

Decision Analysis

LEARNING OBJECTIVES

This chapter describes how to use decision analysis to improve management decisions, thereby enabling you to:

- 1.** Make decisions under certainty by constructing a decision table
- 2.** Make decisions under uncertainty using the maximax criterion, the maximin criterion, the Hurwicz criterion, and minimax regret
- 3.** Make decisions under risk by constructing decision trees, calculating expected monetary value and expected value of perfect information, and analyzing utility
- 4.** Revise probabilities in light of sample information by using Bayesian analysis and calculating the expected value of sample information



Decision Making at the CEO Level

CEOs face major challenges in today's business world. As the international marketplace evolves, competition increases in many cases.

Technology is improving products and process. The political and economic climates both internationally and domestically shift constantly. In the midst of such dynamics, CEOs make decisions about investments, products, resources, suppliers, financing, and many other items. Decision making may be the most important function of management. Successful companies are usually built around successful decisions. Even CEOs of successful companies feel the need to constantly improve the company's position.

In 1994, Ford Motor Company posted a record profit of more than \$4 billion with five of the 10 best-selling vehicles in the United States. Yet, CEO and chairman, Alex Trotman made the decision to merge the North American and European operations into one global unit. The decision was implemented with Ford 2000, a program to design "world cars," with common components that can be sold worldwide with minor style changes to suit local tastes.

In the same year, George Fisher, CEO of Eastman Kodak Company, reversed a decade of diversification for Kodak and led the company in a direction of digital and electronic imaging. He implemented this thrust by aligning digital and electronic imaging with traditional products that emphasize paper and film.

Other CEOs made tough decisions over the years. MCI's chairman, Bert C. Roberts decided to wage a war with the seven Baby Bells over local telephone service. Drew Lewis, CEO and chairman of Union Pacific Corporation, the nation's largest railroad, made a counteroffer to buy the Santa Fe Pacific Corporation when it looked like Burlington Northern would buy Santa Fe Pacific.

CEOs of smaller companies also make tough decisions. The most critical decision-making period for a CEO is likely to be during growth phases. A study of 142 CEOs from small, private companies attempted to ascertain the types of decisions undertaken by top managers. Most of the companies in the study had experienced healthy growth in revenues over the four-year period preceding the study. CEOs in this study suggested that decisions made during growth phases typically are in the areas of expansion, personnel, finances, operations, and planning and control systems. According to respondents in the study, many of these decisions carry system-wide implications for the company that make the decisions quite critical.

CEOs responded that during a growth phase, decisions need to be made about how to handle new business. How is capacity to be expanded? Does the company build, lease, expand its present facility, relocate, automate, and so on? Risk is

inevitably involved in undertaking most of these decisions. Will customer demand continue? Will competitors also increase capacity? How long will the increased demand continue? What is the lost opportunity if the company fails to meet customer demands?

According to the study, another critical area of decision making is personnel. What is the long-term strategy of the company? Should significant layoffs be implemented in an effort to become "lean and mean"? Does the firm need to hire? How does management discover and attract talented managers? How can substandard personnel be released? In the area of production, how does management "level" personnel to match uneven product demand?

A third area of decision making that the study participants considered important was systems, business, and finance. How can the company make operations and procedures more efficient? How are cash flow problems handled? Under what conditions does the company obtain financial backing for capital development?

In the area of marketing, decisions need to be made about pricing, distribution, purchasing, and suppliers. Should the company market overseas? What about vertical integration? Should the company expand into new market segments or with new product lines?

The CEOs in the study enumerated decision choices that represent exciting and sometimes risky opportunities for growing firms. The success or failure of such decision makers often lies in their ability to identify and choose optimal decision pathways for the company.

Managerial and Statistical Questions

1. In any given area of decisions, what choices or options are available to the manager?
2. What occurrences in nature, the marketplace, or the business environment might affect the outcome or payoff for a given decision option?
3. What are some strategies that can be used to help the decision maker determine which option to choose?
4. If risk is involved, can probabilities of occurrence be assigned to various states of nature within each decision option?
5. What are the payoffs for various decision options?
6. Does the manager's propensity toward risk enter into the final decision and, if so, how?

The main focus of this text has been business decision making. In this chapter, we discuss one last category of quantitative techniques for assisting managers in decision making. These techniques, generally referred to as **decision analysis**, are *particularly targeted at clarifying and enhancing the decision-making*

process and can be used in such diverse situations as determining whether and when to drill oil wells, deciding whether and how to expand capacity, deciding whether to automate a facility, and determining what types of investments to make.

In decision analysis, decision-making scenarios are divided into the following three categories.

1. Decision making under certainty
2. Decision making under uncertainty
3. Decision making under risk

In this chapter, we discuss making decisions under each condition, as well as the concepts of utility and Bayesian statistics.



19.1 THE DECISION TABLE AND DECISION MAKING UNDER CERTAINTY

Many decision analysis problems can be viewed as having three variables: decision alternatives, states of nature, and payoffs.

Decision alternatives are *the various choices or options available to the decision maker in any given problem situation*. On most days, financial managers face the choices of whether to invest in blue chip stocks, bonds, commodities, certificates of deposit, money markets, annuities, and other investments. Construction decision makers must decide whether to concentrate on one building job today, spread out workers and equipment to several jobs, or not work today. In virtually every possible business scenario, decision alternatives are available. A good decision maker identifies many options and effectively evaluates them.

States of nature are *the occurrences of nature that can happen after a decision is made that can affect the outcome of the decision and over which the decision maker has little or no control*. These states of nature can be literally natural atmospheric and climatic conditions or they can be such things as the business climate, the political climate, the worker climate, or the condition of the marketplace, among many others. The financial investor faces such states of nature as the prime interest rate, the condition of the stock market, the international monetary exchange rate, and so on. A construction company is faced with such states of nature as the weather, wildcat strikes, equipment failure, absenteeism, and supplier inability to deliver on time. States of nature are usually difficult to predict but are important to identify in the decision-making process.

The **payoffs** of a decision analysis problem are *the benefits or rewards that result from selecting a particular decision alternative*. Payoffs are usually given in terms of dollars. In the financial investment industry, for example, the payoffs can be small, modest, or large, or the investment can result in a loss. Most business decisions involve taking some chances with personal or company money in one form or another. Because for-profit businesses are looking for a return on the dollars invested, the payoffs are extremely important for a successful manager. The trick is to determine which decision alternative to take in order to generate the greatest payoff. Suppose a CEO is examining various environmental decision alternatives. Positive payoffs could include increased market share, attracting and retaining quality employees, consumer appreciation, and governmental support. Negative payoffs might take the form of fines and penalties, lost market share, and lawsuit judgments.

Decision Table

The concepts of decision alternatives, states of nature, and payoffs can be examined jointly by using a **decision table**, or **payoff table**. Table 19.1 shows the structure of a decision table. On the left side of the table are the various decision alternatives, denoted by d_i . Along the top row are the states of nature, denoted by s_j . In the middle of the table are the various payoffs for each decision alternative under each state of nature, denoted by P_{ij} .

TABLE 19.1

Decision Table

		State of Nature				
		s_1	s_2	s_3	...	s_n
Decision Alternative	d_1	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$...	$P_{1,n}$
	d_2	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$...	$P_{2,n}$
	d_3	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$...	$P_{3,n}$

	d_m	P_{m1}	P_{m2}	P_{m3}	...	P_{mn}

where
 s_j = state of nature
 d_i = decision alternative
 P_{ij} = payoff for decision i under state j

TABLE 19.2

Yearly Payoffs on an Investment of \$10,000

		State of the Economy		
		Stagnant	Slow Growth	Rapid Growth
Investment Decision Alternative	Stocks	-\$500	\$700	\$2,200
	Bonds	-\$100	\$600	\$900
	CDs	\$300	\$500	\$750
	Mixture	-\$200	\$650	\$1,300

As an example of a decision table, consider the decision dilemma of the investor shown in Table 19.2. The investor is faced with the decision of where and how to invest \$10,000 under several possible states of nature.

The investor is considering four decision alternatives.

1. Invest in the stock market
2. Invest in the bond market
3. Invest in government certificates of deposit (CDs)
4. Invest in a mixture of stocks and bonds

Because the payoffs are in the future, the investor is unlikely to know ahead of time what the state of nature will be for the economy. However, the table delineates three possible states of the economy.

1. A stagnant economy
2. A slow-growth economy
3. A rapid-growth economy

The matrix in Table 19.2 lists the payoffs for each possible investment decision under each possible state of the economy. Notice that the largest payoff comes with a stock investment under a rapid-growth economic scenario, with a payoff of \$2,200 per year on an investment of \$10,000. The lowest payoff occurs for a stock investment during stagnant economic times, with an annual loss of \$500 on the \$10,000 investment.

Decision Making Under Certainty

The most elementary of the decision-making scenarios is **decision making under certainty**. In making decisions under certainty, *the states of nature are known*. The decision maker needs merely to examine the payoffs under different decision alternatives and select the alternative with the largest payoff. In the preceding example involving the \$10,000 investment, if it is known that the economy is going to be stagnant, the investor would select the decision alternative of CDs, yielding a payoff of \$300. Indeed, each of the other three decision alternatives would result in a loss under stagnant economic conditions. If it is known that the economy is going to have slow growth, the investor would choose stocks as an investment, resulting in a \$700 payoff. If the economy is certain to have rapid growth, the decision maker should opt for stocks, resulting in a payoff of \$2,200. Decision making under certainty is almost the trivial case.



19.2 DECISION MAKING UNDER UNCERTAINTY

In making decisions under certainty, the decision maker knows for sure which state of nature will occur, and he or she bases the decision on the optimal payoff available under that state. **Decision making under uncertainty** occurs *when it is unknown which states of nature will occur and the probability of a state of nature occurring is also unknown*. Hence, the decision maker has virtually no information about which state of nature will occur, and he or she attempts to develop a strategy based on payoffs.

Several different approaches can be taken to making decisions under uncertainty. Each uses a different decision criterion, depending on the decision maker’s outlook. Each of these approaches will be explained and demonstrated with a decision table. Included are the maximax criterion, maximin criterion, Hurwicz criterion, and minimax regret.

In section 19.1, we discussed the decision dilemma of the financial investor who wants to invest \$10,000 and is faced with four decision alternatives and three states of nature. The data for this problem were given in Table 19.2. In decision making under certainty, we selected the optimal payoff under each state of the economy and then, on the basis of which state we were certain would occur, selected a decision alternative. Shown next are techniques to use when we are uncertain which state of nature will occur.

Maximax Criterion

The **maximax criterion** approach is an optimistic approach in which the decision maker bases action on a notion that the best things will happen. The decision maker *isolates the maximum payoff under each decision alternative and then selects the decision alternative that produces the highest of these maximum payoffs*. The name “maximax” means selecting the maximum overall payoff from the maximum payoffs of each decision alternative. Consider the \$10,000 investment problem. The maximum payoff is \$2,200 for stocks, \$900 for bonds, \$750 for CDs, and \$1,300 for the mixture of investments. The maximax criterion approach requires that the decision maker select the maximum payoff of these four.

		State of the Economy			
		Stagnant	Slow Growth	Rapid Growth	Maximum
Investment Decision Alternative	Stocks	−\$500	\$700	\$2,200	\$2,200
	Bonds	−\$100	\$600	\$900	\$900
	CDs	\$300	\$500	\$750	\$750
	Mixture	−\$200	\$650	\$1,300	\$1,300
		maximum of {\$2,200, \$900, \$750, \$1,300} = \$2,200			

Because the maximax criterion results in \$2,200 as the optimal payoff, the decision alternative selected is the stock alternative, which is associated with the \$2,200.

Maximin Criterion

The **maximin criterion** approach to decision making is a pessimistic approach. The assumption is that the worst will happen and attempts must be made to minimize the damage. The decision maker starts by examining the payoffs under each decision alternative and selects the worst, or minimum, payoff that can occur under that decision. Then the decision maker *selects the maximum or best payoff of those minimums selected under each decision alternative*. Thus, the decision maker has maximized the minimums. In the investment problem, the minimum payoffs are −\$500 for stocks, −\$100 for bonds, \$300 for CDs, and −\$200 for the mixture of investments. With the maximin criterion, the decision maker examines the minimum payoffs for each decision alternative given in the last column and selects the maximum of those values.

		State of the Economy			
		Stagnant	Slow Growth	Rapid Growth	Minimum
Investment Decision Alternative	Stocks	-\$500	\$700	\$2,200	-\$500
	Bonds	-\$100	\$600	\$900	-\$100
	CDs	\$300	\$500	\$750	\$300
	Mixture	-\$200	\$650	\$1,300	-\$200
		maximum of {-500, -100, 300, -200} = \$300			

The decision is to invest in CDs because that investment alternative yields the highest, or maximum, payoff under the worst-case scenario.

