for people?
- How many feasible solutions are there in a linear program? Which ones do we need to examine to find the optimal solution?
- Define shadow price (or dual value).
- Explain how to use the iso-cost line in a graphical minimization problem.
- Compare how the corner-point and iso-profit line methods work for solving graphical problems.
- Where a constraint crosses the vertical or horizontal axis, the quantity is fairly obvious. How does one go about finding the quantity coordinates where two constraints cross, not at an axis?
- Suppose a linear programming (maximization) problem has been solved and that the optimal value of the objective function is \$300. Suppose an additional constraint is added to this problem. Explain how this might affect each of the following:
 - The feasible region.
 - The optimal value of the objective function.

Using Software to Solve LP Problems

All LP problems can be solved with the simplex method, using software such as Excel, Excel OM, or POM for Windows.

✕ CREATING YOUR OWN EXCEL SPREADSHEETS

Excel offers the ability to analyze linear programming problems using built-in problem-solving tools. Excel's tool is named *Solver*.

We use Excel to set up the Glickman Electronics problem in Program B.3. The objective and constraints are repeated here:

Objective function: Maximize profit = \$7(No. of x-pods) + \$5(No. of BlueBerrys)

Subject to: $4(\text{x-pods}) + 3(\text{BlueBerrys}) \leq 240$

$2(\text{x-pods}) + 1(\text{BlueBerry}) \leq 100$

Program B.3

Using Excel to Formulate the Glickman Electronics Problem

	x-pods	BlueBerrys		
1 Glickman Electronics				
2 Using Linear Programming to Solve a Product Mix Problem				
3				
4				
5 Variable Values	0	0		
6 Objective Function	\$7.00	\$5.00	\$0.00	Max RHS
7 Subject to:				
8 Electronic Time Available	4	3	0	<= 240
9 Assembly Time Available	2	1	0	<= 100
10				

Annotations:

- The decisions (the number of units to produce) go here. (Points to cells B5:C5)
- These are simply labels. (Points to cells B4:C4)
- Action Copy D6 to D8:D9 (Points to cell D6)
- The objective function value (profit) goes here. (Points to cell D6)
- =B6*\$B\$5+C6*\$C\$5 (Points to cell D6)

PX USING EXCEL OM AND POM FOR WINDOWS

Excel OM and POM for Windows can handle relatively large LP problems. As output, the software provides optimal values for the variables, optimal profit or cost, and sensitivity analysis. In addition, POM for Windows provides graphical output for problems with only two variables.

Solved Problems

Virtual Office Hours help is available in [MyOMLab](#).

SOLVED PROBLEM B.1

Smith's, a Niagara, New York, clothing manufacturer that produces men's shirts and pajamas, has two primary resources available: sewing-machine time (in the sewing department) and cutting-machine time (in the cutting department). Over the next month, owner Barbara Smith can schedule up to 280 hours of work on sewing machines and up to 450 hours of work on cutting machines. Each shirt produced requires

1.00 hour of sewing time and 1.50 hours of cutting time. Producing each pair of pajamas requires .75 hours of sewing time and 2 hours of cutting time.

To express the LP constraints for this problem mathematically, we let:

X_1 = number of shirts produced

X_2 = number of pajamas produced

SOLUTION

First constraint: $1X_1 + .75X_2 \leq 280$ hours of sewing-machine time available—our first scarce resource

Second constraint: $1.5X_1 + 2X_2 \leq 450$ hours of cutting-machine time available—our second scarce resource

Note: This means that each pair of pajamas takes 2 hours of the cutting resource. Smith's accounting department analyzes cost and sales figures and states that each shirt produced will yield a \$4 contribution to profit and that each pair of pajamas will yield a \$3 contribution to profit.

