Planes and Surfaces

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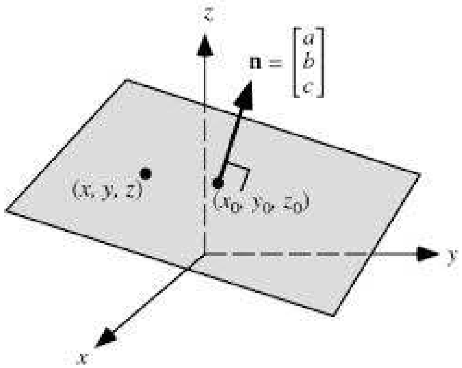
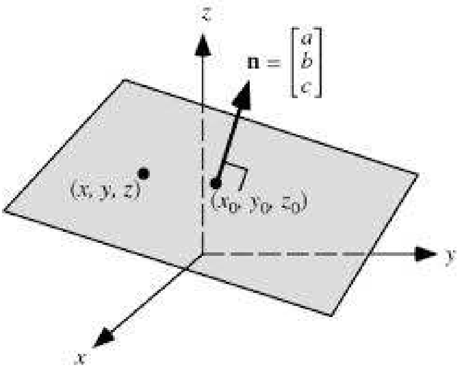

z
n = [a b c]
(x, y, z)
(x0, y0, z0)
y
x



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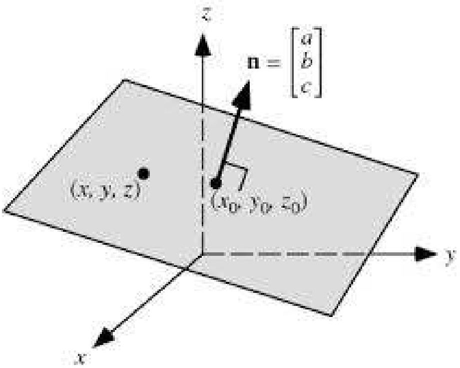

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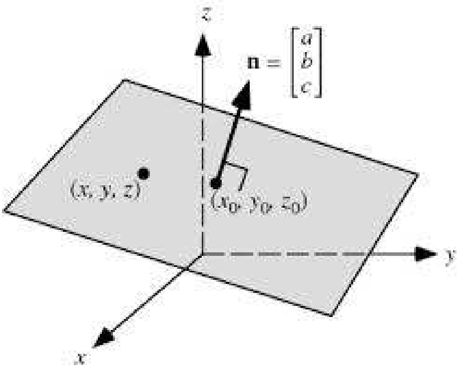

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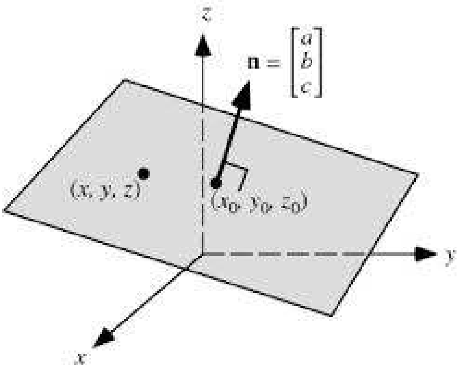

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$$\boxed{a(x-x_0) + b(y - y_0) + c(z - z_0) = 0}$$

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$$\begin{aligned} -31(x - 4) - 20(y + 3)7(z - 1) &= 0 \\ \Rightarrow -31x - 20y + 7z + 57 &= 0. \end{aligned}$$

Note(1): Distance from a point $P(x_0, y_0, z_0)$ to the plane $ax + by + cz + d = 0$ is:

$$h = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

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Note(2): The shortest distance d between skew lines ℓ_1 and ℓ_2 is

$$d = \frac{|(\overrightarrow{P_1Q_2} \times \overrightarrow{P_1Q_2}).\overrightarrow{P_1P_2}|}{\left\|\overrightarrow{P_1Q_1} \times \overrightarrow{P_2Q_2}\right\|}.$$

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Q(1): Find Parametric equations for the line of intersection of the planes

$$\begin{aligned} x + 2y - 9z &= 7 \rightarrow P_1 \\ 2x - 3y + 17z &= 0 \rightarrow P_2. \end{aligned}$$

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Therefore,

$$d = \frac{\sqrt{49 + 100 + 185}}{\sqrt{16 + 1 + 9}} = 2.67.$$

Q(3): Find the shortest distance between the lines ℓ_1 through the points $A(1,-2,3)$, $B(2,0,5)$ and line ℓ_2 through $C(4,1,-1)$, $D(-2,3,4)$. ℓ_1, ℓ_2 are skew.