Hurwicz Criterion

The Hurwicz criterion is an approach somewhere between the maximax and the maximin approaches. The **Hurwicz criterion** approach selects the maximum and the minimum payoff from each decision alternative. A value called alpha (not the same as the probability of a Type I error), which is between 0 and 1, is selected as a weight of optimism. The nearer alpha is to 1, the more optimistic is the decision maker. The use of alpha values near 0 implies a more pessimistic approach. The maximum payoff under each decision alternative is multiplied by alpha and the minimum payoff (pessimistic view) under each decision alternative is multiplied by $1 - \alpha$ (weight of pessimism). These weighted products are summed for each decision alternative, resulting in a weighted value for each decision alternative. The maximum weighted value is selected, and the corresponding decision alternative is chosen.

Following are the data for the investment example, along with the minimum and maximum values.

		State of the Economy				
		Stagnant	Slow Growth	Rapid Growth	Minimum	Maximum
Investment Decision Alternative	Stocks	-\$500	\$700	\$2,200	-\$500	\$2,200
	Bonds	-\$100	\$600	\$900	-\$100	\$900
	CDs	\$300	\$500	\$750	\$300	\$750
	Mixture	-\$200	\$650	\$1,300	-\$200	\$1,300

Suppose we are more optimistic than pessimistic and select $\alpha = .7$ for the weight of optimism. The calculations of weighted values for the decision alternative follow.

$$\begin{aligned}
 \text{Stocks} & (\$2,200)(.7) + (-\$500)(.3) = \$1,390 \\
 \text{Bonds} & (\$900)(.7) + (-\$100)(.3) = \$ 600 \\
 \text{CDs} & (\$750)(.7) + (\$300)(.3) = \$ 615 \\
 \text{Mixture} & (\$1,300)(.7) + (-\$200)(.3) = \$ 850
 \end{aligned}$$

The Hurwicz criterion leads the decision maker to choose the maximum of these values, \$1,390. The result under the Hurwicz criterion with $\alpha = .7$ is to choose stocks as the decision alternative. An advantage of the Hurwicz criterion is that it allows the decision maker the latitude to explore various weights of optimism. A decision maker's outlook might change from scenario to scenario and from day to day. In this case, if we had been fairly pessimistic and chosen an alpha of .2, we would have obtained the following weighted values.

$$\begin{aligned}
 \text{Stocks} & (\$2,200)(.2) + (-\$500)(.8) = \$ 40 \\
 \text{Bonds} & (\$900)(.2) + (-\$100)(.8) = \$100 \\
 \text{CDs} & (\$750)(.2) + (\$300)(.8) = \$390 \\
 \text{Mixture} & (\$1,300)(.2) + (-\$200)(.8) = \$100
 \end{aligned}$$

TABLE 19.3

Decision Alternatives for Various Values of Alpha

		Stocks		Bonds		CDs		Mixture	
α	$1 - \alpha$	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
.0	1.0	2,200	-500	900	-100	750	300	1,300	-200
.1	.9					300			
.2	.8					345			
.3	.7					390			
.4	.6					435			
.5	.5	580		300		480		400	
.6	.4	850		400		525		550	
.7	.3	1,120		500		570		700	
.8	.2	1,390		600		615		850	
.9	.1	1,660		700		660		1,000	
1.0	.0	1,930		800		705		1,150	
		2,200		900		750		1,300	

Note: Circled values indicate the choice for the given value of alpha.

Under this scenario, the decision maker would choose the CD option because it yields the highest weighted payoff (\$390) with $\alpha = .2$.

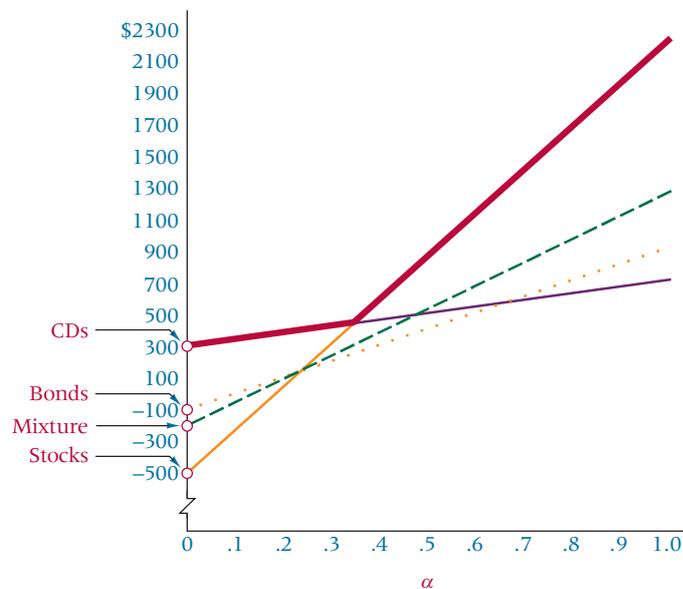
Table 19.3 displays the payoffs obtained by using the Hurwicz criterion for various values of alpha for the investment example. The circled values are the optimum payoffs and represent the decision alternative selection for that value of alpha. Note that for $\alpha = .0, .1, .2,$ and $.3$, the decision is to invest in CDs. For $\alpha = .4$ to 1.0, the decision is to invest in stocks.

Figure 19.1 shows graphically the weighted values for each decision alternative over the possible values of alpha. The thicker line segments represent the maximum of these under each value of alpha. Notice that the graph reinforces the choice of CDs for $\alpha = .0, .1, .2, .3$ and the choice of stocks for $\alpha = .4$ through 1.0.

Between $\alpha = .3$ and $\alpha = .4$, there is a point at which the line for weighted payoffs for CDs intersects the line for weighted payoffs for stocks. By setting the alpha expression with maximum and minimum values of the CD investment equal to that of the stock investment, we can solve for the alpha value at which the intersection occurs. At this value of

FIGURE 19.1

Graph of Hurwicz Criterion Selections for Various Values of Alpha



alpha, the weighted payoffs of the two investments under the Hurwicz criterion are equal, and the decision maker is indifferent as to which one he or she chooses.

$$\begin{aligned}
 \text{Stocks Weighted Payoff} &= \text{CDs Weighted Payoff} \\
 2,200(\alpha) + (-500)(1 - \alpha) &= 750(\alpha) + (300)(1 - \alpha) \\
 2,200\alpha - 500 + 500\alpha &= 750\alpha + 300 - 300\alpha \\
 2,250\alpha &= 800 \\
 \alpha &= .3555
 \end{aligned}$$

At $\alpha = .3555$, both stocks and CDs yield the same payoff under the Hurwicz criterion. For values less than $\alpha = .3555$, CDs are the chosen investment. For $\alpha > .3555$, stocks are the chosen investment. Neither bonds nor the mixture produces the optimum payoff under the Hurwicz criterion for any value of alpha. Notice that in Figure 19.1 the dark line segments represent the optimum solutions. The lines for both bonds and the mixture are beneath these optimum line segments for the entire range of α . In another problem with different payoffs, the results might be different.

Minimax Regret

The strategy of **minimax regret** is based on lost opportunity. Lost opportunity occurs when a decision maker loses out on some payoff or portion of a payoff because he or she chose the wrong decision alternative. For example, if a decision maker selects decision alternative d_i , which pays \$200, and the selection of alternative d_j would have yielded \$300, the opportunity loss is \$100.

$$\$300 - \$200 = \$100$$

In analyzing decision-making situations under uncertainty, an analyst can transform a decision table (payoff table) into an **opportunity loss table**, which can be used to apply the minimax regret criterion. Repeated here is the \$10,000 investment decision table.

		State of the Economy		
		Stagnant	Slow Growth	Rapid Growth
Investment Decision Alternative	Stocks	-\$500	\$700	\$2,200
	Bonds	-\$100	\$600	\$900
	CDs	\$300	\$500	\$750
	Mixture	-\$200	\$650	\$1,300

Suppose the state of the economy turns out to be stagnant. The optimal decision choice would be CDs, which pay off \$300. Any other decision would lead to an opportunity loss. The opportunity loss for each decision alternative other than CDs can be calculated by subtracting the decision alternative payoff from \$300.

$$\begin{aligned}
 \text{Stocks} & \$300 - (-\$500) = \$800 \\
 \text{Bonds} & \$300 - (-\$100) = \$400 \\
 \text{CDs} & \$300 - (\$300) = \$0 \\
 \text{Mixture} & \$300 - (-\$200) = \$500
 \end{aligned}$$

The opportunity losses for the slow-growth state of the economy are calculated by subtracting each payoff from \$700, because \$700 is the maximum payoff that can be obtained from this state; any other payoff is an opportunity loss. These opportunity losses follow.

$$\begin{aligned}
 \text{Stocks} & \$700 - (\$700) = \$0 \\
 \text{Bonds} & \$700 - (\$600) = \$100 \\
 \text{CDs} & \$700 - (\$500) = \$200 \\
 \text{Mixture} & \$700 - (\$650) = \$50
 \end{aligned}$$

STATISTICS IN BUSINESS TODAY

The RadioShack Corporation Makes Decisions

In the 1960s, Charles Tandy founded and built a tightly integrated manufacturing and retailing company, the Tandy Corporation. RadioShack, a retail unit of the Tandy Corporation, has been one of the company's mainstays. However, RadioShack, along with the Tandy Corporation, has seen many changes over the years both because of decisions management made and because of various states of nature that occurred.

In the early days, RadioShack was an outlet for Tandy products with a relatively narrow market niche. In the 1970s, the company made millions on the CB radio craze that hit the United States. In the early 1980s, Radio Shack did well with an inexpensive personal computer. By the mid-1980s, the stores were becoming neglected, with much of the retailing profits being poured back into such unsuccessful manufacturing experiments as low-priced laptop computers and videodisc players.

In 1993, Tandy decided to sell its computer-making operations and placed new emphasis on retailing by bringing in a new president for RadioShack. The resulting series of decisions resulted in a significant positive turnaround for RadioShack. The company placed more emphasis on telephones and cut a deal with the Sprint Corporation to make Sprint its exclusive wireless provider. Sprint, in turn, provided millions of dollars to update RadioShack stores. Since then, RadioShack sold more wireless phones than most of its major rivals. In addition, RadioShack contracted to sell only Compaq computers and RCA audio and video equipment in their stores in exchange for these companies' investment in upgrading the retail outlet facilities. In the year 2000, RadioShack announced its alliance with Verizon Wireless, and the Tandy Corporation became the RadioShack Corporation. In 2001, RadioShack formed an alliance with Blockbuster. In 2002, RadioShack became the only national retailer offering both DIRECTV and DISH Network.

Since then, RadioShack Corporation sold its Incredible Universe stores and its Computer City superstores. These

moves left RadioShack with its 7,000 RadioShack stores as its main presence in the retail arena.

The fast-paced and ever-changing electronics industry presented many decision alternatives to RadioShack. In the early years, the corporation decided to sell mostly Tandy products in RadioShack stores. Then the corporation opened a variety of types and sizes of retail stores, only to sell most of them later. At one point, Tandy invested heavily in manufacturing new items at the expense of retail operations, then it sold its computer manufacturing operations and renewed its focus on retail. Currently, the corporation is putting most of its "eggs" in the RadioShack "basket," with its exclusive agreements with Sprint, Compaq, and RCA and its emphasis on telephones, wireless service, and Internet service. RadioShack could have chosen other decision alternatives that may have led to different outcomes.

Some of the states of nature that occurred include the rise and fall of CB radios, the exponential growth in personal computers and wireless telephones, the development of the Internet as a market and as an outlet for goods and services, a strong U.S. economy, and a growing atmosphere of disgust by large electronics manufacturers with electronics superstores and their deeply discounted merchandise.

The payoffs from some of these decisions for the RadioShack Corporation have been substantial. Some decisions resulted in revenue losses, thereby generating still other decisions. The decision selections, the states of nature, and the resulting payoffs can make the difference between a highly successful company and one that fails.

Today, RadioShack operates more than 7,200 stores nationwide. It is estimated that 94% of all Americans live or work within five minutes of a RadioShack store or dealer. RadioShack's mission is to demystify technology in every neighborhood in the United States.

Source: Adapted from Evan Ramstad, "Inside RadioShack's Surprising Turnaround," *The Wall Street Journal*, 8 June 1999, p. B1. Also, RadioShack available at <http://www.radioshackcorporation.com/>.

The opportunity losses for a rapid-growth state of the economy are calculated similarly.

Stocks	$\$2,200 - (\$2,200) = \$0$
Bonds	$\$2,200 - (\$900) = \$1,300$
CDs	$\$2,200 - (\$750) = \$1,450$
Mixture	$\$2,200 - (\$1,300) = \$900$

Replacing payoffs in the decision table with opportunity losses produces the opportunity loss table, as shown in Table 19.4.

After the opportunity loss table is determined, the decision maker examines the lost opportunity, or regret, under each decision, and selects the maximum regret for consideration. For example, if the investor chooses stocks, the maximum regret or lost opportunity is \$800. If the investor chooses bonds, the maximum regret is \$1,300. If the investor chooses CDs, the maximum regret is \$1,450. If the investor selects a mixture, the maximum regret is \$900.