This information can be used to create the LP *objective function* for this problem:

Objective function: Maximize total contribution to profit = $\$4X_1 + \$3X_2$

SOLVED PROBLEM B.2

We want to solve the following LP problem for Kevin Caskey Wholesale Inc. using the corner-point method:

Objective: Maximize profit = $\$9X_1 + \$7X_2$

Constraints: $2X_1 + 1X_2 \leq 40$

$X_1 + 3X_2 \leq 30$

$X_1, X_2 \geq 0$

SOLUTION

Figure B.10 illustrates these constraints:

Corner-point a: ($X_1 = 0, X_2 = 0$) Profit = 0

Corner-point b: ($X_1 = 0, X_2 = 10$) Profit = $9(0) + 7(10) = \$70$

Corner-point d: ($X_1 = 20, X_2 = 0$) Profit = $9(20) + 7(0) = \$180$

Corner-point c is obtained by solving equations $2X_1 + 1X_2 = 40$ and $X_1 + 3X_2 = 30$ simultaneously. Multiply the second equation by -2 and add it to the first.

$$\begin{array}{r} 2X_1 + 1X_2 = 40 \\ -2X_1 - 6X_2 = -60 \\ \hline -5X_2 = -20 \\ \text{Thus } X_2 = 4 \end{array}$$

And $X_1 + 3(4) = 30$ or $X_1 + 12 = 30$ or $X_1 = 18$

Corner-point c: ($X_1 = 18, X_2 = 4$) Profit = $9(18) + 7(4) = \$190$

Hence the optimal solution is:

$$(x_1 = 18, x_2 = 4) \quad \text{Profit} = \$190$$

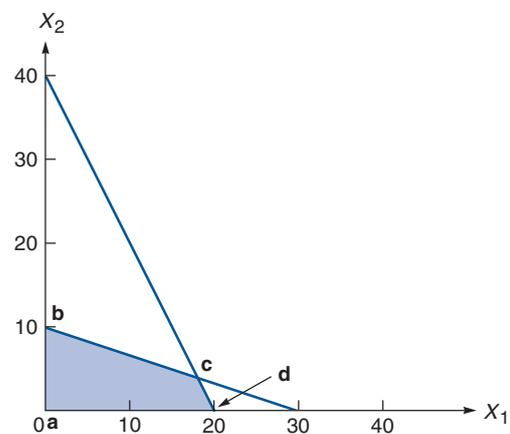


Figure B.10

K. Caskey Wholesale Inc.'s Feasible Region

Problems

Note: **Px** means the problem may be solved with POM for Windows and/or Excel OM.

Problem B.1 relates to Requirements of a Linear Programming Problem

- **B.1** The LP relationships that follow were formulated by Richard Martin at the Long Beach Chemical Company. Which ones are invalid for use in a linear programming problem, and why?

$$\begin{aligned} \text{Maximize} &= 6X_1 + \frac{1}{2}X_1X_2 + 5X_3 \\ \text{Subject to:} & \quad 4X_1X_2 + 2X_3 \leq 70 \\ & \quad 7.9X_1 - 4X_2 \geq 15.6 \\ & \quad 3X_1 + 3X_2 + 3X_3 \geq 21 \\ & \quad 19X_2 - \frac{1}{3}X_3 = 17 \\ & \quad -X_1 - X_2 + 4X_3 = 5 \\ & \quad 4X_1 + 2X_2 + 3\sqrt{X_3} \leq 80 \end{aligned}$$

Problems B.2–B.21 relate to Graphical Solution to a Linear Programming Problem

- **B.2** Solve the following linear programming problem graphically:

$$\begin{aligned} \text{Maximize profit} &= 4X + 6Y \\ \text{Subject to:} & \quad X + 2Y \leq 8 \\ & \quad 5X + 4Y \leq 20 \\ & \quad X, Y \geq 0 \quad \mathbf{Px} \end{aligned}$$

- **B.3** Solve the following linear programming problem graphically:

$$\begin{aligned} \text{Maximize profit} &= X + 10Y \\ \text{Subject to:} & \quad 4X + 3Y \leq 36 \\ & \quad 2X + 4Y \leq 40 \\ & \quad Y \geq 3 \\ & \quad X, Y \geq 0 \quad \mathbf{Px} \end{aligned}$$

- **B.4** Consider the following linear programming problem:

$$\begin{aligned} \text{Maximize profit} &= 30X_1 + 10X_2 \\ \text{Subject to:} & \quad 3X_1 + X_2 \leq 300 \\ & \quad X_1 + X_2 \leq 200 \\ & \quad X_1 \leq 100 \\ & \quad X_2 \geq 50 \\ & \quad X_1 - X_2 \leq 0 \\ & \quad X_1, X_2 \geq 0 \end{aligned}$$

- a) Solve the problem graphically.
b) Is there more than one optimal solution? Explain. **Px**

- **B.5** The Attaran Corporation manufactures two electrical products: portable air conditioners and portable heaters. The assembly process for each is similar in that both require a certain amount of wiring and drilling. Each air conditioner takes 3 hours of wiring and 2 hours of drilling. Each heater must go through 2 hours of wiring and 1 hour of drilling. During the next production period, 240 hours of wiring time are available and up to 140 hours of drilling time may be used. Each air conditioner sold yields a profit of \$25. Each heater assembled may be sold for a \$15 profit.