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$$(\overrightarrow{AB} \times \overrightarrow{CD}).\overrightarrow{AC} = 18 - 51 - 56 = -89.$$

Q(3): Find the shortest distance between the lines ℓ_1 through the points $A(1,-2,3)$, $B(2,0,5)$ and line ℓ_2 through $C(4,1,-1)$, $D(-2,3,4)$. ℓ_1, ℓ_2 are skew.

Solution: the shortest distance is given by

$$h = \frac{|(\overrightarrow{AB} \times \overrightarrow{CD}).\overrightarrow{AC}|}{\left\|\overrightarrow{AB} \times \overrightarrow{CD}\right\|}.$$

Now $\overrightarrow{AB} = \langle 1,2,2 \rangle$, $\overrightarrow{CD} = \langle -6,2,5 \rangle$ and $\overrightarrow{AC} = \langle 3,3,-4 \rangle$. So

$$\overrightarrow{AB} \times \overrightarrow{CD} = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ -6 & 2 & 5 \end{vmatrix} = 6i - 17j + 14k.$$

Now we copmute:

$$(\overrightarrow{AB} \times \overrightarrow{CD}).\overrightarrow{AC} = 18 - 51 - 56 = -89.$$

Finally

$$h = \frac{|-89|}{\sqrt{36 + 289 + 196}}$$

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Now we copmute:

$$(\overrightarrow{AB} \times \overrightarrow{CD}).\overrightarrow{AC} = 18 - 51 - 56 = -89.$$

Finally

$$h = \frac{|-89|}{\sqrt{36 + 289 + 196}} = 3.9.$$

Q(4): Use the dot product to find the dstance from $A(2, -6, 1)$ to the line through $B(3, 4, -2)$ and $C(7, -1, 5)$.

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Solution:

We have $\overrightarrow{BA}$

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Solution:

We have $\overrightarrow{BA} = \langle -1, -10, 3 \rangle$ and $\overrightarrow{BC}$

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We have $\overrightarrow{BA} = \langle -1, -10, 3\rangle$ and $\overrightarrow{BC} = \langle 7, -1, 5\rangle$. We have

$$|AD| =$$

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$$|AD| = \text{Comp}_{\overrightarrow{BC}}^{\overrightarrow{BA}} = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{\left\|\overrightarrow{BC}\right\|} = \frac{67}{\sqrt{90}}.$$

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The distance is:

$$|AD|$$

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The distance is:

$$|AD| = \sqrt{110 - \frac{67^2}{90}} = 7.75.$$

Check that:

$$|AD| = \frac{\left\|\overrightarrow{BA} \times \overrightarrow{BC}\right\|}{\left\|\overrightarrow{BC}\right\|}.$$

Definition

let c be a curve in a plane and ℓ be a line that is not a parallel to the plane. The set of points on all lines that are parallel to ℓ and intersect with c is a *cylinder*.

