

GLOBAL
EDITION 

Chapter 5

Discrete Probability Distributions

Business Statistics

A First Course

SEVENTH EDITION

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Objectives

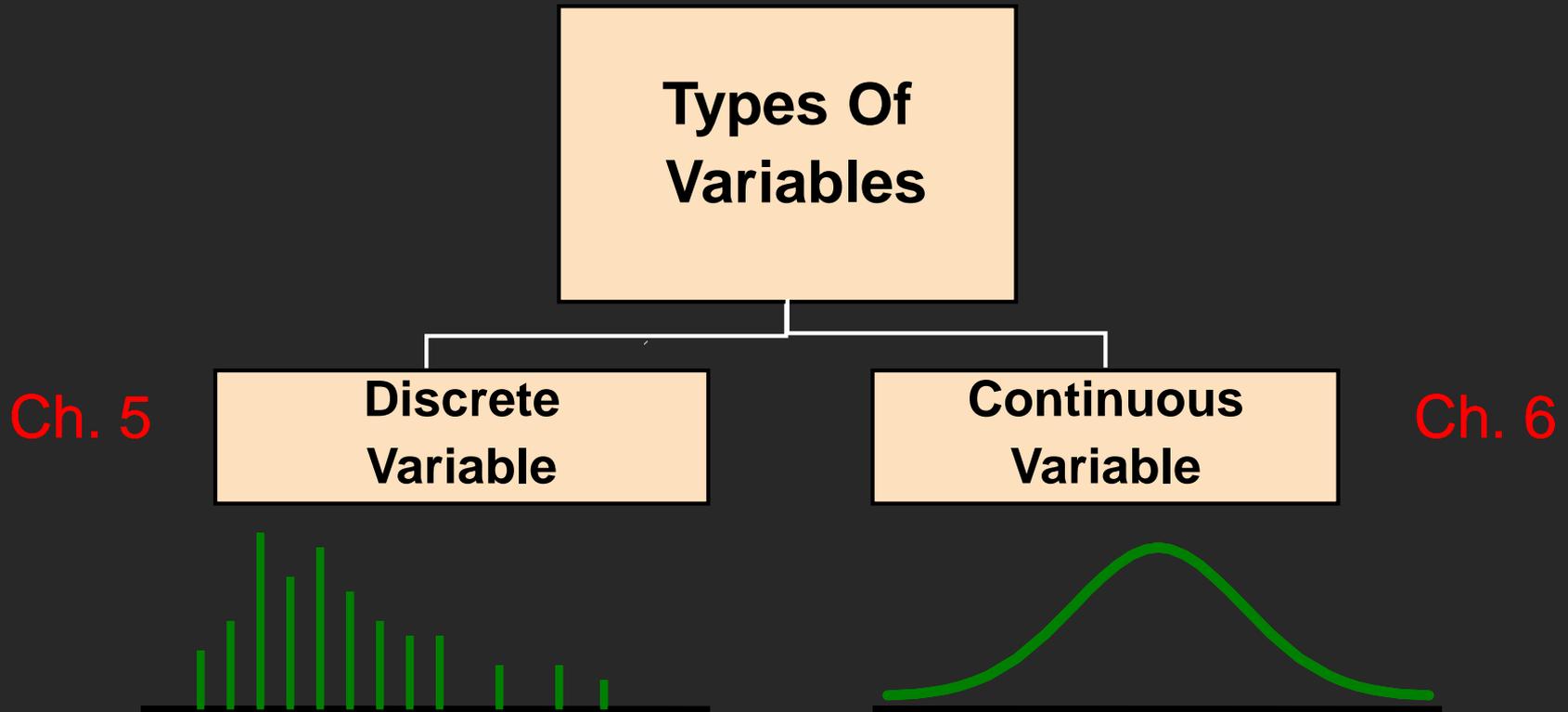
In this chapter, you learn:

- The properties of a probability distribution.
- To compute the expected value and variance of a probability distribution.
- To compute probabilities from binomial, and Poisson distributions.
- To use the binomial, and Poisson distributions to solve business problems

Definitions

- **Discrete** variables produce outcomes that come from a counting process (e.g. number of classes you are taking).
- **Continuous** variables produce outcomes that come from a measurement (e.g. your annual salary, or your weight).

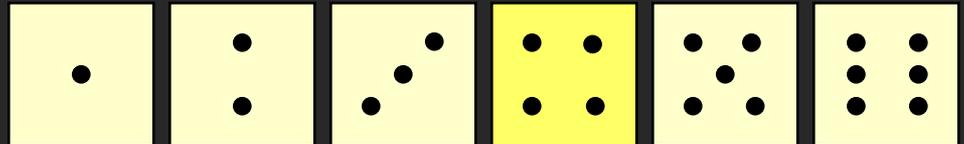
Types Of Variables



Discrete Variables

- Can only assume a countable number of values

Examples:



- Roll a die twice

Let X be the number of times 4 occurs
(then X could be 0, 1, or 2 times)

- Toss a coin 5 times.