Formulate and solve this LP production-mix situation, and find the best combination of air conditioners and heaters that yields the highest profit. **Px**

- **B.6** The Chris Beehner Company manufactures two lines of designer yard gates, called model A and model B. Every gate requires blending a certain amount of steel and zinc; the company

has available a total of 25,000 lb of steel and 6,000 lb of zinc. Each model A gate requires a mixture of 125 lb of steel and 20 lb of zinc, and each yields a profit of \$90. Each model B gate requires 100 lb of steel and 30 lb of zinc and can be sold for a profit of \$70.

Find by graphical linear programming the best production mix of yard gates. **Px**

- **B.7** Green Vehicle Inc. manufactures electric cars and small delivery trucks. It has just opened a new factory where the C1 car and the T1 truck can both be manufactured. To make either vehicle, processing in the assembly shop and in the paint shop are required. It takes 1/40 of a day and 1/60 of a day to paint a truck of type T1 and a car of type C1 in the paint shop, respectively. It takes 1/50 of a day to assemble either type of vehicle in the assembly shop.

A T1 truck and a C1 car yield profits of \$300 and \$220, respectively, per vehicle sold.

- a) Define the objective function and constraint equations.
b) Graph the feasible region.
c) What is a maximum-profit daily production plan at the new factory?
d) How much profit will such a plan yield, assuming whatever is produced is sold? **Px**

- **B.8** The Lifang Wu Corporation manufactures two models of industrial robots, the Alpha 1 and the Beta 2. The firm employs 5 technicians, working 160 hours each per month, on its assembly line. Management insists that full employment (that is, *all* 160 hours of time) be maintained for each worker during next month's operations. It requires 20 labor-hours to assemble each Alpha 1 robot and 25 labor-hours to assemble each Beta 2 model. Wu wants to see at least 10 Alpha 1s and at least 15 Beta 2s produced during the production period. Alpha 1s generate a \$1,200 profit per unit, and Beta 2s yield \$1,800 each.

Determine the most profitable number of each model of robot to produce during the coming month. **Px**

- **B.9** Consider the following LP problem developed at Zafar Malik's Carbondale, Illinois, optical scanning firm:

$$\begin{aligned} \text{Maximize profit} &= \$1X_1 + \$1X_2 \\ \text{Subject to:} & \quad 2X_1 + 1X_2 \leq 100 \\ & \quad 1X_1 + 2X_2 \leq 100 \end{aligned}$$

- a) What is the optimal solution to this problem? Solve it graphically.
b) If a technical breakthrough occurred that raised the profit per unit of X_1 to \$3, would this affect the optimal solution?
c) Instead of an increase in the profit coefficient X_1 to \$3, suppose that profit was overestimated and should only have been \$1.25. Does this change the optimal solution? **Px**

- **B.10** A craftsman named William Barnes builds two kinds of birdhouses, one for wrens and a second for bluebirds. Each wren birdhouse takes 4 hours of labor and 4 units of lumber. Each bluebird house requires 2 hours of labor and 12 units of lumber. The craftsman has available 60 hours of labor and 120 units of lumber. Wren houses yield a profit of \$6 each, and bluebird houses yield a profit of \$15 each.

- a) Write out the objective and constraints.
b) Solve graphically. **Px**

TABLE B.3 Data for Problem B.40

FOOD	COST/SERVING	CALORIES/SERVING	% PROTEIN	% CARBS	% FAT	FRUIT/VEG
Applesauce	\$0.30	100	0%	100%	0%	Y
Canned corn	\$0.40	150	20%	80%	0%	Y
Fried chicken	\$0.90	250	55%	5%	40%	N
French fries	\$0.20	400	5%	35%	60%	N
Mac and cheese	\$0.50	430	20%	30%	50%	N
Turkey breast	\$1.50	300	67%	0%	33%	N
Garden salad	\$0.90	100	15%	40%	45%	Y

Formulate and solve as a linear problem. Print out your formulation in Excel showing the objective function coefficients and constraint matrix in standard form.