TABLE 19.4
Opportunity Loss Table

		State of the Economy		
		Stagnant	Slow Growth	Rapid Growth
Investment Decision Alternative	Stocks	\$800	\$0	\$0
	Bonds	\$400	\$100	\$1,300
	CDs	\$0	\$200	\$1,450
	Mixture	\$500	\$50	\$900

In making a decision based on a minimax regret criterion, the decision maker examines the maximum regret under each decision alternative and selects the minimum of these. The result is the stocks option, which has the minimum regret of \$800. An investor who wants to minimize the maximum regret under the various states of the economy will choose to invest in stocks under the minimax regret strategy.

DEMONSTRATION PROBLEM 19.1

A manufacturing company is faced with a capacity decision. Its present production facility is running at nearly maximum capacity. Management is considering the following three capacity decision alternatives.

1. No expansion
2. Add on to the present facility
3. Build a new facility

The managers believe that if a large increase occurs in demand for their product in the near future, they will need to build a new facility to compete and capitalize on more efficient technological and design advances. However, if demand does not increase, it might be more profitable to maintain the present facility and add no capacity. A third decision alternative is to add on to the present facility, which will suffice for a moderate increase in demand and will be cheaper than building an entirely new facility. A drawback of adding to the old facility is that if there is a large demand for the product, the company will be unable to capitalize on new technologies and efficiencies, which cannot be built into the old plant.

The following decision table shows the payoffs (in \$ millions) for these three decision alternatives for four different possible states of demand for the company's product (less demand, same demand, moderate increase in demand, and large increase in demand). Use these data to determine which decision alternative would be selected by the maximax criterion and the maximin criterion. Use $\alpha = .4$ and the Hurwicz criterion to determine the decision alternative. Calculate an opportunity loss table and determine the decision alternative by using the minimax regret criterion.

		State of Demand			
		Less	No Change	Moderate Increase	Large Increase
Capacity Decision	No Expansion	-\$3	\$2	\$3	\$6
	Add On	-\$40	-\$28	\$10	\$20
	Build a New Facility	-\$210	-\$145	-\$5	\$55

Solution

The maximum and minimum payoffs under each decision alternative follow.

	Maximum	Minimum
No Expansion	\$ 6	-\$ 3
Add On	\$20	-\$ 40
Build a New Facility	\$55	-\$210

Using the maximax criterion, the decision makers select the maximum of the maximum payoffs under each decision alternative. This value is the maximum of $\{\$6, \$20, \$55\} = \55 , or the selection of the decision alternative of building a new facility and maximizing the maximum payoff (\$55).

Using the maximin criterion, the decision makers select the maximum of the minimum payoffs under each decision alternative. This value is the maximum of $\{-\$3, -\$40, -\$210\} = -\3 . They select the decision alternative of no expansion and maximize the minimum payoff (-\$3).

Following are the calculations for the Hurwicz criterion with $\alpha = .4$.

No Expansion	$\$6(.4) + (-\$3)(.6)$	=	\$0.60
Add On	$\$20(.4) + (-\$40)(.6)$	=	-\$16.00
Build a New Facility	$\$55(.4) + (-\$210)(.6)$	=	-\$104.00

Using the Hurwicz criterion, the decision makers would select no expansion as the maximum of these weighted values (\$0.60).

Following is the opportunity loss table for this capacity choice problem. Note that each opportunity loss is calculated by taking the maximum payoff under each state of nature and subtracting each of the other payoffs under that state from that maximum value.

		State of Demand			
		Less	No Change	Moderate Increase	Large Increase
Capacity Decision	No Expansion	\$0	\$0	\$7	\$49
	Add On	\$37	\$30	\$0	\$35
	Build a New Facility	\$207	\$147	\$15	\$0

Using the minimax regret criterion on this opportunity loss table, the decision makers first select the maximum regret under each decision alternative.

Decision Alternative	Maximum Regret
No Expansion	49
Add On	37
Build a New Facility	207

Next, the decision makers select the decision alternative with the minimum regret, which is to add on, with a regret of \$37.

19.2 PROBLEMS

19.1 Use the decision table given here to complete parts (a) through (d).

		State of Nature		
		s_1	s_2	s_3
Decision Alternative	d_1	250	175	-25
	d_2	110	100	70
	d_3	390	140	-80

- Use the maximax criterion to determine which decision alternative to select.
- Use the maximin criterion to determine which decision alternative to select.
- Use the Hurwicz criterion to determine which decision alternative to select. Let $\alpha = .3$ and then let $\alpha = .8$ and compare the results.

- d. Compute an opportunity loss table from the data. Use this table and a minimax regret criterion to determine which decision alternative to select.

19.2 Use the decision table given here to complete parts (a) through (d).

		State of Nature			
		s_1	s_2	s_3	s_4
Decision Alternative	d_1	50	70	120	110
	d_2	80	20	75	100
	d_3	20	45	30	60
	d_4	100	85	-30	-20
	d_5	0	-10	65	80

- a. Use the maximax criterion to determine which decision alternative to select.
 b. Use the maximin criterion to determine which decision alternative to select.
 c. Use the Hurwicz criterion to determine which decision alternative to select. Let $\alpha = .5$.
 d. Compute an opportunity loss table from the data. Use this table and a minimax regret criterion to determine which decision alternative to select.
- 19.3 Election results can affect the payoff from certain types of investments. Suppose a brokerage firm is faced with the prospect of investing \$20 million a few weeks before the national election for president of the United States. They feel that if a Republican is elected, certain types of investments will do quite well; but if a Democrat is elected, other types of investments will be more desirable. To complicate the situation, an independent candidate, if elected, is likely to cause investments to behave in a different manner. Following are the payoffs for different investments under different political scenarios. Use the data to reach a conclusion about which decision alternative to select. Use both the maximax and maximin criteria and compare the answers.

		Election Winner		
		Republican	Democrat	Independent
Investment	A	60	15	-25
	B	10	25	30
	C	-10	40	15
	D	20	25	5

19.4 The introduction of a new product into the marketplace is quite risky. The percentage of new product ideas that successfully make it into the marketplace is as low as 1%. Research and development costs must be recouped, along with marketing and production costs. However, if a new product is warmly received by customers, the payoffs can be great. Following is a payoff table (decision table) for the production of a new product under different states of the market. Notice that the decision alternatives are to not produce the product at all, produce a few units of the product, and produce many units of the product. The market may be not receptive to the product, somewhat receptive to the product, and very receptive to the product.

- a. Use this matrix and the Hurwicz criterion to reach a decision. Let $\alpha = .6$.
 b. Determine an opportunity loss table from this payoff table and use minimax regret to reach a decision.

		State of the Market		
		Not Receptive	Somewhat Receptive	Very Receptive
Production Alternative	Don't Produce	-50	-50	-50
	Produce Few	-200	300	400
	Produce Many	-600	100	1000



19.3 DECISION MAKING UNDER RISK

In Section 19.1 we discussed making decisions in situations where it is certain which states of nature will occur. In section 19.2, we examined several strategies for making decisions when it is uncertain which state of nature will occur. In this section we examine decision making under risk. **Decision making under risk** occurs *when it is uncertain which states of nature will occur but the probability of each state of nature occurring has been determined*. Using these probabilities, we can develop some additional decision-making strategies.

In preceding sections, we discussed the dilemma of how best to invest \$10,000. Four investment decision alternatives were identified and three states of the economy seemed possible (stagnant economy, slow-growth economy, and rapid-growth economy). Suppose we determine that there is a .25 probability of a stagnant economy, a .45 probability of a slow-growth economy, and a .30 probability of a rapid-growth economy. In a decision table, or payoff table, we place these probabilities next to each state of nature. Table 19.5 is a decision table for the investment example shown in Table 19.1 with the probabilities given in parentheses.

Decision Trees

Another way to depict the decision process is through the use of decision trees. **Decision trees** have a □ node to represent decision alternatives and a ○ node to represent states of nature. If probabilities are available for states of nature, they are assigned to the line segment following the state-of-nature node symbol, ○. Payoffs are displayed at the ends of the decision tree limbs. Figure 19.2 is a decision tree for the financial investment example given in Table 19.5.

Expected Monetary Value (EMV)

One strategy that can be used in making decisions under risk is the **expected monetary value (EMV)** approach. A person who uses this approach is sometimes referred to as an **EMVer**. The expected monetary value of each decision alternative is calculated by multiplying the probability of each state of nature by the state’s associated payoff and summing these products across the states of nature for each decision alternative, producing an expected monetary value for each decision alternative. The decision maker compares the expected monetary values for the decision alternatives and selects the alternative with the highest expected monetary value.

As an example, we can compute the expected monetary value for the \$10,000 investment problem displayed in Table 19.5 and Figure 19.2 with the associated probabilities. We use the following calculations to find the expected monetary value for the decision alternative *Stocks*.

$$\begin{aligned} \text{Expected Value for Stagnant Economy} &= (.25)(-\$500) = -\$125 \\ \text{Expected Value for Slow-Growth Economy} &= (.45)(\$700) = \$315 \\ \text{Expected Value for Rapid-Growth Economy} &= (.30)(\$2,200) = \$660 \end{aligned}$$

The expected monetary value of investing in stocks is

$$-\$125 + \$315 + \$660 = \$850$$

The calculations for determining the expected monetary value for the decision alternative *Bonds* follow.

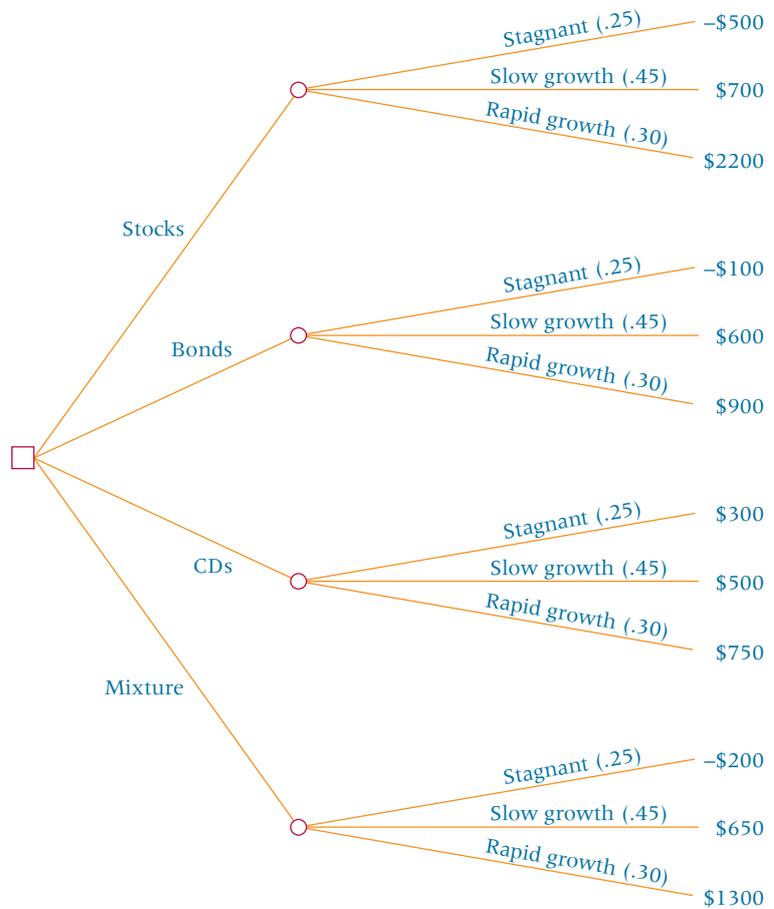
TABLE 19.5

Decision Table with State of Nature Probabilities

		State of the Economy		
		Stagnant (.25)	Slow Growth (.45)	Rapid Growth (.30)
Investment Decision Alternative	<i>Stocks</i>	-\$500	\$700	\$2,200
	<i>Bonds</i>	-\$100	\$600	\$ 900
	<i>CDs</i>	\$300	\$500	\$ 750
	<i>Mixture</i>	-\$200	\$650	\$1,300

FIGURE 19.2

Decision Tree for the Investment Example



Expected Value for Stagnant Economy = $(.25)(-\$100) = -\25
 Expected Value for Slow-Growth Economy = $(.45)(\$600) = \270
 Expected Value for Rapid-Growth Economy = $(.30)(\$900) = \270

The expected monetary value of investing in bonds is

$$-\$25 + \$270 + \$270 = \$515$$

The expected monetary value for the decision alternative *CDs* is found by the following calculations.

Expected Value for Stagnant Economy = $(.25)(\$300) = \75
 Expected Value for Slow-Growth Economy = $(.45)(\$500) = \225
 Expected Value for Rapid-Growth Economy = $(.30)(\$750) = \225

The expected monetary value of investing in CDs is

$$\$75 + \$225 + \$225 = \$525$$

The following calculations are used to find the expected monetary value for the decision alternative *Mixture*.

Expected Value for Stagnant Economy = $(.25)(-\$200) = -\50.00
 Expected Value for Slow-Growth Economy = $(.45)(\$650) = \292.50
 Expected Value for Rapid-Growth Economy = $(.30)(\$1,300) = \390.00

The expected monetary value of investing in a mixture is

$$-\$50 + \$292.50 + \$390 = \$632.50$$

A decision maker using expected monetary value as a strategy will choose the maximum of the expected monetary values computed for each decision alternative.