Let X be the number of heads
(then $X = 0, 1, 2, 3, 4, \text{ or } 5$)

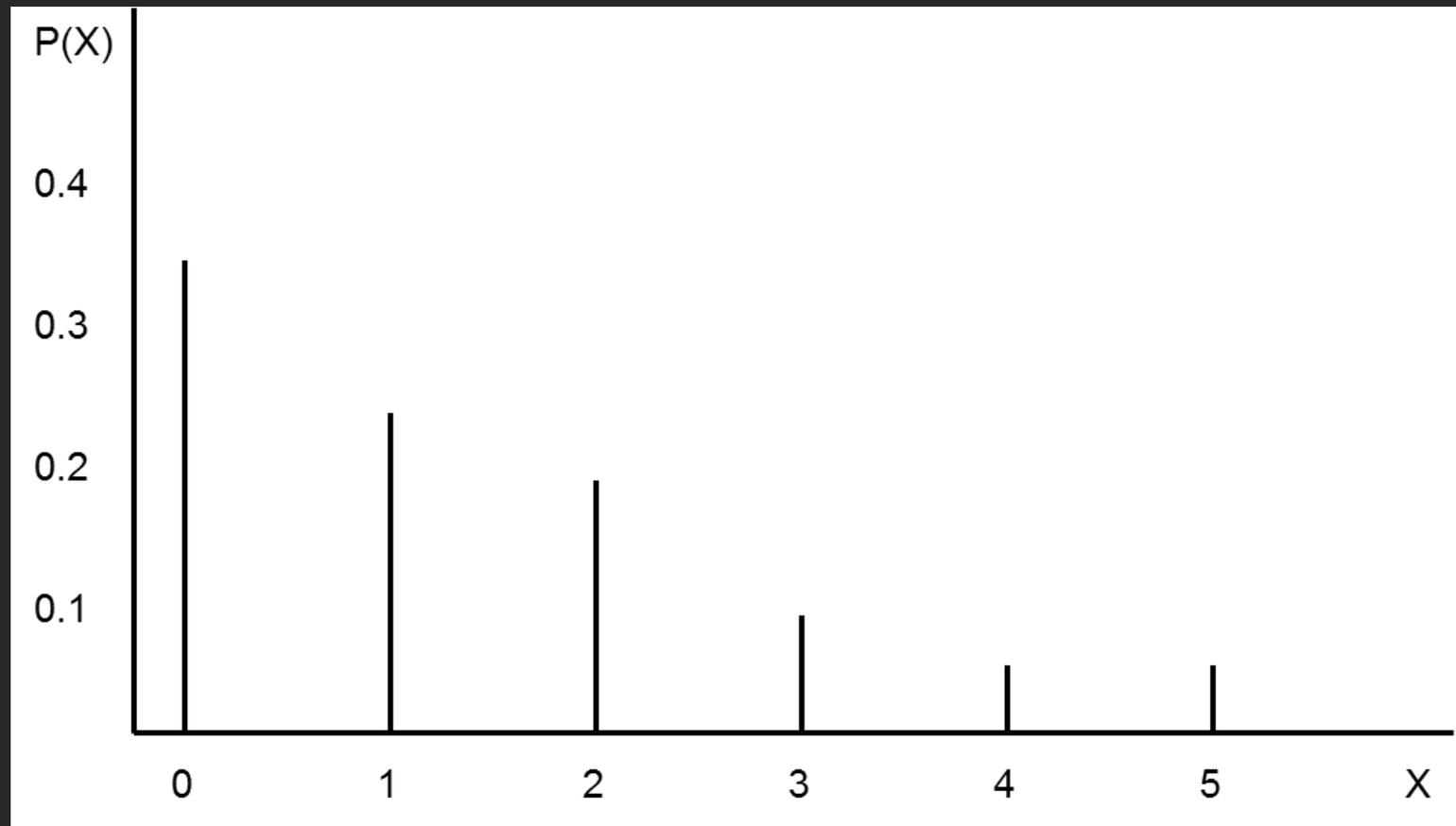


Probability Distribution For Discrete Variables

- A **probability distribution for a discrete variable** is a mutually exclusive listing of all possible numerical outcomes for that variable and a probability of occurrence associated with each outcome.

Interruptions Per Day In Computer Network	Probability
0	0.35
1	0.25
2	0.20
3	0.10
4	0.05
5	0.05

Probability Distributions Are Often Represented Graphically



Discrete Variables Expected Value (Measuring Center)

- Expected Value (or mean) of a discrete variable (Weighted Average)

$$\mu = E(X) = \sum_{i=1}^N x_i P(X = x_i)$$

Interrupti ons (x_i)	Probability $P(X = x_i)$	$x_i P(X = x_i)$
0	0.35	(0)(0.35) = 0.00
1	0.25	(1)(0.25) = 0.25
2	0.20	(2)(0.20) = 0.40
3	0.10	(3)(0.10) = 0.30
4	0.05	(4)(0.05) = 0.20
5	0.05	(5)(0.05) = 0.25
	1.00	$\mu = E(X) = 1.40$