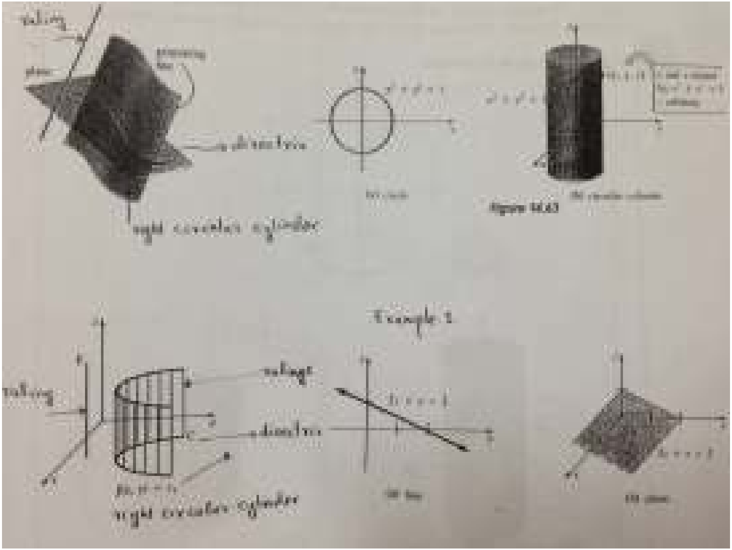
98.

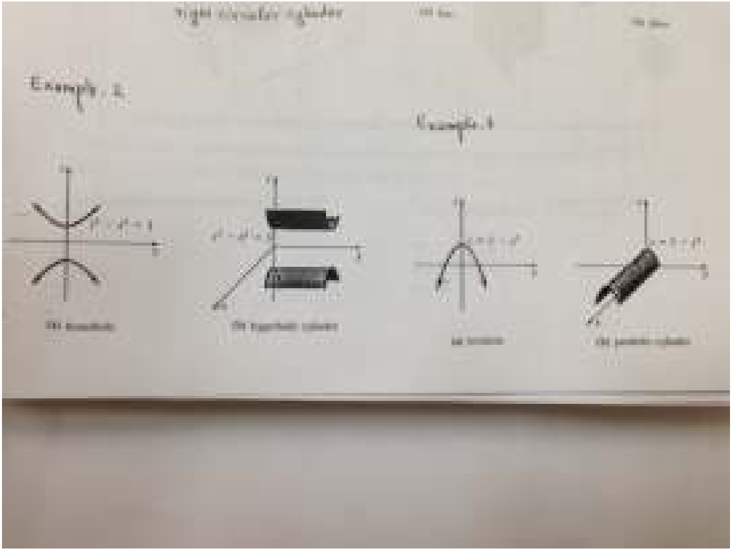
10.6 SURFACES

Cylinder

Def

Let C be a curve in a plane, and let l be a line that is not in a parallel plane. The set of points on all lines that are parallel to l and intersect C is a cylinder.


right circular cylinder
right circular cylinder



Example 2 Graph $x^2 + y^2 = 4$ in $\mathbb{R}^2$ and $\mathbb{R}^3$.

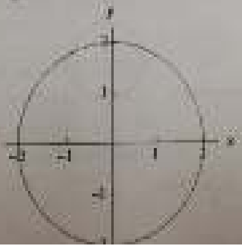
Solution

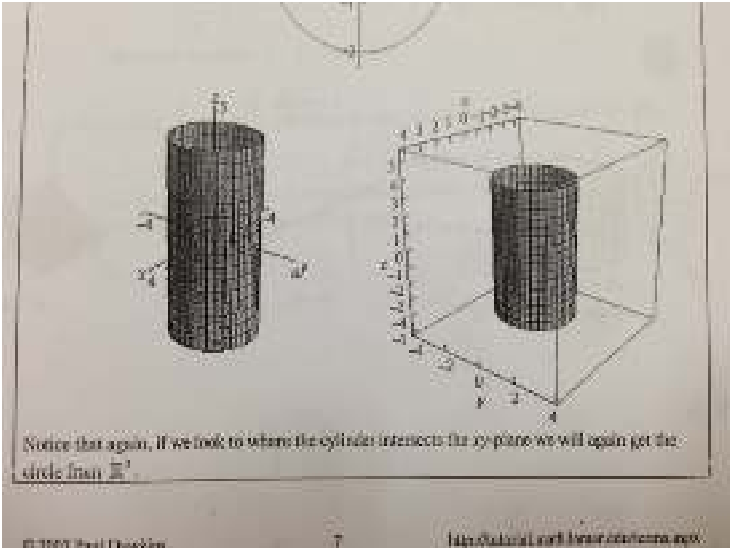
As with the previous example this won't have a 3-D graph since there are two variables.