Maximum of $\{\$850, \$515, \$525, \$632.5\} = \$850$

The maximum of the expected monetary values is \$850, which is produced from a stock investment. An EMV chooses to invest in stocks on the basis of this information.

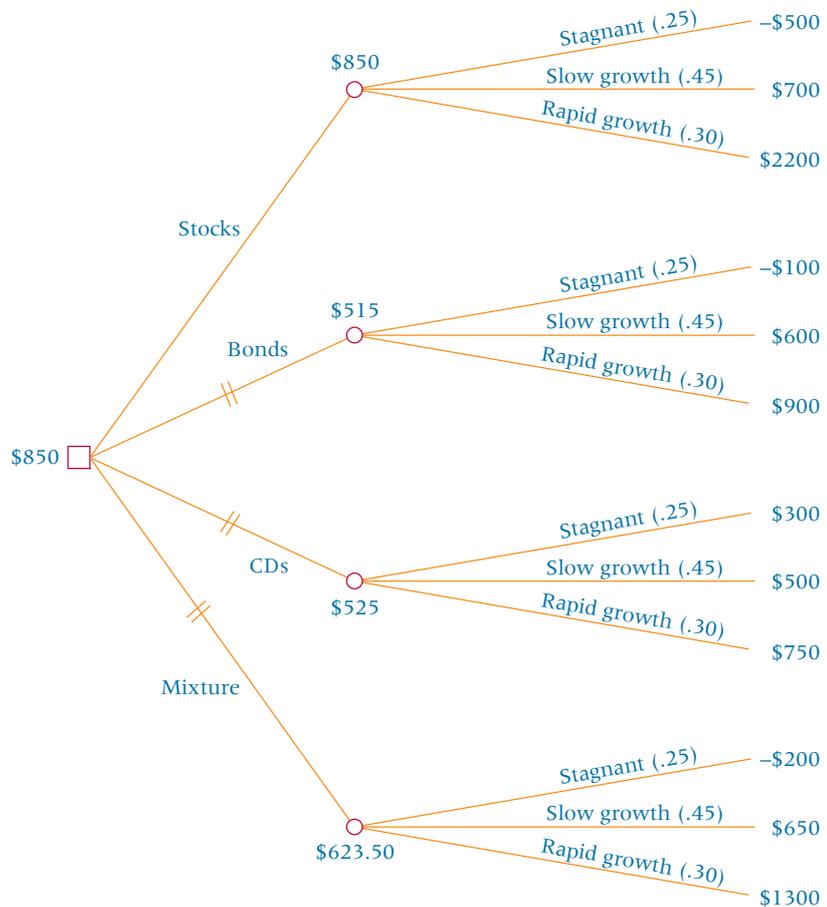
This process of expected monetary value can be depicted on decision trees like the one in Figure 19.2. Each payoff at the end of a branch of the tree is multiplied by the associated probability of that state of nature. The resulting products are summed across all states for a given decision choice, producing an expected monetary value for that decision alternative. These expected monetary values are displayed on the decision tree at the chance or state-of-nature nodes, ○.

The decision maker observes these expected monetary values. The optimal expected monetary value is the one selected and is displayed at the decision node in the tree, □. The decision alternative pathways leading to lesser, or nonoptimal, monetary values are marked with a double vertical line symbol, ||, to denote rejected decision alternatives. Figure 19.3 depicts the EMV analysis on the decision tree in Figure 19.2.

The strategy of expected monetary value is based on a long-run average. If a decision maker could “play this game” over and over with the probabilities and payoffs remaining the same, he or she could *expect* to earn an average of \$850 in the long run by choosing to invest in stocks. The reality is that for any *one* occasion, the investor will earn payoffs of either -\$500, \$700, or \$2,200 on a stock investment, depending on which state of the economy occurs. The investor will not earn \$850 at any *one* time on this decision, but he or she could average a profit of \$850 if the investment continued through time. With an investment of this size, the investor will potentially have the chance to make this decision several times. Suppose, on the other hand, an investor has to decide whether to spend \$5 million to drill an oil well. Expected monetary values might not mean as much to the decision maker if he or she has only enough financial support to make this decision once.

FIGURE 19.3

Expected Monetary Value for the Investment Example



**DEMONSTRATION
PROBLEM 19.2**

Recall the capacity decision scenario presented in Demonstration Problem 19.1. Suppose probabilities have been determined for the states of demand such that there is a .10 probability that demand will be less, a .25 probability that there will be no change in demand, a .40 probability that there will be a moderate increase in demand, and a .25 probability that there will be a large increase in demand. Use the data presented in the problem, which are restated here, and the included probabilities to compute expected monetary values and reach a decision conclusion based on these findings.

		State of Demand			
		Less (.10)	No Change (.25)	Moderate Increase (.40)	Large Increase (.25)
Capacity Decision	No Expansion	-\$3	\$2	\$3	\$6
	Add On	-\$40	-\$28	\$10	\$20
	Build a New Facility	-\$210	-\$145	-\$5	\$55

Solution

The expected monetary value for no expansion is

$$(-\$3)(.10) + (\$2)(.25) + (\$3)(.40) + (\$6)(.25) = \$2.90$$

The expected monetary value for adding on is

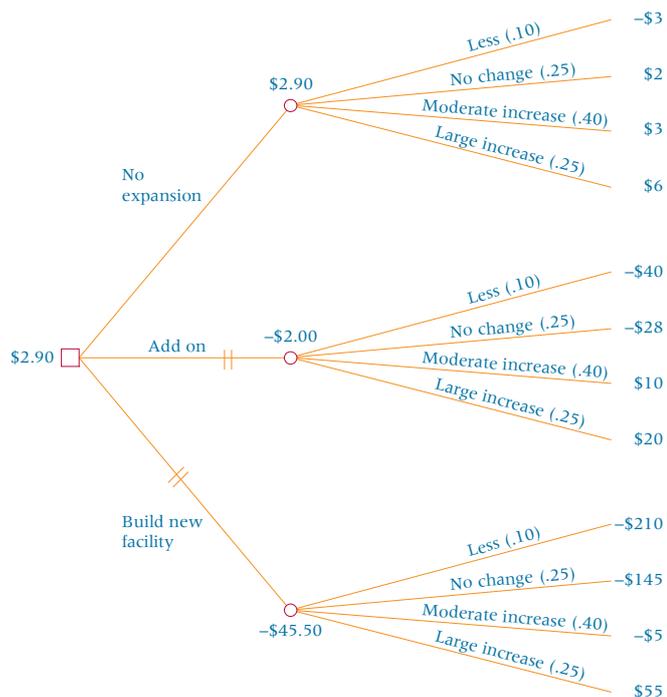
$$(-\$40)(.10) + (-\$28)(.25) + (\$10)(.40) + (\$20)(.25) = -\$2.00$$

The expected monetary value for building a new facility is

$$(-\$210)(.10) + (-\$145)(.25) + (-\$5)(.40) + (\$55)(.25) = -\$45.50$$

The decision maker who uses the EMV criterion will select the no-expansion decision alternative because it results in the highest long-run average payoff, \$2.90. It is possible that the decision maker will only have one chance to make this decision at this company. In such a case, the decision maker will not average \$2.90 for selecting no expansion but rather will get a payoff of -\$3.00, \$2.00, \$3.00, or \$6.00, depending on which state of demand follows the decision.

This analysis can be shown through the use of a decision tree.



Expected Value of Perfect Information

What is the value of knowing which state of nature will occur and when? The answer to such a question can provide insight into how much it is worth to pay for market or business research. The **expected value of perfect information** is *the difference between the payoff that would occur if the decision maker knew which states of nature would occur and the expected monetary payoff from the best decision alternative when there is no information about the occurrence of the states of nature.*

Expected Value of Perfect Information

$$= \text{Expected Monetary Payoff with Perfect Information} \\ - \text{Expected Monetary Value without Information}$$

As an example, consider the \$10,000 investment example with the probabilities of states of nature shown.

		State of the Economy		
		Stagnant (.25)	Slow Growth (.45)	Rapid Growth (.30)
Investment Decision Alternative	Stocks	-\$500	\$700	\$2,200
	Bonds	-\$100	\$600	\$900
	CDs	\$300	\$500	\$750
	Mixture	-\$200	\$650	\$1,300

The following expected monetary values were computed for this problem.

Stocks	\$850
Bonds	515
CDs	525
Mixture	632.50

The investment of stocks was selected under the expected monetary value strategy because it resulted in the maximum expected payoff of \$850. This decision was made with no information about the states of nature. Suppose we could obtain information about the states of the economy; that is, we *know* which state of the economy will occur. Whenever the state of the economy is stagnant, we would invest in CDs and receive a payoff of \$300. Whenever the state of the economy is slow growth, we would invest in stocks and earn \$700. Whenever the state of the economy is rapid growth, we would also invest in stocks and earn \$2,200. Given the probabilities of each state of the economy occurring, we can use these payoffs to compute an expected monetary payoff of perfect information.

Expected Monetary Payoff of Perfect Information

$$= (\$300)(.25) + (\$700)(.45) + (\$2,200)(.30) \\ = \$1,050$$

The difference between this expected monetary payoff with perfect information (\$1,050) and the expected monetary payoff with no information (\$850) is the value of perfect information (\$1,050 - \$850 = \$200). It would not be economically wise to spend more than \$200 to obtain perfect information about these states of nature.

DEMONSTRATION PROBLEM 19.3

Compute the value of perfect information for the capacity problem discussed in Demonstration Problems 19.1 and 19.2. The data are shown again here.

		State of Demand			
		No Change Less (.10)	No Change (.25)	Moderate Increase (.40)	Large Increase (.25)
Capacity Decision	No Expansion	-\$3	\$2	\$3	\$6
	Add On	-\$40	-\$28	\$10	\$20
	Build a New Facility	-\$210	-\$145	-\$5	\$55

Solution

The expected monetary value (payoff) under no information computed in Demonstration Problem 19.2 was \$2.90 (recall that all figures are in \$ millions). If the decision makers had perfect information, they would select no expansion for the state of less demand, no expansion for the state of no change, add on for the state of moderate increase, and build a new facility for the state of large increase. The expected payoff of perfect information is computed as

$$(-\$3)(.10) + (\$2)(.25) + (\$10)(.40) + (\$55)(.25) = \$17.95$$

The expected value of perfect information is

$$\$17.95 - \$2.90 = \$15.05$$

In this case, the decision makers might be willing to pay up to \$15.05 (\$ million) for perfect information.

Utility

As pointed out in the preceding section, expected monetary value decisions are based on long-run averages. Some situations do not lend themselves to expected monetary value analysis because these situations involve relatively large amounts of money and one-time decisions. Examples of these one-time decisions might be drilling an oil well, building a new production facility, merging with another company, ordering 100 new 737s, or buying a professional sports franchise. In analyzing the alternatives in such decisions, a concept known as utility can be helpful.

Utility is the degree of pleasure or displeasure a decision maker has in being involved in the outcome selection process given the risks and opportunities available. Suppose a person has the chance to enter a contest with a 50–50 chance of winning \$100,000. If the person wins the contest, he or she wins \$100,000. If the person loses, he or she receives \$0. There is no cost to enter this contest. The expected payoff of this contest for the entrant is

$$(\$100,000)(.50) + (\$0)(.50) = \$50,000$$

In thinking about this contest, the contestant realizes that he or she will never get \$50,000. The \$50,000 is the long-run average payoff if the game is played over and over. Suppose contest administrators offer the contestant \$30,000 not to play the game. Would the player take the money and drop out of the contest? Would a certain payoff of \$30,000 outdraw a .50 chance at \$100,000? The answer to this question depends, in part, on the person's financial situation and on his or her propensity to take risks. If the contestant is a multimillionaire, he or she might be willing to take big risks and even refuse \$70,000 to drop out of the contest, because \$70,000 does not significantly increase his or her worth. On the other hand, a person on welfare who is offered \$20,000 not to play the contest might take the money because \$20,000 is worth a great deal to him or her. In addition, two different people on welfare might have different risk-taking profiles. One might be a risk taker who, in spite of a need for money, is not willing to take less than \$70,000 or \$80,000 to pull out of a contest. The same could be said for the wealthy person.

Utility theory provides a mechanism for determining whether a person is a risk taker, a risk avoider, or an EMVer for a given decision situation. Consider the contest just described. A person receives \$0 if he or she does not win the contest and \$100,000 if he or she does win the contest. How much money would it take for a contestant to be indifferent between participating in the contest and dropping out? Suppose we examine three possible contestants, X, Y, and Z.

X is indifferent between receiving \$20,000 and a .50 chance of winning the contest. For any amount more than \$20,000, X will take the money and not play the game. As we stated before, a .50 chance of winning yields an expected payoff of \$50,000. Suppose we increase the chance of winning to .80, so that the expected monetary payoff is \$80,000. Now X is indifferent between receiving \$50,000 and playing the game and will drop out of the game for any amount more than \$50,000. In virtually all cases, X is willing to take less money than the expected payoff to quit the game. X is referred to as a **risk avoider**. Many of us are

risk avoiders. For this reason, we pay insurance companies to cover our personal lives, our homes, our businesses, our cars, and so on, even when we know the odds are in the insurance companies' favor. We see the potential to lose at such games as unacceptable, so we bail out of the games for less than the expected payoff and pay out more than the expected cost to avoid the game.