Discrete Variables: Measuring Dispersion

- Variance of a discrete variable

$$\sigma^2 = \sum_{i=1}^N [x_i - E(X)]^2 P(X = x_i)$$

- Standard Deviation of a discrete variable

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^N [x_i - E(X)]^2 P(X = x_i)}$$

where:

$E(X)$ = Expected value of the discrete variable X

x_i = the i^{th} outcome of X

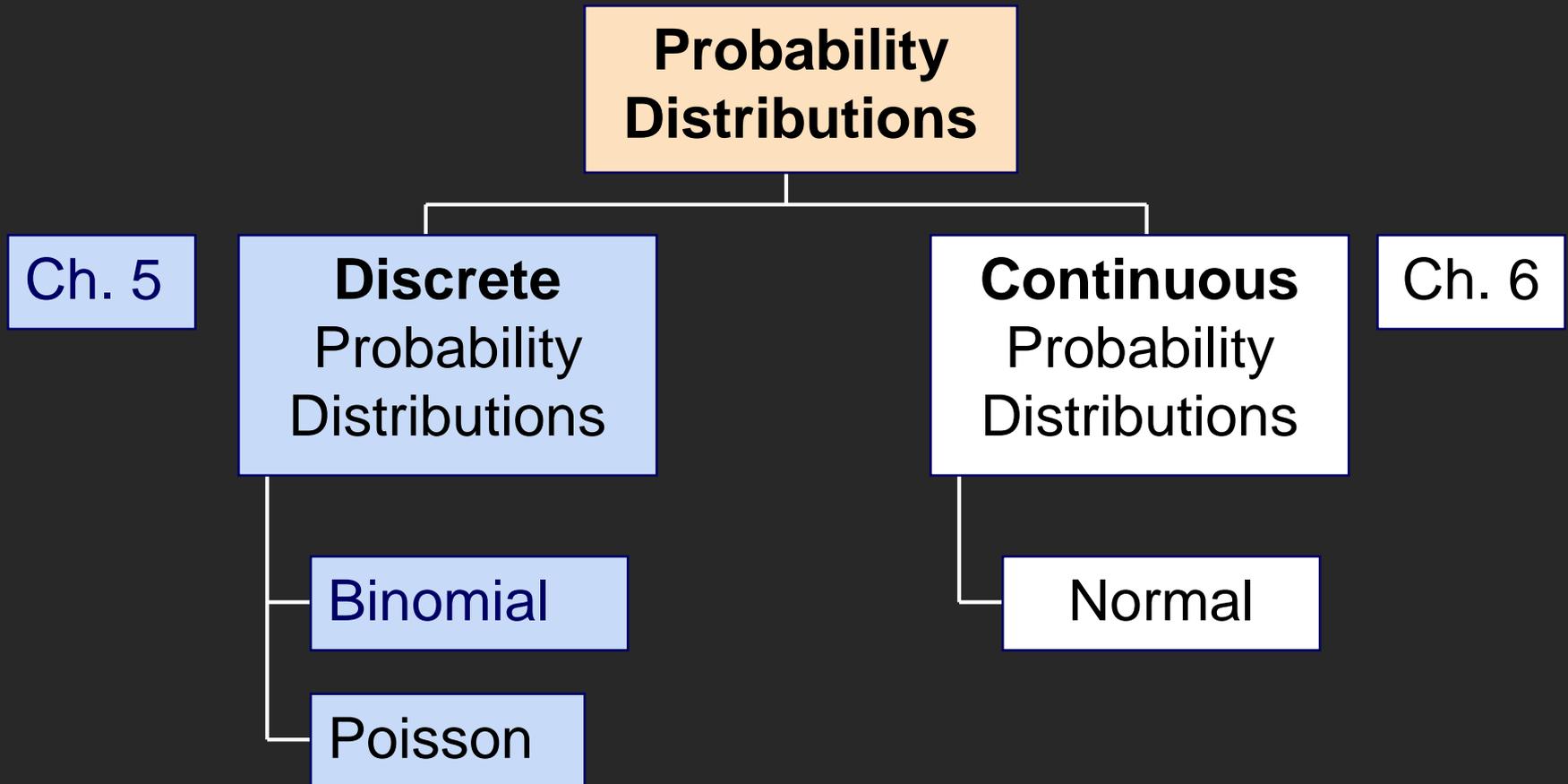
$P(X=x_i)$ = Probability of the i^{th} occurrence of X

Discrete Variables: Measuring Dispersion

(continued)

Interruptions Per Day In Computer Network (x_i)	Probability $P(X = x_i)$	$[x_i - E(X)]^2$	$[x_i - E(X)]^2 P(X = x_i)$
0	0.35	$(0 - 1.4)^2 = 1.96$	$(1.96)(0.35) = 0.686$
1	0.25	$(1 - 1.4)^2 = 0.16$	$(0.16)(0.25) = 0.040$
2	0.20	$(2 - 1.4)^2 = 0.36$	$(0.36)(0.20) = 0.072$
3	0.10	$(3 - 1.4)^2 = 2.56$	$(2.56)(0.10) = 0.256$
4	0.05	$(4 - 1.4)^2 = 6.76$	$(6.76)(0.05) = 0.338$
5	0.05	$(5 - 1.4)^2 = 12.96$	$(12.96)(0.05) = 0.648$
			$\sigma^2 = 2.04, \sigma = 1.4283$