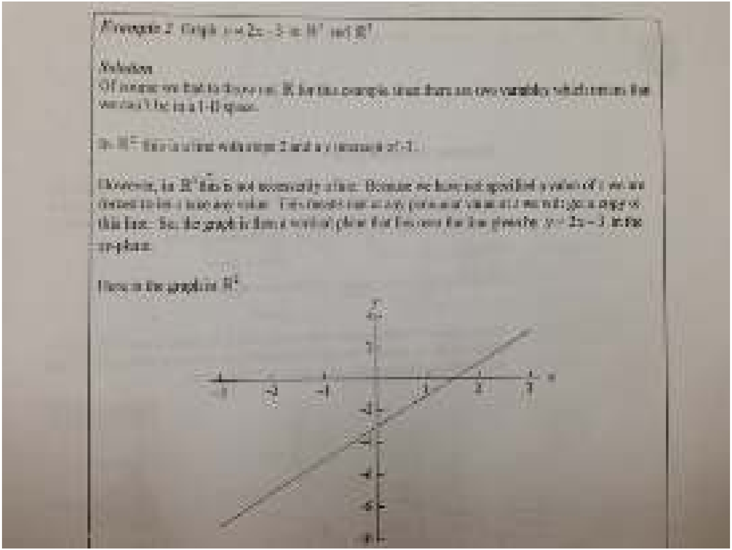
In $\mathbb{R}^2$ this is a circle centered at the origin with radius 2.

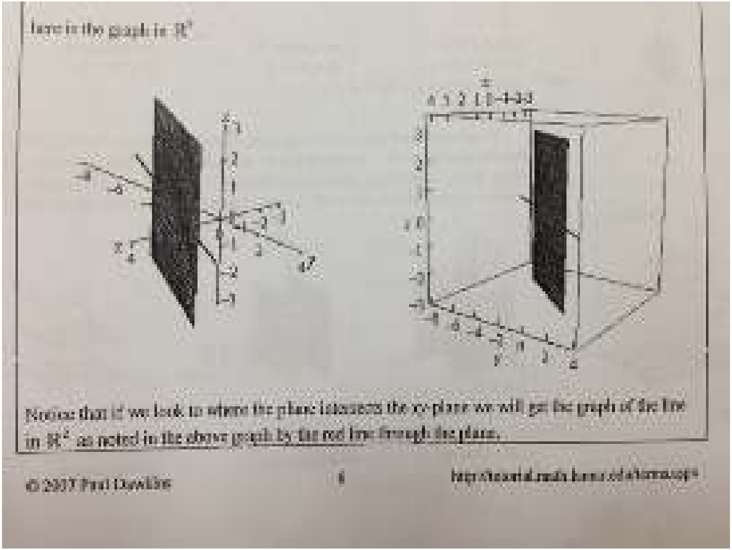
In $\mathbb{R}^3$, however, as with the previous example this may or may not be a circle. Since we have not specified z in any way we must assume that z can take on any value. In other words, at any value of z this equation must be satisfied and so at any value z we have a circle of radius 2 centered on the z-axis. This means that we have a cylinder of radius 2 centered on the z-axis.

Here are the graphs for this example.




Notice that again, if we look to where the cylinder intersects the xy-plane we will again get the
circle from ℝ².




Here is the graph in $\mathbb{R}^3$

Notice that if we look to where the plane intersects the xy-plane we will get the graph of the line in $\mathbb{R}^2$ as noted in the above graph by the red line through the plane.

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Sketching Planes in Space:

Sketching Planes in Space: If a plane in space intersect one of the coordinate planes, we call *the line of intersection the trace of* the given plane in the coordinate plane.