Y, on the other hand, loves such contests. It would take about \$70,000 to get Y not to play the game with a .50 chance of winning \$100,000, even though the expected payoff is only \$50,000. Suppose Y were told that there was only a .20 chance of winning the game. How much would it take for Y to become indifferent to playing? It might take \$40,000 for Y to be indifferent, even though the expected payoff for a .20 chance is only \$20,000. Y is a **risk taker** and enjoys playing risk-taking games. It always seems to take more than the expected payoff to get Y to drop out of the contest.

Z is an EMVer. Z is indifferent between receiving \$50,000 and having a .50 chance of winning \$100,000. To get Z out of the contest if there is only a .20 chance of winning, the contest directors would have to offer Z about \$20,000 (the expected value). Likewise, if there were an .80 chance of winning, it would take about \$80,000 to get Z to drop out of the contest. Y makes a decision by going with the long-run averages even in one-time decisions.

Figure 19.4 presents a graph with the likely shapes of the utility curves for X, Y, and Z. This graph is constructed for the game using the payoff range of \$0 to \$100,000; in-between values can be offered to the players in an effort to buy them out of the game. These units are displayed along what is normally the x axis. Along the y axis are the probabilities of winning the game, ranging from .0 to 1.0. A straight line through the middle of the values represents the EMV responses. If a person plays the game with a .50 chance of winning, he or she is indifferent to taking \$50,000 not to play and playing. For .20, it is \$20,000. For .80, it is \$80,000.

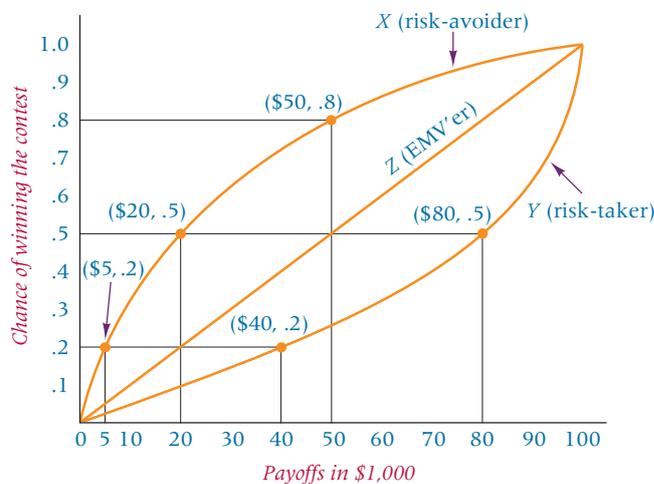
Notice in the graph that where the chance of winning is .50, contestant X is willing to drop out of the game for \$20,000. This point, (\$20,000, .50), is above the EMV line. When the chance is .20, X will drop out for \$5,000; for a chance of .80, X will drop out for \$50,000. Both of these points, (\$5,000, .20) and (\$50,000, .80), are above the EMV line also.

Y, in contrast, requires \$80,000 to be indifferent to a .50 chance of winning. Hence, the point (\$80,000, .50) is below the EMV line. Contest officials will have to offer Y at least \$40,000 for Y to become indifferent to a .20 chance of winning. This point, (\$40,000, .20), also is below the EMV line.

X is a risk avoider and Y is a risk taker. Z is an EMVer. In the utility graph in Figure 19.4, the risk avoider's curve is above the EMV line and the risk taker's curve is below the line.

As discussed earlier in the chapter, in making decisions under uncertainty risk takers might be more prone to use the maximax criterion and risk avoiders might be more prone to use the maximin criterion. The Hurwicz criterion allows the user to introduce his or her propensity toward risk into the analysis by using alpha.

FIGURE 19.4
Risk Curves for Game Players



Much information has been compiled and published about utility theory. The objective here is to give you a brief introduction to it through this example, thus enabling you to see that there are risk takers and risk avoiders along with EMVers. A more detailed treatment of this topic is beyond the scope of this text.

19.3 PROBLEMS

19.5 Use the following decision table to construct a decision tree.

		State of Nature				
		s_1	s_2	s_3	s_4	s_5
Decision Alternative	d_1	50	20	15	5	1
	d_2	75	50	20	-5	-20
	d_3	15	12	10	8	6

19.6 Suppose the probabilities of the states of nature occurring for Problem 19.5 are $s_1 = .15$, $s_2 = .25$, $s_3 = .30$, $s_4 = .10$, and $s_5 = .20$. Use these probabilities and expected monetary values to reach a conclusion about the decision alternatives in Problem 19.5.

19.7 How much is the expected monetary payoff with perfect information in Problem 19.5? From this answer and the decision reached in Problem 19.6, what is the value of perfect information?

19.8 Use the following decision table to complete parts (a) through (c).

		State of Nature		
		$s_1 (.40)$	$s_2 (.35)$	$s_3 (.25)$
Decision Alternative	d_1	150	250	500
	d_2	100	200	400
	d_3	75	150	700
	d_4	125	450	650

- Draw a decision tree to represent this payoff table.
 - Compute the expected monetary values for each decision and label the decision tree to indicate what the final decision would be.
 - Compute the expected payoff of perfect information. Compare this answer to the answer determined in part (b) and compute the value of perfect information.
- 19.9 A home buyer is completing application for a home mortgage. The buyer is given the option of “locking in” a mortgage loan interest rate or waiting 60 days until closing and locking in a rate on the day of closing. The buyer is not given the option of locking in at any time in between. If the buyer locks in at the time of application and interest rates go down, the loan will cost the buyer \$150 per month more ($-\150 payoff) than it would have if he or she had waited and locked in later. If the buyer locks in at the time of application and interest rates go up, the buyer has saved money by locking in at a lower rate. The amount saved under this condition is a payoff of $+\$200$. If the buyer does not lock in at application and rates go up, he or she must pay more interest on the mortgage loan; the payoff is $-\$250$. If the buyer does not lock in at application and rates go down, he or she has reduced the interest amount and the payoff is $+\$175$. If the rate does not change at all, there is a \$0 payoff for locking in at the time of application and also a \$0 payoff for not locking in at that time. There is a probability of .65 that the interest rates will rise by the end of the 60-day period, a .30 probability that they will fall, and a .05 probability that they will remain constant. Construct a decision table from this information.

Compute the expected monetary values from the table and reach a conclusion about the decision alternatives. Compute the value of perfect information.

- 19.10** A CEO faces a tough human resources decision. Because the company is currently operating in a budgetary crisis, the CEO will either lay off 1,000 people, lay off 5,000 people, or lay off no one. One of the problems for the CEO is that she cannot foretell what the business climate will be like in the coming months. If the CEO knew there would be a rapid rise in demand for the company’s products, she might be inclined to hold off on layoffs and attempt to retain the workforce. If the business climate worsens, however, big layoffs seem to be the reasonable decision. Shown here are payoffs for each decision alternative under each state of the business climate. Included in the payoffs is the cost (loss of payoff) to the company when workers are laid off. The probability of each state occurring is given. Use the table and the information given to compute expected monetary values for each decision alternative and make a recommendation based on these findings. What is the most the CEO should be willing to pay for information about the occurrence of the various states of the business climate?

		State of Business Climate		
		Improved (.10)	About the Same (.40)	Worsened (.50)
Decision Alternative	No Layoffs	100	-300	-1700
	Lay Off 1000	-100	100	-700
	Lay Off 5000	-200	300	600

- 19.11** A person has a chance to invest \$50,000 in a business venture. If the venture works, the investor will reap \$200,000. If the venture fails, the investor will lose his money. It appears that there is about a .50 probability of the venture working. Using this information, answer the following questions.
- What is the expected monetary value of this investment?
 - If this person decides not to undertake this venture, is he an EMVer, a risk avoider, or a risk taker? Why?
 - You would have to offer at least how much money to get a risk taker to quit pursuing this investment?



19.4 REVISING PROBABILITIES IN LIGHT OF SAMPLE INFORMATION

In Section 19.3 we discussed decision making under risk in which the probabilities of the states of nature were available and included in the analysis. Expected monetary values were computed on the basis of payoffs and probabilities. In this section we include the additional aspect of sample information in the decision analysis. If decision makers opt to purchase or in some other manner garner sample information, the probabilities of states of nature can be revised. The revisions can be incorporated into the decision-making process, resulting—one hopes—in a better decision.

In Chapter 4 we examined the use of Bayes’ rule in the revision of probabilities, in which we started with a prior probability and revised it on the basis of some bit of information. Bayesian analysis can be applied to decision making under risk analysis to revise the prior probabilities of the states of nature, resulting in a clearer picture of the decision options. Usually the securing of additional information through sampling entails a cost. After the discussion of the revision of probabilities, we will examine the worth of sampling information. Perhaps the best way to explain the use of Bayes’ rule in revising prior state-of-nature probabilities is by using an example.

Let us examine the revision of probabilities by using Bayes’ rule with sampling information in the context of the \$10,000 investment example discussed earlier in the chapter.

Because the problem as previously stated is too complex to provide a clear example of this process, it is reduced here to simpler terms. The problem is still to determine how best to invest \$10,000 for the next year. However, only two decision alternatives are available to the investor: bonds and stocks. Only two states of the investment climate can occur: no growth or rapid growth.

There is a .65 probability of no growth in the investment climate and a .35 probability of rapid growth. The payoffs are \$500 for a bond investment in a no-growth state, \$100 for a bond investment in a rapid-growth state, -\$200 for a stock investment in a no-growth state, and a \$1,100 payoff for a stock investment in a rapid-growth state. Table 19.6 presents the decision table (payoff table) for this problem.

Figure 19.5 shows a decision tree with the decision alternatives, the payoffs, the states of nature, the probabilities, and the expected monetary values of each decision alternative. The expected monetary value for the bonds decision alternative is

$$EMV(\text{bonds}) = \$500(.65) + \$100(.35) = \$360$$

The expected monetary value for the stocks decision alternative is

$$EMV(\text{stocks}) = -\$200(.65) + \$1,100(.35) = \$255$$

An EMV'er would select the bonds decision alternative because the expected monetary value is \$360, which is higher than that of the stocks alternative (\$255).

Suppose the investor has a chance to obtain some information from an economic expert about the future state of the investment economy. This expert does not have a perfect record of forecasting, but she has predicted a no-growth economy about .80 of the time when such a state actually occurred. She has been slightly less successful in predicting rapid-growth economies, with a .70 probability of success. The following table shows her success and failure rates in forecasting these two states of the economy.

	Actual State of the Economy	
	No Growth (s_1)	Rapid Growth (s_2)
Forecaster predicts no growth (F_1)	.80	.30
Forecaster predicts rapid growth (F_2)	.20	.70

When the state of the economy is no growth, the forecaster will predict no growth .80 of the time, but she will predict rapid growth .20 of the time. When the state of the economy is rapid growth, she will predict rapid growth .70 of the time and no growth .30 of the time.

Using these conditional probabilities, we can revise prior probabilities of the states of the economy by using Bayes' rule, restated here from Chapter 4.

$$P(X_i|Y) = \frac{P(X_i) \cdot P(Y|X_i)}{P(X_1) \cdot P(Y|X_1) + P(X_2) \cdot P(Y|X_2) + \dots + P(X_n) \cdot P(Y|X_n)}$$

TABLE 19.6
Decision Table for the Investment Example

Decision Alternative		State of Nature	
		No Growth (.65)	Rapid Growth (.35)
Bonds		\$500	\$ 100
Stocks		-\$200	\$1,100

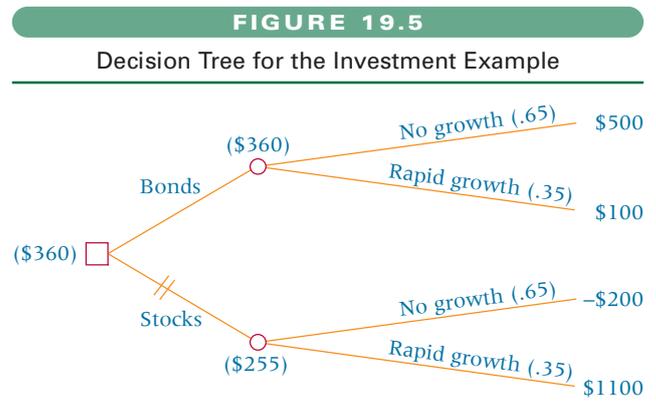


TABLE 19.7

Revision Based on a Forecast of No Growth (F_1)

State of Economy	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Revised Probabilities
No growth (s_1)	$P(s_1) = .65$	$P(F_1 s_1) = .80$	$P(F_1 \cap s_1) = .520$	$.520/.625 = .832$
Rapid growth (s_2)	$P(s_2) = .35$	$P(F_1 s_2) = .30$	$P(F_1 \cap s_2) = .105$	$.105/.625 = .168$
$P(F_1) = .625$				

Applying the formula to the problem, we obtain the revised probabilities shown in the following tables. Suppose the forecaster predicts no growth (F_1). The prior probabilities of the states of the economy are revised as shown in Table 19.7.