Probability Distributions



Binomial Probability Distribution

- A fixed number of observations, n
- e.g., 15 tosses of a coin; ten light bulbs taken from a warehouse
- Each observation is categorized as to whether or not the “event of interest” occurred
 - e.g., head or tail in each toss of a coin; defective or not defective light bulb
 - Since these two categories are mutually exclusive and collectively exhaustive
 - When the probability of the event of interest is represented as π , then the probability of the event of interest not occurring is $1 - \pi$

Binomial Probability Distribution

(continued)

- **Constant probability** for the event of interest occurring (π) for each observation
 - Probability of getting a tail is the same each time we toss the coin
- Observations are **independent**
 - The outcome of one observation does not affect the outcome of the other
 - Two sampling methods deliver independence
 - Infinite population without replacement
 - Finite population with replacement

Possible Applications for the Binomial Distribution

- A manufacturing plant labels items as either defective or acceptable
- A firm bidding for contracts will either get a contract or not
- A marketing research firm receives survey responses of “yes I will buy” or “no I will not”
- New job applicants either accept the offer or reject it

The Binomial Distribution

Counting Techniques

- Suppose the event of interest is obtaining heads on the toss of a fair coin. You are to toss the coin three times. In how many ways can you get two heads?
- Possible ways: **HHT**, **HTH**, **THH**, so there are three ways you can get two heads.
- This situation is fairly simple. We need to be able to count the number of ways for more complicated situations.

Counting Techniques Rule of Combinations

- The number of combinations of selecting x objects out of n objects is

$${}_n C_x = \frac{n!}{x!(n-x)!}$$

where:

$$n! = (n)(n-1)(n-2) \cdots (2)(1)$$

$$x! = (X)(X-1)(X-2) \cdots (2)(1)$$

$$0! = 1 \quad (\text{by definition})$$

Counting Techniques Rule of Combinations

- How many possible 3 scoop combinations could you create at an ice cream parlor if you have 31 flavors to select from and no flavor can be used more than once in the 3 scoops?
- The total choices is $n = 31$, and we select $X = 3$.

$${}_{31}C_3 = \frac{31!}{3!(31-3)!} = \frac{31!}{3!28!} = \frac{31 \cdot 30 \cdot 29 \cdot 28!}{3 \cdot 2 \cdot 1 \cdot 28!} = 31 \cdot 5 \cdot 29 = 4,495$$

Binomial Distribution Formula

$$P(X=x | n, \pi) = \frac{n!}{x! (n-x)!} \pi^x (1-\pi)^{n-x}$$

$P(X=x|n,\pi)$ = probability of x events of interest in n trials, with the probability of an “event of interest” being π for each trial

x = number of “events of interest” in sample, ($x = 0, 1, 2, \dots, n$)

n = sample size (number of trials or observations)

π = probability of “event of interest”

Example: Flip a coin four times, let $x = \#$ heads:

$$n = 4$$

$$\pi = 0.5$$

$$1 - \pi = (1 - 0.5) = 0.5$$

$$X = 0, 1, 2, 3, 4$$

Binomial Distribution Summary Measures

- Mean

$$\mu = E(X) = n\pi$$

- Variance and Standard Deviation

$$\sigma^2 = n\pi(1 - \pi)$$

$$\sigma = \sqrt{n\pi(1 - \pi)}$$

Where n = sample size

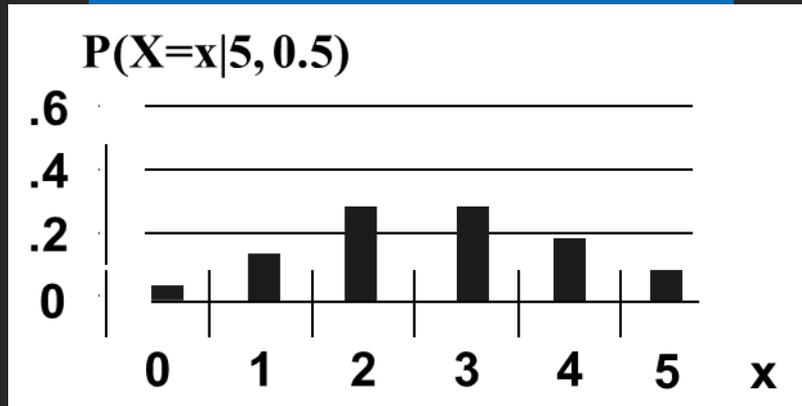
π = probability of the event of interest for any trial