- ◆ Display, on a separate page, the full *Answer Report* as generated by Excel Solver.
- ◆ Highlight and label as *Z* the objective value for the optimal solution on the Answer Report.
- ◆ Highlight the nonzero decision variables for the optimal solution on the Answer Report.
- ◆ Display, on a separate page, the full *Sensitivity Report* as generated by Excel Solver. **Px**

Problems B.41–B.42 relate to Integer and Binary Variables

•• **B.41** Rollins Publishing needs to decide what textbooks from the following table to publish.

TEXT-BOOK	DEMAND	FIXED COST	VARIABLE COST	SELLING PRICE
Book 1	9,000	\$12,000	\$19	\$40
Book 2	8,000	\$21,000	\$28	\$60
Book 3	5,000	\$15,000	\$30	\$52
Book 4	6,000	\$10,000	\$20	\$34
Book 5	7,000	\$18,000	\$20	\$45

For each book, the maximum demand, fixed cost of publishing, variable cost, and selling price are provided. Rollins has the capacity to publish a total of 20,000 books.

a) Formulate this problem to determine which books should be selected and how many of each should be published to maximize profit.

b) Solve using computer software. **Px**

•• **B.42** Porter Investments needs to develop an investment portfolio for Mrs. Singh from the following list of possible investments:

INVESTMENT	COST	EXPECTED RETURN
A	\$10,000	\$ 700
B	\$12,000	\$1,000
C	\$ 3,500	\$ 390
D	\$ 5,000	\$ 500
E	\$ 8,500	\$ 750
F	\$ 8,000	\$ 640
G	\$ 4,000	\$ 300

Mrs. Singh has a total of \$60,000 to invest. The following conditions must be met: (1) If investment F is chosen, then investment G must also be part of the portfolio, (2) at least four investments should be chosen, and (3) of investments A and B, exactly one must be included. Formulate and solve this problem using LP software to determine which stocks should be included in Mrs. Singh’s portfolio. **Px**

CASE STUDIES

Quain Lawn and Garden, Inc.

Bill and Jeanne Quain spent a career as a husband-and-wife real estate investment partnership in Atlantic City, New Jersey. When they finally retired to a 25-acre farm in nearby Cape May County, they became ardent amateur gardeners. Bill planted shrubs and fruit trees, and Jeanne spent her hours potting all sizes of plants. When the volume of shrubs and plants reached the point that the Quains began to think of their hobby in a serious vein, they built a greenhouse adjacent to their home and installed heating and watering systems.

By 2012, the Quains realized their retirement from real estate had really only led to a second career—in the plant and shrub business—and they filed for a New Jersey business license. Within

a matter of months, they asked their attorney to file incorporation documents and formed the firm Quain Lawn and Garden, Inc.

Early in the new business’s existence, Bill Quain recognized the need for a high-quality commercial fertilizer that he could blend himself, both for sale and for his own nursery. His goal was to keep his costs to a minimum while producing a top-notch product that was especially suited to the New Jersey climate.

Working with chemists at Rutgers University, Quain blended “Quain-Grow.” It consists of four chemical compounds, C-30, C-92, D-21, and E-11. The cost per pound for each compound is indicated in the table on the next page:

CHEMICAL COMPOUND	COST PER POUND
C-30	\$.12
C-92	.09
D-21	.11
E-11	.04

The specifications for Quain-Grow are established as:

- Chemical E-11 must constitute at least 15% of the blend.
- C-92 and C-30 must together constitute at least 45% of the blend.

- D-21 and C-92 can together constitute no more than 30% of the blend.
- Quain-Grow is packaged and sold in 50-lb bags.

Discussion Questions

- Formulate an LP problem to determine what blend of the four chemicals will allow Quain to minimize the cost of a 50-lb bag of the fertilizer.
- Solve to find the best solution.

Scheduling Challenges at Alaska Airlines

Video Case

Good airline scheduling is essential to delivering outstanding customer service with high plane utilization rates. Airlines must schedule pilots, flight attendants, aircraft, baggage handlers, customer service agents, and ramp crews. At Alaska Airlines, it all begins with seasonal flight schedules that are developed 330 days in advance.