$P(F_1)$ is computed as follows.

$$P(F_1) = P(F_1 \cap s_1) + P(F_1 \cap s_2) = .520 + .105 = .625$$

The revised probabilities are computed as follows

$$P(s_1|F_1) = \frac{P(F_1 \cap s_1)}{P(F_1)} = \frac{.520}{.625} = .832$$

$$P(s_2|F_1) = \frac{P(F_1 \cap s_2)}{P(F_1)} = \frac{.105}{.625} = .168$$

The prior probabilities of the states of the economy are revised as shown in Table 19.8 for the case in which the forecaster predicts rapid growth (F_2).

These revised probabilities can be entered into a decision tree that depicts the option of buying information and getting a forecast, as shown in Figure 19.6. Notice that the first node is a decision node to buy the forecast. The next node is a state-of-nature node, where the forecaster will predict either a no-growth economy or a rapid-growth economy. It is a state of nature because the decision maker has no control over what the forecast will be. As a matter of fact, the decision maker has probably paid for this “independent” forecast. Once a forecast is made, the decision maker is faced with the decision alternatives of investing in bonds or investing in stocks. At the end of each investment alternative branch is a state of the economy of either no growth or rapid growth. The four revised probabilities calculated in Tables 19.7 and 19.8 are assigned to these states of economy. The payoffs remain the same. The probability of the forecaster predicting no growth comes from the sum of the joint probabilities in Table 19.7. This value of $P(F_1) = .625$ is assigned a position on the first set of states of nature (forecast). The probability of the forecaster predicting rapid growth comes from summing the joint probabilities in Table 19.8. This value of $P(F_2) = .375$ is also assigned a position on the first set of states of nature (forecasts).

The decision maker can make a choice from this decision tree after the expected monetary values are calculated.

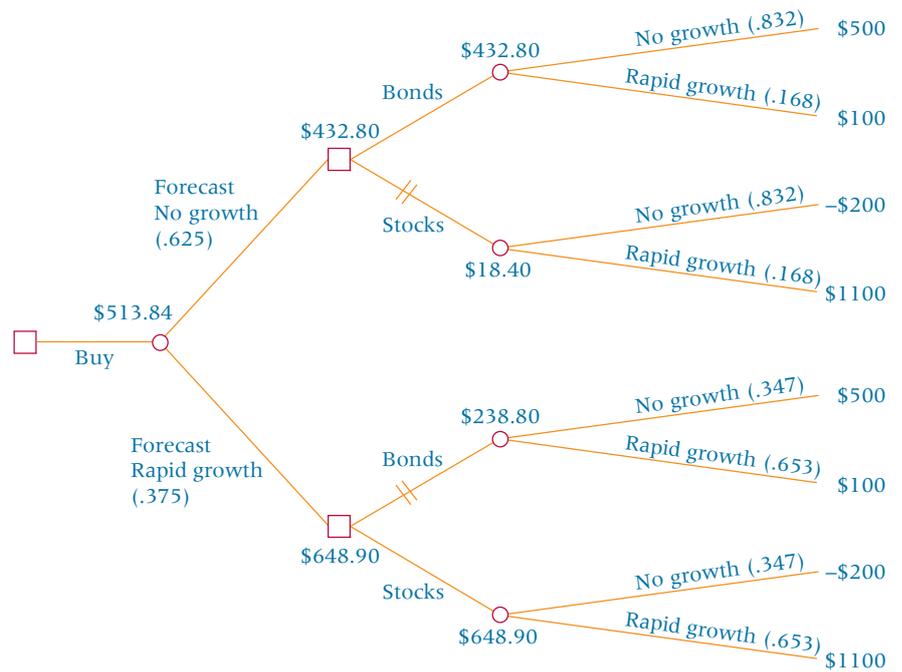
In Figure 19.6, the payoffs are the same as in the decision table without information. However, the probabilities of no-growth and rapid-growth states have been revised. Multiplying the payoffs by these revised probabilities and summing them for each investment produces expected monetary values at the state-of-economy nodes. Moving back to the decision nodes preceding these values, the investor has the opportunity to invest in either bonds or stocks. The investor examines the expected monetary values and selects the investment with the highest value. For the decision limb in which the forecaster predicted no growth, the investor selects the bonds investment, which yields an expected monetary value of \$432.80 (as opposed to \$18.40 from stocks). For the decision limb in which the

TABLE 19.8

Revision Based on a Forecast of Rapid Growth (F_2)

State of Economy	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Revised Probabilities
No growth (s_1)	$P(s_1) = .65$	$P(F_2 s_1) = .20$	$P(F_2 \cap s_1) = .130$	$.130/.375 = .347$
Rapid growth (s_2)	$P(s_2) = .35$	$P(F_2 s_2) = .70$	$P(F_2 \cap s_2) = .245$	$.245/.375 = .653$
$P(F_2) = .375$				

FIGURE 19.6
Decision Tree for the Investment Example after Revision of Probabilities



forecaster predicted rapid growth, the investor selects the stocks investment, which yields an expected monetary value of \$648.90 (as opposed to \$238.80 for bonds).

The investor is thus faced with the opportunity to earn an expected monetary value of \$432.80 if the forecaster predicts no growth or \$648.90 if the forecaster predicts rapid growth. How often does the forecaster predict each of these states of the economy to happen? Using the sums of the joint probabilities from Tables 19.7 and 19.8, the decision maker gets the probabilities of each of these forecasts.

$$P(F_1) = .625 \text{ (no growth)}$$

$$P(F_2) = .375 \text{ (rapid growth)}$$

Entering these probabilities into the decision tree at the first probability node with the forecasts of the states of the economy and multiplying them by the expected monetary value of each state yields an overall expected monetary value of the opportunity.

$$\text{EMV for Opportunity} = \$432.80(.625) + \$648.90(.375) = \$513.84$$

Expected Value of Sample Information

The preceding calculations for the investment example show that the expected monetary value of the opportunity is \$513.84 with sample information, but it is only \$360 without sample information, as shown in Figure 19.5. Using the sample information appears to profit the decision maker.

$$\text{Apparent Profit of Using Sample Information} = \$513.84 - \$360 = \$153.84$$

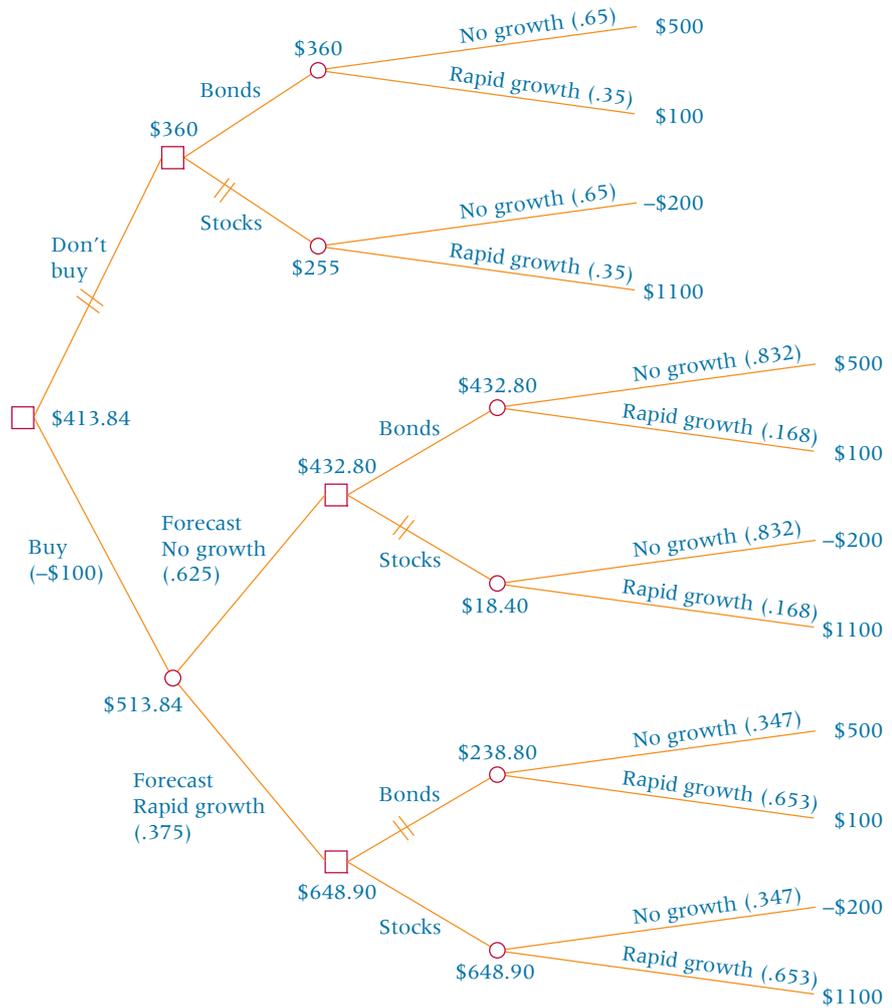
How much did this sample information cost? If the sample information is not free, less than \$153.84 is gained by using it. How much is it worth to use sample information? Obviously, the decision maker should not pay more than \$153.84 for sample information because an expected \$360 can be earned without the information. In general, the **expected value of sample information** is worth no more than *the difference between the expected monetary value with the information and the expected monetary value without the information.*

EXPECTED VALUE OF SAMPLE INFORMATION

$$\begin{aligned} \text{Expected Value of Sample Information} \\ &= \text{Expected Monetary Value with Information} \\ &\quad - \text{Expected Monetary Value without Information} \end{aligned}$$

FIGURE 19.7

Decision Tree for the Investment Example—All Options Included



Suppose the decision maker had to pay \$100 for the forecaster’s prediction. The expected monetary value of the decision with information shown in Figure 19.6 is reduced from \$513.84 to \$413.84, which is still superior to the \$360 expected monetary value without sample information. Figure 19.7 is the decision tree for the investment information with the options of buying the information or not buying the information included. The tree is constructed by combining the decision trees from Figures 19.5 and 19.6 and including the cost of buying information (\$100) and the expected monetary value with this purchased information (\$413.84).

DEMONSTRATION PROBLEM 19.4

In Demonstration Problem 19.1, the decision makers were faced with the opportunity to increase capacity to meet a possible increase in product demand. Here we reduced the decision alternatives and states of nature and altered the payoffs and probabilities. Use the following decision table to create a decision tree that displays the decision alternatives, the payoffs, the probabilities, the states of demand, and the expected monetary payoffs. The decision makers can buy information about the states of demand for \$5 (recall that amounts are in \$ millions). Incorporate this fact into your decision. Calculate the expected value of sampling information for this problem.

The decision alternatives are: no expansion or build a new facility. The states of demand and prior probabilities are: less demand (.20), no change (.30), or large increase (.50).

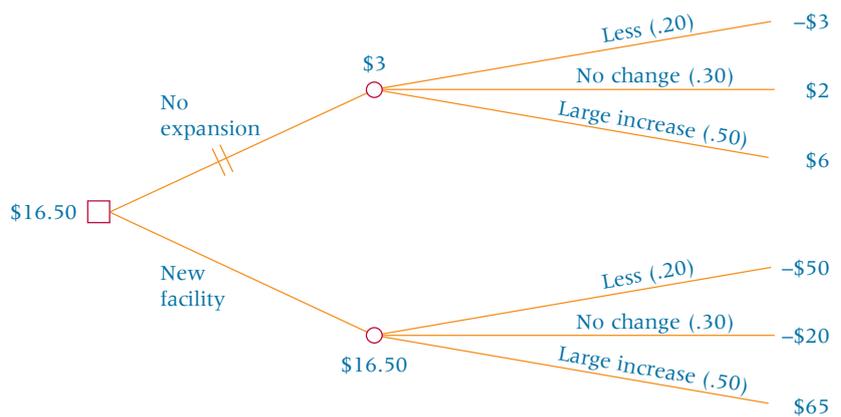
		State of Demand		
		Less (.20)	No Change (.30)	Large Increase (.50)
Decision	No Expansion	-\$ 3	\$ 2	\$ 6
Alternative	New Facility	-\$50	-\$20	\$65

The state-of-demand forecaster has historically not been accurate 100% of the time. For example, when the demand was less, the forecaster correctly predicted it .75 of the time. When there was no change in demand, the forecaster correctly predicted it .80 of the time. Sixty-five percent of the time the forecaster correctly forecast large increases when large increases occurred. Shown next are the probabilities that the forecaster will predict a particular state of demand under the actual states of demand.