Sketching Planes in Space

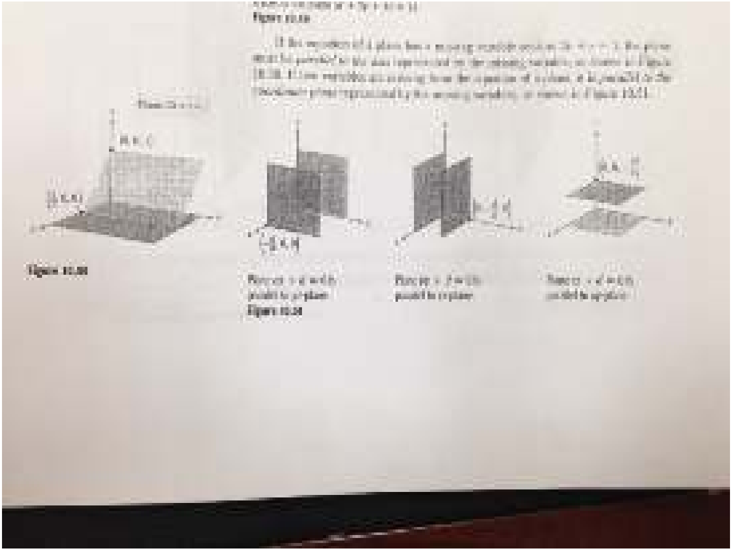
If a plane in space intersects one of the coordinate planes, we call the line of intersection the trace of the given plane in the coordinate plane. To sketch a plane in space, it is helpful to find its points of intersection with the coordinate axes and its traces in the coordinate planes. For example, consider the plane given by

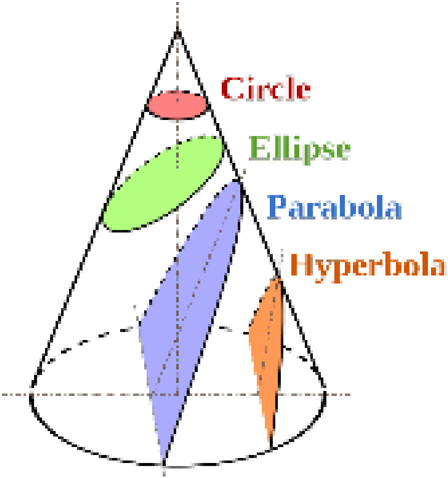
$$3x + 2y + 4z = 12. \qquad \text{Equation of plane}$$

We find the xy-trace by letting $z = 0$ and sketching the line

$$3x + 2y = 12 \qquad xy\text{-trace}$$

in the xy-plane. This line intersects the x-axis at $(4, 0, 0)$ and the y-axis at $(0, 6, 0)$. In Figure 10.49, we continue this process by finding the yz-trace and the xz-trace, and then shading in the triangular region lying in the first octant.




Circle
Ellipse
Parabola
Hyperbola

The equations for Ellipse is:

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$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \quad \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1; \quad \frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$$