$(1 - \pi)$ = probability of no event of interest for any trial

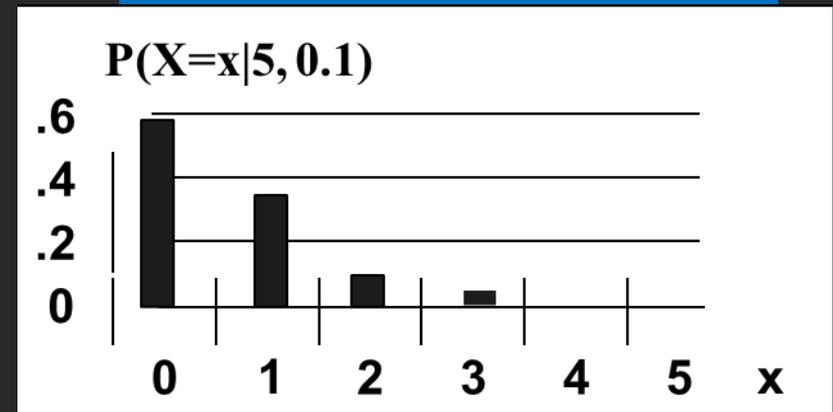
The Binomial Distribution Shape

- The shape of the binomial distribution depends on the values of π and n

■ $n = 5$ and $\pi = .5$



■ $n = 5$ and $\pi = .1$

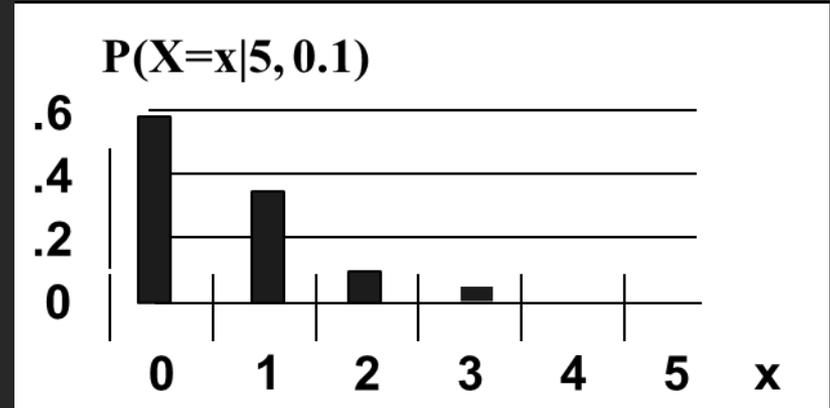


The Binomial Distribution Characteristics

Examples

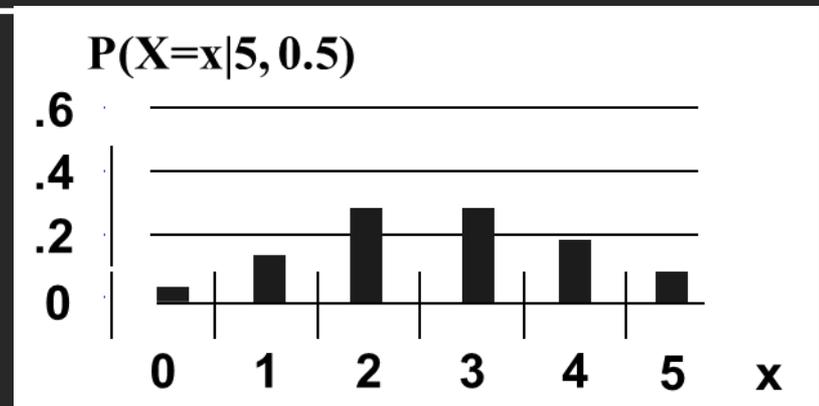
$$\mu = n\pi = (5)(.1) = 0.5$$

$$\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{(5)(.1)(1-.1)} \\ = 0.6708$$



$$\mu = n\pi = (5)(.5) = 2.5$$

$$\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{(5)(.5)(1-.5)} \\ = 1.118$$



Example: Calculating a Binomial Probability

What is the probability of one success in five observations if the probability of an event of interest is 0.1?

$$x = 1, n = 5, \text{ and } \pi = 0.1$$

$$\begin{aligned} P(X = 1 | 5, 0.1) &= \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x} \\ &= \frac{5!}{1!(5-1)!} (0.1)^1 (1-0.1)^{5-1} \\ &= (5)(0.1)(0.9)^4 \\ &= 0.32805 \end{aligned}$$

The Binomial Distribution: Example

Suppose the probability of purchasing a defective computer is 0.02.

What is the probability of purchasing 2 defective computers in a group of 10?

$$x = 2, n = 10, \text{ and } \pi = 0.02$$

$$\begin{aligned} P(X = 2 | 10, 0.02) &= \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x} \\ &= \frac{10!}{2!(10-2)!} (.02)^2 (1-.02)^{10-2} \\ &= (45)(.0004)(.8508) \\ &= .01531 \end{aligned}$$