		State of Demand		
		Less	No Change	Large Increase
Forecast	Less	.75	.10	.05
	No Change	.20	.80	.30
	Large Increase	.05	.10	.65

Solution

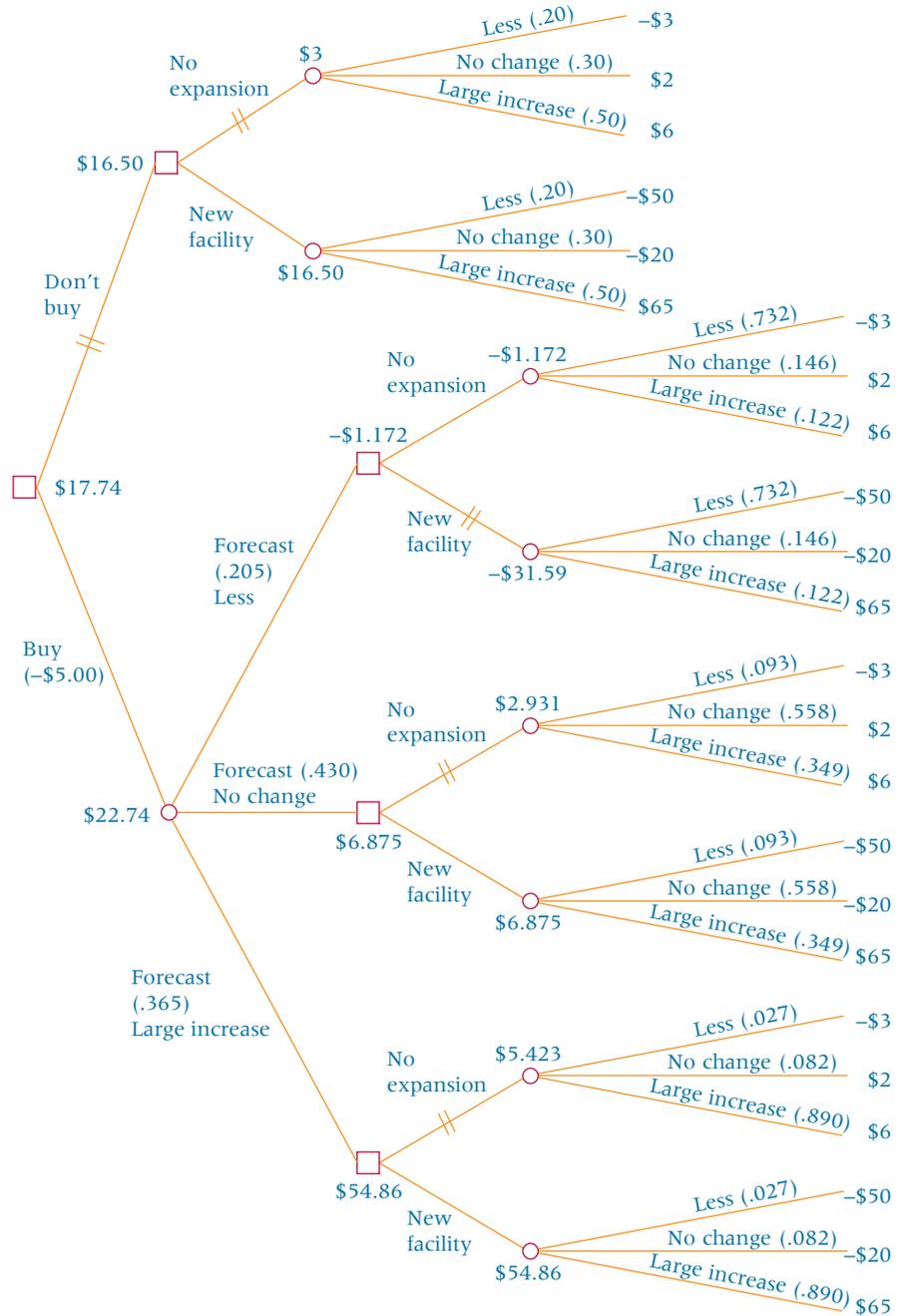
The following figure is the decision tree for this problem when no sample information is purchased.



In light of sample information, the prior probabilities of the three states of demand can be revised. Shown here are the revisions for F_1 (forecast of less demand), F_2 (forecast of no change in demand), and F_3 (forecast of large increase in demand).

State of Demand	Prior Probability	Conditional Probability	Joint Probability	Revised Probability
For Forecast of Less Demand (F_1)				
Less (s_1)	.20	$P(F_1 s_1) = .75$	$P(F_1 \cap s_1) = .150$	$.150/.205 = .732$
No change (s_2)	.30	$P(F_1 s_2) = .10$	$P(F_1 \cap s_2) = .030$	$.030/.205 = .146$
Large increase (s_3)	.50	$P(F_1 s_3) = .05$	$P(F_1 \cap s_3) = .025$	$.025/.205 = .122$
			$P(F_1) = .205$	
For Forecast of No Change in Demand (F_2)				
Less (s_1)	.20	$P(F_2 s_1) = .20$	$P(F_2 \cap s_1) = .040$	$.040/.430 = .093$
No change (s_2)	.30	$P(F_2 s_2) = .80$	$P(F_2 \cap s_2) = .240$	$.240/.430 = .558$
Large increase (s_3)	.50	$P(F_2 s_3) = .30$	$P(F_2 \cap s_3) = .150$	$.150/.430 = .349$
			$P(F_2) = .430$	
For Forecast of Large Increase in Demand (F_3)				
Less (s_1)	.20	$P(F_3 s_1) = .05$	$P(F_3 \cap s_1) = .010$	$.010/.365 = .027$
No change (s_2)	.30	$P(F_3 s_2) = .10$	$P(F_3 \cap s_2) = .030$	$.030/.365 = .082$
Large increase (s_3)	.50	$P(F_3 s_3) = .65$	$P(F_3 \cap s_3) = .325$	$.325/.365 = .890$
			$P(F_3) = .365$	

From these revised probabilities and other information, the decision tree containing alternatives and states using sample information can be constructed. The following figure is the decision tree containing the sample information alternative *and* the portion of the tree for the alternative of no sampling information.



If the decision makers calculate the expected monetary value after buying the sample information, they will see that the value is \$17.74. The final expected monetary value with sample information is calculated as follows.

$$\text{EMV at Buy Node: } - \$1.172(.205) + \$6.875(.430) + \$54.86(.365) = \$22.74$$

However, the sample information cost \$5. Hence, the net expected monetary value at the buy node is

$$\$22.74 \text{ (EMV)} - \$5.00 \text{ (cost of information)} = \$17.74 \text{ (net expected monetary value)}$$

The worth of the sample information is

$$\begin{aligned}
 &\text{Expected Monetary Value of Sample Information} \\
 &= \text{Expected Monetary Value with Sample Information} \\
 &\quad - \text{Expected Monetary Value without Sample Information} \\
 &= \$22.74 - \$16.50 = \$6.24
 \end{aligned}$$

19.4 PROBLEMS

19.12 Shown here is a decision table from a business situation. The decision maker has an opportunity to purchase sample information in the form of a forecast. With the sample information, the prior probabilities can be revised. Also shown are the probabilities of forecasts from the sample information for each state of nature. Use this information to answer parts (a) through (d).

		State of Nature	
		$s_1 (.30)$	$s_2 (.70)$
Alternative	d_1	\$350	-\$100
	d_2	-\$200	\$325

		State of Nature	
		s_1	s_2
Forecast	s_1	.90	.25
	s_2	.10	.75

- Compute the expected monetary value of this decision without sample information.
 - Compute the expected monetary value of this decision with sample information.
 - Use a tree diagram to show the decision options in parts (a) and (b).
 - Calculate the value of the sample information.
- 19.13 a.** A car rental agency faces the decision of buying a fleet of cars, all of which will be the same size. It can purchase a fleet of small cars, medium cars, or large cars. The smallest cars are the most fuel efficient and the largest cars are the greatest fuel users. One of the problems for the decision makers is that they do not know whether the price of fuel will increase or decrease in the near future. If the price increases, the small cars are likely to be most popular. If the price decreases, customers may demand the larger cars. Following is a decision table with these decision alternatives, the states of nature, the probabilities, and the payoffs. Use this information to determine the expected monetary value for this problem.

		State of Nature	
		Fuel Decrease (.60)	Fuel Increase (.40)
Decision Alternative	Small Cars	-\$225	\$425
	Medium Cars	\$125	-\$150
	Large Cars	\$350	-\$400

- The decision makers have an opportunity to purchase a forecast of the world oil markets that has some validity in predicting gasoline prices. The following matrix gives the probabilities of these forecasts being correct for various states of nature. Use this information to revise the prior probabilities and recompute the expected monetary value on the basis of sample information. What is the

expected value of sample information for this problem? Should the agency decide to buy the forecast?

		State of Nature	
		Fuel Decrease	Fuel Increase
Forecast	Fuel Decrease	.75	.15
	Fuel Increase	.25	.85

- 19.14 a. A small group of investors is considering planting a tree farm. Their choices are (1) don't plant trees, (2) plant a small number of trees, or (3) plant a large number of trees. The investors are concerned about the demand for trees. If demand for trees declines, planting a large tree farm would probably result in a loss. However, if a large increase in the demand for trees occurs, not planting a tree farm could mean a large loss in revenue opportunity. They determine that three states of demand are possible: (1) demand declines, (2) demand remains the same as it is, and (3) demand increases. Use the following decision table to compute an expected monetary value for this decision opportunity.

		State of Demand		
		Decline (.20)	Same (.30)	Increase (.50)
Decision Alternative	Don't Plant	\$20	\$0	-\$40
	Small Tree Farm	-\$90	\$10	\$175
	Large Tree Farm	-\$600	-\$150	\$800

- b. Industry experts who believe they can forecast what will happen in the tree industry contact the investors. The following matrix shows the probabilities with which it is believed these "experts" can foretell tree demand. Use these probabilities to revise the prior probabilities of the states of nature and recompute the expected value of sample information. How much is this sample information worth?

		State of Demand		
		Decrease	Same	Increase
Forecast	Decrease	.70	.02	.02
	Same	.25	.95	.08
	Increase	.05	.03	.90

- 19.15 a. Some oil speculators are interested in drilling an oil well. The rights to the land have been secured and they must decide whether to drill. The states of nature are that oil is present or that no oil is present. Their two decision alternatives are drill or don't drill. If they strike oil, the well will pay \$1 million. If they have a dry hole, they will lose \$100,000. If they don't drill, their payoffs are \$0 when oil is present and \$0 when it is not. The probability that oil is present is .11. Use this information to construct a decision table and compute an expected monetary value for this problem.
- b. The speculators have an opportunity to buy a geological survey, which sometimes helps in determining whether oil is present in the ground. When the geologists say there is oil in the ground, there actually is oil .20 of the time. When there is oil in the ground, .80 of the time the geologists say there is no oil. When there is no oil in the ground, .90 of the time the geologists say there is no oil. When there is no oil in the ground, .10 of the time the geologists say there is oil. Use this information to revise the prior probabilities of oil being present in the ground and compute the expected monetary value based on sample information. What is the value of the sample information for this problem?



Decision Making at the CEO Level

The study of CEOs revealed that decision making takes place in many different areas of business. No matter what the decision concerns, it is critical for the CEO or manager to identify the decision alternatives. Sometimes decision alternatives are not obvious and can be identified only after considerable examination and brainstorming. Many different alternatives are available to decision makers in personnel, finance, operations, and so on. Alternatives can sometimes be obtained from worker suggestions and input. Others are identified through consultants or experts in particular fields. Occasionally, a creative and unobvious decision alternative is derived that proves to be the most successful choice.

Alex Trotman at Ford Motor, in a reorganization decision, chose the alternative of combining two operations into one unit. Other alternatives might have been to combine other operations into a unit (rather than the North American and European), create more units, or not reorganize at all. At Kodak, CEO George Fisher made the decision that the company would adopt digital and electronic imaging wholeheartedly. In addition, he determined that these new technologies would be interwoven with their paper and film products in such a manner as to be “seamless.” Fisher had other alternatives available such as not entering the arena of digital and electronic imaging or entering it but keeping it separated from the paper and film operation. Union Pacific was faced with a crisis as it watched Burlington Northern make an offer to buy Santa Fe Pacific. The CEO chose to propose a counteroffer. He could have chosen to not enter the fray.

CEOs need to identify as many states of nature that can occur under the decision alternatives as possible. What might happen to sales? Will product demand increase or decrease? What is the political climate for environmental or international monetary regulation? What will occur next in the business cycle? Will there be inflation? What will the competitors do? What new inventions or developments will occur? What is the investment climate? Identifying as many of these states as possible helps the decision maker examine decision alternatives in light of those states and calculate payoffs accordingly.

Many different states of nature may arise that will affect the outcome of CEO decisions made in the 1990s. Ford Motor may find that the demand for a “world car” does not materialize so slowly that the company wastes their effort for many years, materializes as Trotman foresaw, or materializes even faster. The world economy might undergo a depression, a slowdown, a constant growth, or even an accelerated rate of growth. Political conditions in countries of the world might make an American “world car” unacceptable. The governments of the countries that would be markets for such a car might cause the countries to become more a part of the world

economy, stay about the same, slowly withdraw from the world scene, or become isolated.