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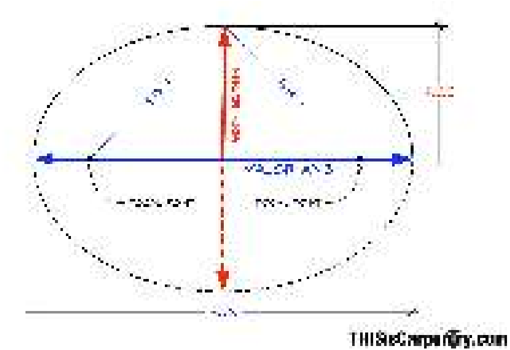
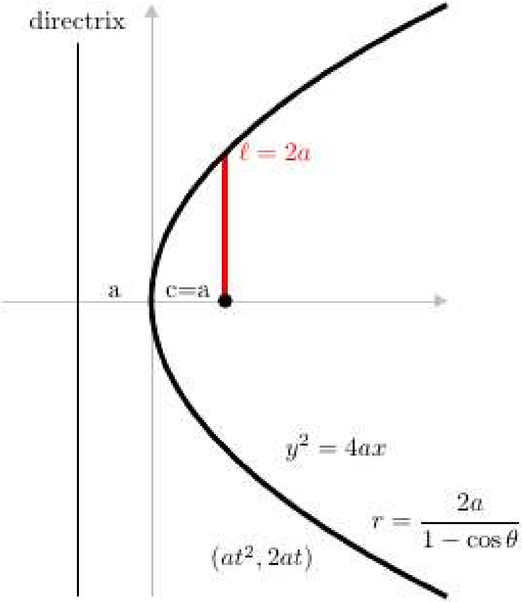
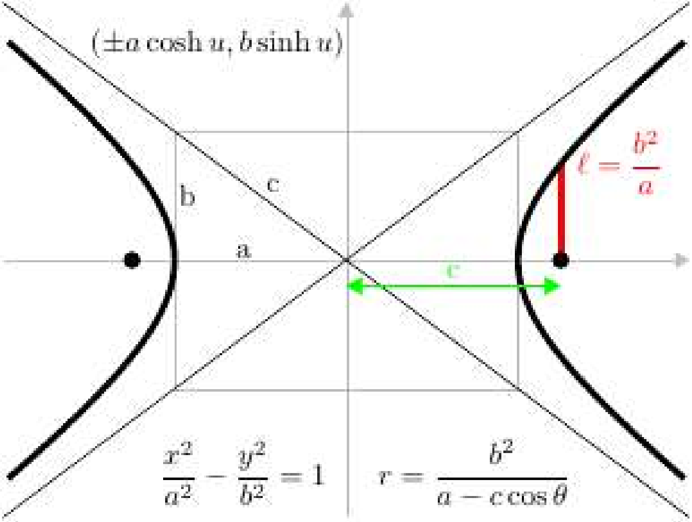



directrix
$\ell = 2a$
a
c=a
$y^2 = 4ax$
$r = \dfrac{2a}{1-\cos\theta}$
$(at^2, 2at)$


$(\pm a \cosh u, b \sinh u)$
b
c
a
$\ell = \frac{b^2}{a}$
c
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
$r = \frac{b^2}{a - c\cos\theta}$

For more information check:
http://tutorial.math.lamar.edu/Classes/CalcIII/QuadricSurfaces.aspx

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http://tutorial.math.lamar.edu/Classes/CalcIII/QuadricSurfaces.aspx

The graph of a second-degree equation in x, y, z:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Cx + Hy + Iz + J = 0$$

is a *quadric surface*.

For more information check:
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There are three types of quadric surfaces:

1. Ellipsoids.

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is a *quadric surface*.
There are three types of quadric surfaces:

1. Ellipsoids.
2. Hyperboloids.
3. Paraboloids.

Elipsoid surface equation has the form:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

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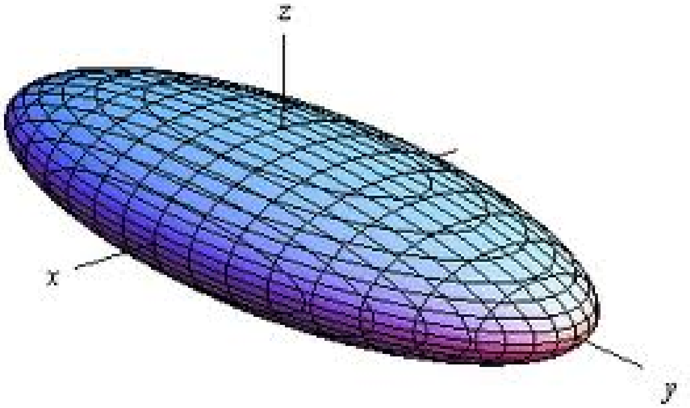
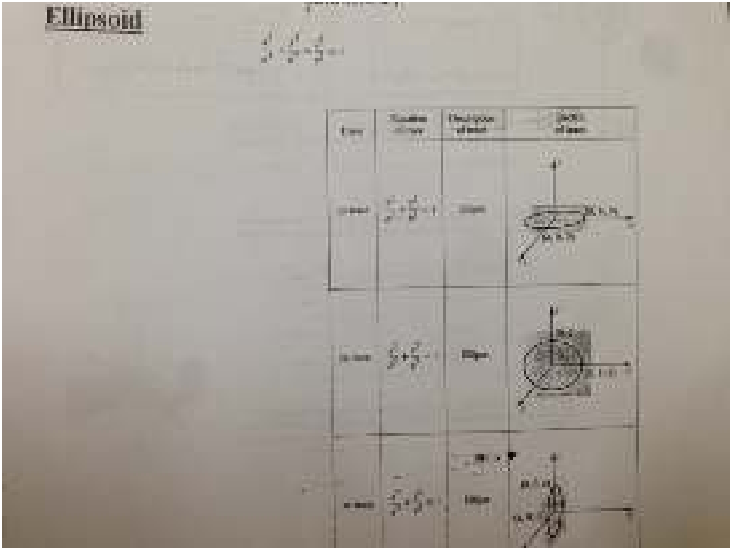
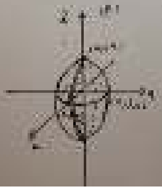