The Binomial Distribution Using Binomial Tables (Available On Line)

n = 10									
x	...	$\pi=.20$	$\pi=.25$	$\pi=.30$	$\pi=.35$	$\pi=.40$	$\pi=.45$	$\pi=.50$	
0	...	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010	10
1	...	0.2684	0.1877	0.1211	0.0725	0.0403	0.0207	0.0098	9
2	...	0.3020	0.2816	0.2335	0.1757	0.1209	0.0763	0.0439	8
3	...	0.2013	0.2503	0.2668	0.2522	0.2150	0.1665	0.1172	7
4	...	0.0881	0.1460	0.2001	0.2377	0.2508	0.2384	0.2051	6
5	...	0.0264	0.0584	0.1029	0.1536	0.2007	0.2340	0.2461	5
6	...	0.0055	0.0162	0.0368	0.0689	0.1115	0.1596	0.2051	4
7	...	0.0008	0.0031	0.0090	0.0212	0.0425	0.0746	0.1172	3
8	...	0.0001	0.0004	0.0014	0.0043	0.0106	0.0229	0.0439	2
9	...	0.0000	0.0000	0.0001	0.0005	0.0016	0.0042	0.0098	1
10	...	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0010	0
	...	$\pi=.80$	$\pi=.75$	$\pi=.70$	$\pi=.65$	$\pi=.60$	$\pi=.55$	$\pi=.50$	x

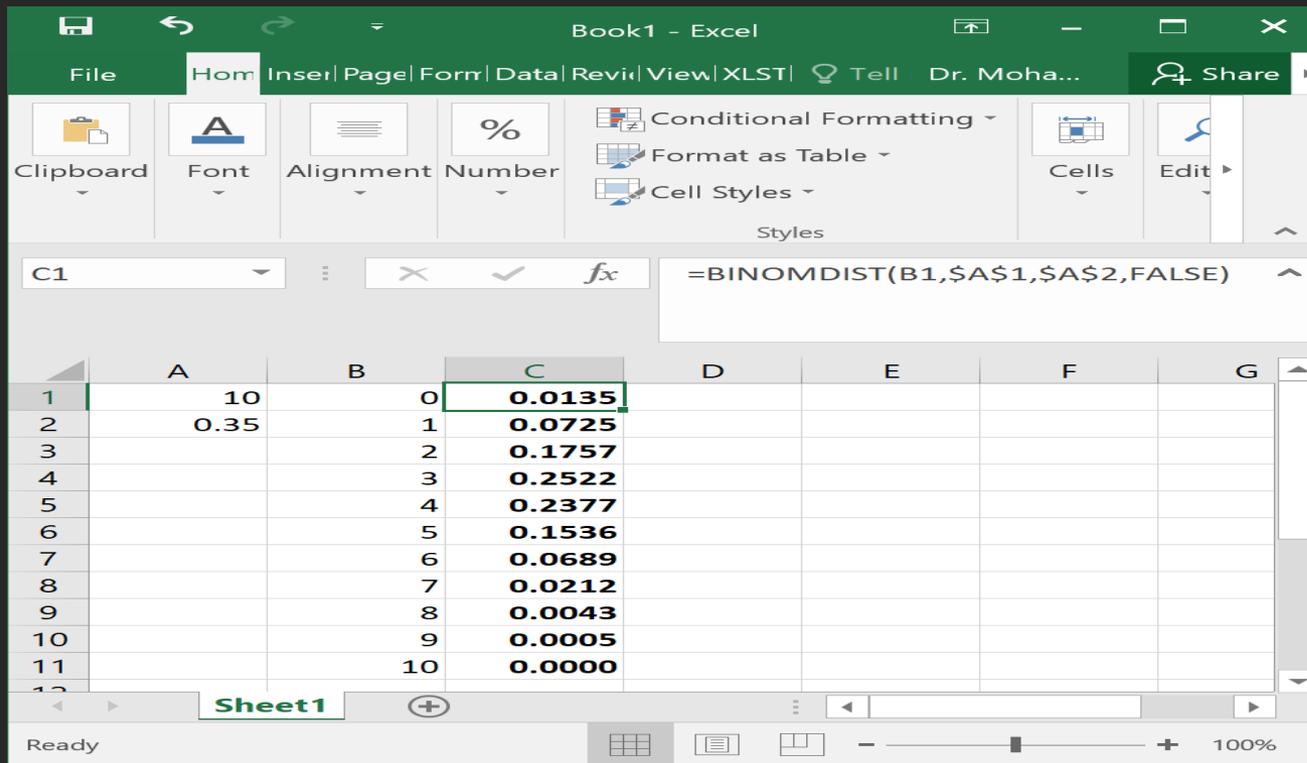
Examples:

$n = 10, \pi = 0.35, x = 3: P(X = 3|10, 0.35) = 0.2522$

$n = 10, \pi = 0.75, x = 8: P(X = 8|10, 0.75) = 0.2816$

Using Excel

=BINOM.DIST(B1,A1,A2,FALSE)



Poisson Distribution

- You use the **Poisson distribution** when you are interested in the number of times an event occurs in a given area of opportunity.
- An **area of opportunity** is a continuous unit or interval of time, volume, or such area in which more than one occurrence of an event can occur.
 - The number of scratches in a car's paint
 - The number of mosquito bites on a person
 - The number of computer crashes in a day

The Poisson Distribution

(continued)

- Apply the Poisson Distribution when:
 - You wish to count the number of times an event occurs in a given area of opportunity
 - The probability that an event occurs in one area of opportunity is the same for all areas of opportunity
 - The number of events that occur in one area of opportunity is independent of the number of events that occur in the other areas of opportunity
 - The probability that two or more events occur in an area of opportunity approaches zero as the area of opportunity becomes smaller
 - **The average number of events per unit is λ (lambda)**

Poisson Distribution Formula

$$P(X = x | \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}$$

where:

x = number of events in an area of opportunity

λ = expected number of events

e = base of the natural logarithm system (2.71828...)