Revenue and marketing goals drive the potential routing decisions, but thousands of constraints impact these schedules. Using SABRE scheduling optimizer software, Alaska considers the number of planes available, seat capacity, ranges, crew availability, union contracts that dictate hours that crews can fly, and maintenance regulations that regularly take planes out of service, just to name a few. Alaska's scheduling department sends preliminary schedules to the human resources, maintenance, operations, customer service, marketing, and other departments for feedback before finalizing flight schedules.

Alaska Airlines' historic mission is to serve its extremely loyal customer base in the remote and unreachable small towns in Alaska. Serving many airports in Alaska is especially complex because the airline requires its pilots to have special skills to deal with extremely adverse weather, tight mountain passes, and short runways. Some airports lack full-time TSA agents or strong ground support and may not even be open 24 hours per day. In some cases, runways are not plowed because the village plow is busy clearing the roads for school buses. Navigational technology developed by Alaska Airlines has significantly reduced weather-related cancelled flights as Alaska can now land where many other carriers cannot.

After the SABRE optimizer schedules thousands of flights, scheduling activity turns to the next step: crew optimizing. The crew optimizer (Alaska uses Jeppesen software developed by Boeing and based on linear programming) attempts to eliminate unnecessary layovers and crew idle time while adhering to FAA and union restrictions. Alaska leads the industry in pilot "hard time" (i.e., the amount of time a pilot is being paid when passengers are actually being moved). After the crew requirements for every flight are determined, the 3,000 flight attendants and 1,500 pilots rank their preferred routings on a monthly basis. Personnel are assigned to each flight using seniority and feasibility.

Interestingly, not every pilot or flight attendant always bids on the Hawaii routes (about 20% of all flights), the long-haul East Coast routes, or the Mexico flights. Some prefer the flying challenge of the "milk run" flights to Ketchikan, Sitka, Juneau, Fairbanks, Anchorage, and back to Seattle, which are in keeping with the culture and contact with local residents.

As an airline that accentuates risk taking and empowers employees to think "out of the box," Alaska recently decided to experiment with a schedule change on its Seattle-to-Chicago route. Given crew restrictions on flying hours per day, the flight had previously included a crew layover in Chicago. When a company analyst documented the feasibility of running the same crew on the two 4-hour legs of the round trip (which implied an *extremely* tight turnaround schedule in Chicago), his data indicated that on 98.7% of the round trip flights, the crew would not "time out." His boss gave the go-ahead.

Discussion Questions*

- Why is scheduling for Alaska more complex than for other airlines?
- What operational considerations may prohibit Alaska from adding flights and more cities to its network?
- What were the risks of keeping the same crew on the Seattle—Chicago—Seattle route?
- Estimate the direct costs to the airline should the crew "time out" and not be able to fly its Boeing 737 back to Seattle from Chicago on the same day. These direct variable costs should include moving and parking the plane overnight along with hotel and meal costs for the crew and passengers. Do you think this is more advantageous than keeping a spare crew in Chicago?

*You may wish to view the video that accompanies this case before addressing these questions.

- Additional Case Studies:** Visit [MyOMLab](#) for these free case studies:

Chase Manhattan Bank: This scheduling case involves finding the optimal number of full-time versus part-time employees at a bank.

Coastal States Chemical: The company must prepare for a shortage of natural gas.

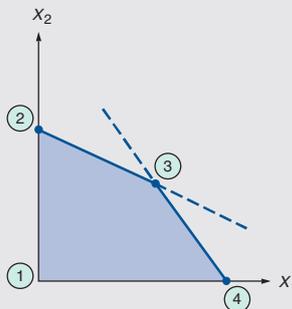
Endnotes

- Iso* means "equal" or "similar." Thus, an iso-profit line represents a line with all profits the same, in this case \$210.
- See, for example, Barry Render, Ralph M. Stair, Michael Hanna, and T. Hale, *Quantitative Analysis for Management*, 12th ed. (Pearson Education, Inc., Upper Saddle River, NJ, 2013):

Chapters 7–9; or Raju Balakrishnan, Barry Render, and Ralph M. Stair, *Managerial Decision Modeling with Spreadsheets*, 3rd ed. (Pearson Education, Inc., Upper Saddle River, NJ, 2012): Chapters 2–4.