Ellipsoid

100.

[illegible]



[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[a] Hyperbolids of One Sheet:

The equation has the form:

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$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

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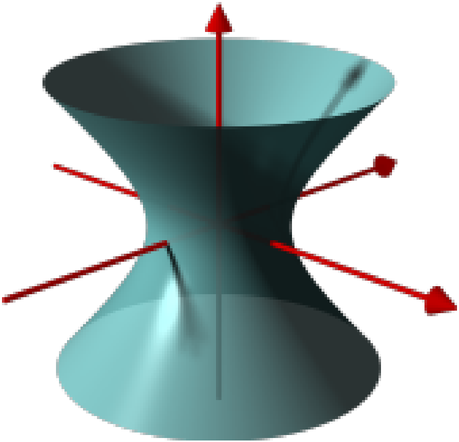

[b] Hyperbolids of Two Sheets:

The equation has the form:

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The equation has the form:

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, a > 0, b > 0, c > 0$$

[b] Hyperbolids of Two Sheets:

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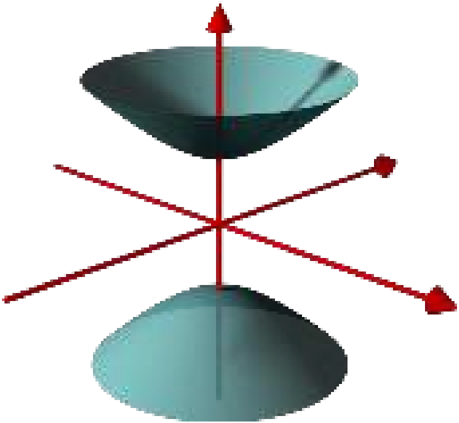

[c] Cone:

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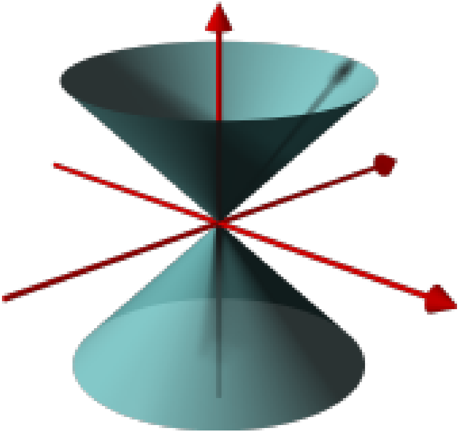

The equation has the form:

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$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$$

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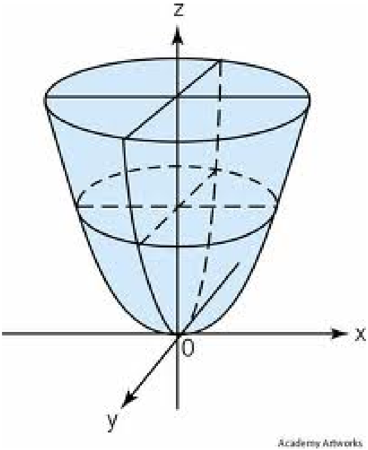

The equation has the form:

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$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = cz$$

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$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = cz$$