Poisson Distribution Summary Measures

- Mean

$$\mu = \lambda$$

- Variance and Standard Deviation

$$\sigma^2 = \lambda$$

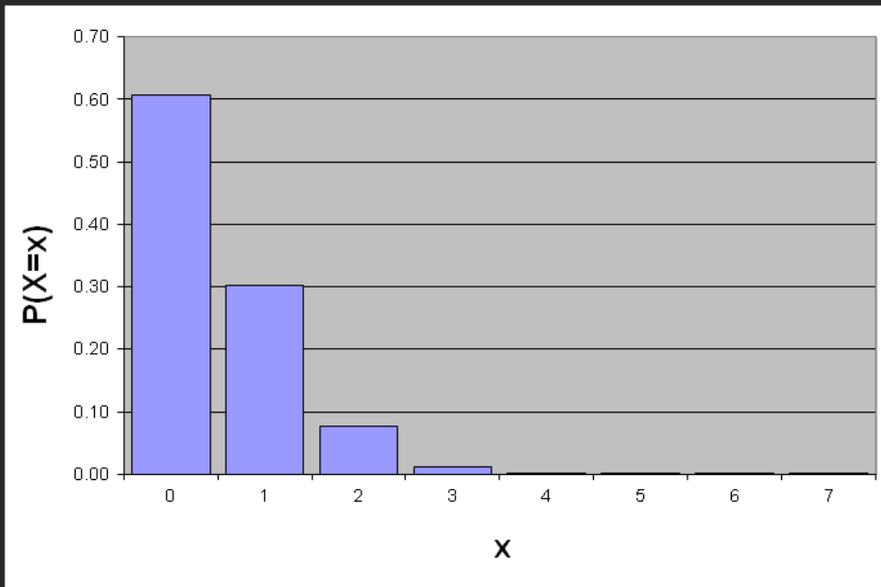
$$\sigma = \sqrt{\lambda}$$

where λ = expected number of events

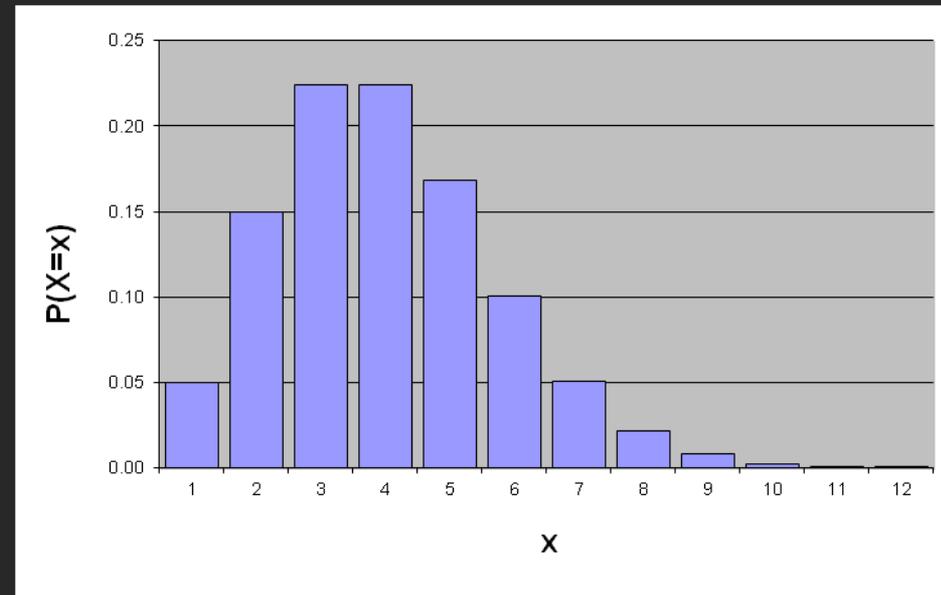
Poisson Distribution Shape

- The shape of the Poisson Distribution depends on the parameter λ :

$$\lambda = 0.50$$



$$\lambda = 3.00$$



Using Poisson Tables (Available On Line)

x	λ								
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494
4	0.0000	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111
5	0.0000	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Example: Find $P(X = 2 \mid \lambda = 0.50)$

$$P(X = 2 \mid 0.50) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.50} (0.50)^2}{2!} = 0.0758$$

Using Excel

=POISSON.DIST(B1,A1,FALSE)

	A	B	C	D	E	F
1	0.5	0	0.6065			
2		1	0.3033			
3		2	0.0758			
4		3	0.0126			
5		4	0.0016			
6		5	0.0002			
7		6	0.0000			
8		7	0.0000			