Module B *Rapid Review*

Main Heading	Review Material	MyOMLab
WHY USE LINEAR PROGRAMMING? (p. 700)	<ul style="list-style-type: none"> ■ Linear programming (LP)—A mathematical technique designed to help operations managers plan and make decisions relative to allocation of resources. 	Concept Questions: 1.1–1.4 VIDEO B.1 Scheduling Challenges at Alaska Airlines
REQUIREMENTS OF A LINEAR PROGRAMMING PROBLEM (p. 701)	<ul style="list-style-type: none"> ■ Objective function—A mathematical expression in linear programming that maximizes or minimizes some quantity (often profit or cost, but any goal may be used). ■ Constraints—Restrictions that limit the degree to which a manager can pursue an objective. <p>All LP problems have four properties in common:</p> <ol style="list-style-type: none"> 1. LP problems seek to <i>maximize</i> or <i>minimize</i> some quantity. We refer to this property as the <i>objective function</i> of an LP problem. 2. The presence of restrictions, or <i>constraints</i>, limits the degree to which we can pursue our objective. We want, therefore, to maximize or minimize a quantity (the objective function) subject to limited resources (the constraints). 3. There must be <i>alternative courses of action</i> to choose from. 4. The objective and constraints in linear programming problems must be expressed in terms of <i>linear equations</i> or inequalities. 	Concept Questions: 2.1–2.4 Problem: B.1
FORMULATING LINEAR PROGRAMMING PROBLEMS (pp. 701–702)	<p>One of the most common linear programming applications is the <i>product-mix problem</i>. Two or more products are usually produced using limited resources. For example, a company might like to determine how many units of each product it should produce to maximize overall profit, given its limited resources.</p> <p>An important aspect of linear programming is that certain interactions will exist between variables. The more units of one product that a firm produces, the fewer it can make of other products.</p>	Concept Questions: 3.1–3.4 Virtual Office Hours for Solved Problem: B.1 ACTIVE MODEL B.1
GRAPHICAL SOLUTION TO A LINEAR PROGRAMMING PROBLEM (pp. 702–705)	<ul style="list-style-type: none"> ■ Graphical solution approach—A means of plotting a solution to a two-variable problem on a graph. ■ Decision variables—Choices available to a decision maker. <p>Constraints of the form $X \geq 0$ are called <i>nonnegativity constraints</i>.</p> <ul style="list-style-type: none"> ■ Feasible region—The set of all feasible combinations of decision variables. Any point inside the feasible region represents a <i>feasible solution</i>, while any point outside the feasible region represents an <i>infeasible solution</i>. ■ Iso-profit line method—An approach to identifying the optimum point in a graphic linear programming problem. The line that touches a particular point of the feasible region will pinpoint the optimal solution. ■ Corner-point method—Another method for solving graphical linear programming problems. <p>The mathematical theory behind linear programming states that an optimal solution to any problem will lie at a <i>corner point</i>, or an <i>extreme point</i>, of the feasible region. Hence, it is necessary to find only the values of the variables at each corner; the optimal solution will lie at one (or more) of them. This is the corner-point method.</p>	Concept Questions: 4.1–4.4 Problems: B.2–B.21 Virtual Office Hours for Solved Problem: B.2
SENSITIVITY ANALYSIS (pp. 705–708)	<ul style="list-style-type: none"> ■ Parameter—A numerical value that is given in a model. ■ Sensitivity analysis—An analysis that projects how much a solution may change if there are changes in the variables or input data. <p>Sensitivity analysis is also called <i>postoptimality analysis</i>. There are two approaches to determining just how sensitive an optimal solution is to changes: (1) a trial-and-error approach and (2) the analytic postoptimality method.</p>	Concept Questions: 5.1–5.4 Problems: B.22–B.24