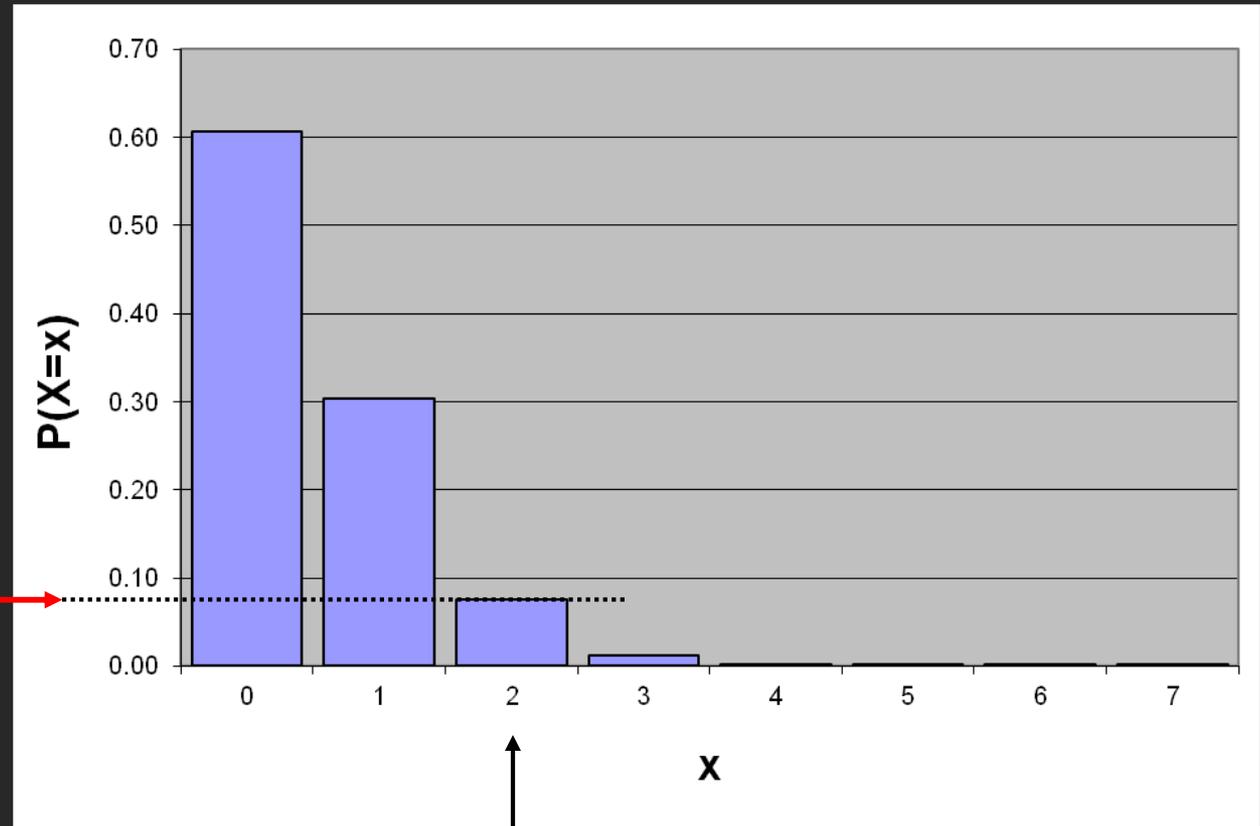
Average: 0.1250 Count: 8 Sum: 1.0000

Graph of Poisson Probabilities

Graphically:

$\lambda = 0.50$

X	$\lambda = 0.50$
0	0.6065
1	0.3033
2	0.0758
3	0.0126
4	0.0016
5	0.0002
6	0.0000
7	0.0000



$$P(X = 2 \mid \lambda=0.50) = 0.0758$$

The Poisson Distribution: Example

- A bank is interested in studying the number of people who use the ATM located outside its office late at night.
 - On average, 1.6 customers walk up to the ATM during any 10 minute interval between 9pm and midnight.
1. What is the probability of exactly 3 customers using th ATM during any 10 minute interval?
 2. What is the probability of 3 or fewer customers using th ATM during any 10 minute interval?

The Poisson Distribution: Example

(continued)

- We have the discrete random variable: **number of people who use the ATM**

$$\lambda = 1.6$$

1. Find $P(X = 3 \mid \lambda = 1.6)$?
2. Find $P(X \leq 3 \mid \lambda = 1.6)$?

The Poisson Distribution: Example

(continued)

$$P(X = 3 | 1.6) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-1.6} (1.6)^3}{3!} = 0.1378$$

$$P(X \leq 3 | \lambda = 1.6) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$\frac{e^{-1.6} (1.6)^0}{0!} + \frac{e^{-1.6} (1.6)^1}{1!} + \frac{e^{-1.6} (1.6)^2}{2!} + \frac{e^{-1.6} (1.6)^3}{3!}$$

$$= 0.2019 + 0.3230 + 0.2584 + 0.1378$$

$$= 0.9212$$

	A	B	C	D	E	F	G
1	1.6	0	0.2019				
2		1	0.3230				
3		2	0.2584				
4		3	0.1378				
5							

Chapter Summary

In this chapter we covered:

- The properties of a probability distribution.
- To compute the expected value and variance of a probability distribution.
- To compute probabilities from binomial, and Poisson distributions.
- To use the binomial, and Poisson distributions to solve business problems