Module B **Rapid Review** *continued*

MyOMLab

Main Heading	Review Material	
	<p>To use the analytic postoptimality method, after an LP problem has been solved, we determine a range of changes in problem parameters that will not affect the optimal solution or change the variables in the solution. LP software has this capability.</p> <p>While using the information in a sensitivity report to answer what-if questions, we assume that we are considering a change to only a <i>single</i> input data value at a time. That is, the sensitivity information does not generally apply to simultaneous changes in several input data values.</p> <ul style="list-style-type: none"> ▪ Shadow price (or dual value)—The value of one additional unit of a scarce resource in LP. <p>The shadow price is valid as long as the right-hand side of the constraint stays in a range within which all current corner points continue to exist. The information to compute the upper and lower limits of this range is given by the entries labeled Allowable Increase and Allowable Decrease in the sensitivity report.</p>	
SOLVING MINIMIZATION PROBLEMS (pp. 708–709)	<ul style="list-style-type: none"> ▪ Iso-cost—An approach to solving a linear programming minimization problem graphically. <p>The iso-cost line approach to solving minimization problems is analogous to the iso-profit approach for maximization problems, but successive iso-cost lines are drawn <i>inward</i> instead of outward.</p>	<p>Concept Questions: 6.1–6.3</p> <p>Problems: B.25–B.33</p> <p>Virtual Office Hours for Solved Problem: B.3</p>
LINEAR PROGRAMMING APPLICATIONS (pp. 710–713)	<p>The <i>diet problem</i>, known in agricultural applications as the <i>feed-mix problem</i>, involves specifying a food or feed ingredient combination that will satisfy stated nutritional requirements at a minimum cost level.</p> <p><i>Labor scheduling problems</i> address staffing needs over a specific time period. They are especially useful when managers have some flexibility in assigning workers to jobs that require overlapping or interchangeable talents.</p>	<p>Concept Questions: 7.1–7.3</p> <p>Problems: B.34–B.40</p>
THE SIMPLEX METHOD OF LP (p. 713)	<ul style="list-style-type: none"> ▪ Simplex method—An algorithm for solving linear programming problems of all sizes. <p>The simplex method is actually a set of instructions with which we examine corner points in a methodical fashion until we arrive at the best solution—highest profit or lowest cost. Computer programs (such as Excel OM and POM for Windows) and Excel’s Solver add-in are available to solve linear programming problems via the simplex method.</p>	<p>Concept Questions: 8.1–8.2</p> <p>Virtual Office Hours for Solved Problem: C.1 (note that this Module C video is an LP application of the transportation problem)</p>
INTEGER AND BINARY VARIABLES (pp. 713–715)	<ul style="list-style-type: none"> ▪ Binary variables—Decision variables that can only take on the value of 0 or 1. <p>Using computer software, decision variables for linear programs can be forced to be integer or even binary. Binary variables extend the flexibility of linear programs to include such options as mutually exclusive alternatives, either-or constraints, contingent decisions, fixed-charge problems, and threshold levels.</p>	<p>Concept Questions: 9.1–9.4</p> <p>Problems: B.41–B.42</p>

Self Test

▪ **Before taking the self-test**, refer to the learning objectives listed at the beginning of the module and the key terms listed at the end of the module.

- LO B.1** Which of the following is *not* a valid LP constraint formulation?
 a) $3X + 4Y \leq 12$ b) $2X \times 2Y \leq 12$
 c) $3Y + 2Z = 18$ d) $100 \geq X + Y$
 e) $2.5X + 1.5Z = 30.6$
- LO B.2** Using a *graphical solution procedure* to solve a maximization problem requires that we:
 a) move the iso-profit line up until it no longer intersects with any constraint equation.
 b) move the iso-profit line down until it no longer intersects with any constraint equation.
 c) apply the method of simultaneous equations to solve for the intersections of constraints.
 d) find the value of the objective function at the origin.
- LO B.3** Consider the following linear programming problem:
 Maximize $4X + 10Y$
 Subject to: $3X + 4Y \leq 480$
 $4X + 2Y \leq 360$
 $X, Y \geq 0$
- The feasible corner points are (48,84), (0,120), (0,0), and (90,0). What is the maximum possible value for the objective function?
 a) 1,032 b) 1,200 c) 360 d) 1,600 e) 840
- LO B.4** A zero shadow price for a resource ordinarily means that:
 a) the resource is scarce.
 b) the resource constraint was redundant.
 c) the resource has not been used up.
 d) something is wrong with the problem formulation.
 e) none of the above.
- LO B.5** For these two constraints, which point is in the feasible region of this minimization problem?
 $14x + 6y \geq 42$ and $x + y \geq 3$
 a) $x = -1, y = 1$ b) $x = 0, y = 4$
 c) $x = 2, y = 1$ d) $x = 5, y = 1$
 e) $x = 2, y = 0$
- LO B.6** When applying LP to diet problems, the objective function is usually designed to:
 a) maximize profits from blends of nutrients.
 b) maximize ingredient blends.
 c) minimize production losses.
 d) maximize the number of products to be produced.
 e) minimize the costs of nutrient blends.

Answers: LO B.1. b; LO B.2. a; LO B.3. b; LO B.4. c; LO B.5. d; LO B.6. e.