States of nature can impact a CEO’s decision in other ways. The rate of growth and understanding of technology is uncertain in many ways and can have a great effect on the decision to embrace digital and electronic imaging. Will the technology develop in time for the merging of these new technologies and the paper and film operations? Will there be suppliers who can provide materials and parts? What about the raw materials used in digital and electronic imaging? Will there be an abundance, a shortage, or an adequate supply of raw materials? Will the price of raw materials fluctuate widely, increase, decrease, or remain constant?

The decision maker should recognize whether he or she is a risk avoider or a risk taker. Does propensity toward risk vary by situation? Should it? How do the board of directors and stockholders view risk? Will the employees respond to risk taking or avoidance?

Successful CEOs may well incorporate risk taking, risk avoidance, and expected value decision making into their decisions. Perhaps the successful CEOs know when to take risks and when to pull back. In Union Pacific’s decision to make a counteroffer for Santa Fe Pacific, risk taking is evident. Of course with the possibility of Burlington Northern growing into a threateningly large competitor, it could be argued that making a counteroffer was actually a risk averse decision. Certainly, the decision by a successful company like Ford Motor, which had five of the top 10 vehicles at the time, to reorganize in an effort to make a “world car” is risk taking. Kodak’s decision to embrace digital and electronic imaging and merge it with their paper and film operations is a risk-taking venture.

If successful, the payoffs from these CEO decisions could be great. The current success of Ford Motor may just scratch the surface if the company successfully sells their “world car” in the twenty-first century. On the other hand, the company could experience big losses or receive payoffs somewhere in between. Union Pacific’s purchasing of Santa Fe Pacific could greatly increase their market share and result in huge dividends to the company. A downturn in transportation, the unforeseen development of some more efficient mode of shipping, inefficient or hopelessly irreversibly poor management in Santa Fe Pacific, or other states of nature could result in big losses to Union Pacific. MCI’s decision to wage a war with the Baby Bells did not result in immediate payoffs because of a slowing down of growth due to efforts by AT&T in the long-distance market and MCI’s difficulty in linking to a large cable company.

CEOs are not always able to visualize all decision alternatives. However, creative, inspired thinking along with the brainstorming of others and an extensive investigation of the facts and figures can successfully result in the identification of

most of the possibilities. States of nature are unknown and harder to discern. However, awareness, insight, understanding, and knowledge of economies, markets, governments, and competitors can greatly aid a decision maker in considering possible states of nature that may impact the payoff of a deci-

sion along with the likelihood that such states of nature might occur. The payoffs for CEOs range from the loss of thousands of jobs including his/her own, loss of market share, and bankruptcy of the company, to worldwide growth, record stockholder dividends, and fame.

ETHICAL CONSIDERATIONS

Ethical considerations occasionally arise in decision analysis situations. The techniques presented in this chapter are for aiding the decision maker in selecting from among decision alternatives in light of payoffs and expected values. Payoffs do not always reflect all costs, including ethical considerations. The decision maker needs to decide whether to consider ethics in examining decision alternatives. What are some decision alternatives that raise ethical questions?

Some decision alternatives are environmentally damaging in the form of ground, air, or water pollution. Other choices endanger the health and safety of workers. In the area of human resources, some decision alternatives include eliminating jobs and laying off workers. Should the ethical issues involved in these decisions be factored into payoffs? For example, what effects would a layoff have on families and communities? Does a business have any moral obligation toward its workers and its community that should be taken into consideration in payoffs? Does a

decision alternative involve producing a product that is detrimental to a customer or a customer's family? Some marketing decisions might involve the use of false or misleading advertising. Are distorted or misleading advertising attacks on competing brands unethical even when the apparent payoff is great?

States of nature are usually beyond the control of the decision maker; therefore, it seems unlikely that unethical behavior would be connected with a state of nature. However, obtaining sample or perfect information under which to make decisions about states of nature has the usual potential for unethical behavior in sampling.

In many cases, payoffs other than the dollar values assigned to a decision alternative should be considered. In using decision analysis with ethical behavior, the decision maker should attempt to factor into the payoffs the cost of pollution, safety features, human resource loss, and so on. Unethical behavior is likely to result in a reduction or loss of payoff.

SUMMARY

Decision analysis is a branch of quantitative management in which mathematical and statistical approaches are used to assist decision makers in reaching judgments about alternative opportunities. Three types of decisions are (1) decisions made under certainty, (2) decisions made under uncertainty, and (3) decisions made with risk. Several aspects of the decision-making situation are decision alternatives, states of nature, and payoffs. Decision alternatives are the options open to decision makers from which they can choose. States of nature are situations or conditions that arise after the decision has been made over which the decision maker has no control. The payoffs are the gains or losses that the decision maker will reap from various decision alternatives. These three aspects (decision alternatives, states of nature, and payoffs) can be displayed in a decision table or payoff table.

Decision making under certainty is the easiest of the three types of decisions to make. In this case, the states of nature are known, and the decision maker merely selects the decision alternative that yields the highest payoff.

Decisions are made under uncertainty when the likelihoods of the states of nature occurring are unknown. Four approaches to making decisions under uncertainty are maximax

criterion, maximin criterion, Hurwicz criterion, and minimax regret. The maximax criterion is an optimistic approach based on the notion that the best possible outcomes will occur. In this approach, the decision maker selects the maximum possible payoff under each decision alternative and then selects the maximum of these. Thus, the decision maker is selecting the maximum of the maximums.

The maximin criterion is a pessimistic approach. The assumption is that the worst case will happen under each decision alternative. The decision maker selects the minimum payoffs under each decision alternative and then picks the maximum of these as the best solution. Thus, the decision maker is selecting the best of the worst cases, or the maximum of the minimums.

The Hurwicz criterion is an attempt to give the decision maker an alternative to maximax and maximin that is somewhere between an optimistic and a pessimistic approach. With this approach, decision makers select a value called alpha between 0 and 1 to represent how optimistic they are. The maximum and minimum payoffs for each decision alternative are examined. The alpha weight is applied to the maximum payoff under each decision alternative and $1 - \alpha$ is applied to the minimum payoff. These two weighted values are combined

for each decision alternative, and the maximum of these weighted values is selected.

Minimax regret is calculated by examining opportunity loss. An opportunity loss table is constructed by subtracting each payoff from the maximum payoff under each state of nature. This step produces a lost opportunity under each state. The maximum lost opportunity from each decision alternative is determined from the opportunity table. The minimum of these values is selected, and the corresponding decision alternative is chosen. In this way, the decision maker has reduced or minimized the regret, or lost opportunity.

In decision making with risk, the decision maker has some prior knowledge of the probability of each occurrence of each state of nature. With these probabilities, a weighted payoff referred to as expected monetary value (EMV) can be calculated for each decision alternative. A person who makes decisions based on these EMVs is called an EMVer. The expected monetary value is essentially the average payoff that would occur if the decision process were to be played out over a long period of time with the probabilities holding constant.

The expected value of perfect information can be determined by comparing the expected monetary value if the states of nature are known to the expected monetary value. The difference in the two is the expected value of perfect information.

Utility refers to a decision maker's propensity to take risks. People who avoid risks are called risk avoiders. People who are prone to take risks are referred to as risk takers. People who use EMV generally fall between these two categories. Utility curves can be sketched to ascertain or depict a decision maker's tendency toward risk.

By use of Bayes' theorem, the probabilities associated with the states of nature in decision making under risk can be revised when new information is obtained. This information can be helpful to the decision maker. However, it usually carries a cost for the decision maker. This cost can reduce the payoff of decision making with sample information. The expected monetary value with sample information can be compared to the expected monetary value without it to determine the value of sample information.

KEY TERMS

decision alternatives
 decision analysis
 decision making under certainty
 decision making under risk
 decision making under uncertainty

decision table
 decision trees
 EMVer
 expected monetary value (EMV)
 expected value of perfect information

expected value of sample information
 Hurwicz criterion
 maximax criterion
 maximin criterion
 minimax regret

opportunity loss table
 payoffs
 payoff table
 risk avoider
 risk taker
 states of nature

FORMULA

Bayes' Rule

$$P(X_i|Y) = \frac{P(X_i) \cdot P(Y|X_i)}{P(X_1) \cdot P(Y|X_1) + P(X_2) \cdot P(Y|X_2) + \dots + P(X_n) \cdot P(Y|X_n)}$$

SUPPLEMENTARY PROBLEMS

CALCULATING THE STATISTICS

19.16 Use the following decision table to complete parts (a) through (d).

	State of Nature	
	s ₁	s ₂
Decision Alternative d ₁	50	100
d ₂	-75	200
d ₃	25	40
d ₄	75	10

a. Use the maximax criterion to determine which decision alternative to select.

- b. Use the maximin criterion to determine which decision alternative to select.
- c. Use the Hurwicz criterion to determine which decision alternative to select. Let α = .6.
- d. Compute an opportunity loss table from these data. Use this table and a minimax regret criterion to determine which decision alternative to select.

19.17 Use the following decision table to complete parts (a) through (c).

	State of Nature			
	s ₁ (.30)	s ₂ (.25)	s ₃ (.20)	s ₄ (.25)
Decision Alternative d ₁	400	250	300	100
d ₂	300	-100	600	200

- a. Draw a decision tree to represent this decision table.
- b. Compute the expected monetary values for each decision and label the decision tree to indicate what the final decision would be.
- c. Compute the expected payoff of perfect information. Compare this answer to the answer determined in part (b) and compute the value of perfect information.

19.18 Shown here is a decision table. A forecast can be purchased by the decision maker. The forecaster is not correct 100% of the time. Also given is a table containing the probabilities of the forecast being correct under different states of nature. Use the first table to compute the expected monetary value of this decision without sample information. Use the second table to revise the prior probabilities of the various decision alternatives. From this and the first table, compute the expected monetary value with sample information. Construct a decision tree to represent the options, the payoffs, and the expected monetary values. Calculate the value of sample information.

		State of Nature	
		$s_1 (.40)$	$s_2 (.60)$
Decision Alternative	d_1	\$200	\$150
	d_2	-\$75	\$450
	d_3	\$175	\$125

		State of Nature	
		s_1	s_2
Forecast	s_1	.90	.30
	s_2	.10	.70

TESTING YOUR UNDERSTANDING

19.19 Managers of a manufacturing firm decided to add Christmas tree ornaments to their list of production items. However, they have not decided how many to produce because they are uncertain about the level of demand. Shown here is a decision table that has been constructed to help the managers in their decision situation. Use this table to answer parts (a) through (c).

		State of Demand		
		Small	Moderate	Large
Decision Alternative (Produce)	Small Number	\$200	\$250	\$300
	Modest Number	\$100	\$300	\$600
	Large Number	-\$300	\$400	\$2,000

- a. Use maximax and maximin criteria to evaluate the decision alternatives.

- b. Construct an opportunity loss table and use minimax regret to select a decision alternative.
- c. Compare the results of the maximax, maximin, and minimax regret criteria in selecting decision alternatives.

19.20 Some companies use production learning curves to set pricing strategies. They price their product lower than the initial cost of making the product; after some period of time, the learning curve takes effect and the product can be produced for less than its selling price. In this way, the company can penetrate new markets with aggressive pricing strategies and still make a long-term profit.

A company is considering using the learning curve to set its price on a new product. There is some uncertainty as to how soon, if at all, the production operation will learn to make the product more quickly and efficiently. If the learning curve does not drop enough or the initial price is too low, the company will be operating at a loss on this product. If the product is priced too high, the sales volume might be too low to justify production. Shown here is a decision table that contains as its states of nature several possible learning-curve scenarios. The decision alternatives are three different pricing strategies. Use this table and the Hurwicz criterion to make a decision about the pricing strategies with each given value of alpha.

		State of Nature		
		No Learning	Slow Learning	Fast Learning
Decision Alternative	Price Low	-\$700	-\$400	\$1,200
	Price Medium	-\$300	-\$100	\$550
	Price High	\$100	\$125	\$150

- a. $\alpha = .10$
- b. $\alpha = .50$
- c. $\alpha = .80$
- d. Compare and discuss the decision choices in parts (a) through (c).

19.21 An entertainment company owns two amusement parks in the South. They are faced with the decision of whether to open the parks in the winter. If they choose to open the parks in the winter, they can leave the parks open during regular hours (as in the summer) or they can open only on the weekends. To some extent, the payoffs from opening the park hinge on the type of weather that occurs during the winter season. Following are the payoffs for various decision options about opening the park for two different weather scenarios: mild weather and severe weather. Use the information to construct a decision tree. Determine the expected monetary value and the value of perfect information.

		State of the Weather	
		Mild (.75)	Severe (.25)
Decision Alternative	Open Regular Hours	\$2,000	-\$2,500
	Open Weekends Only	\$1,200	-\$200
	Not Open at All	-\$300	\$100

19.22 A U.S. manufacturing company decided to consider producing a particular model of one of its products just for sale in Germany. Because of the German requirements, the product must be made specifically for German consumption and cannot be sold in the United States. Company officials believe the market for the product is highly price-sensitive. Because the product will be manufactured in the United States and exported to Germany, the biggest variable factor in being price competitive is the exchange rate between the two countries. If the dollar is strong, German consumers will have to pay more for the product in marks. If the dollar becomes weaker against the mark, Germans can buy more U.S. products for their money. The company officials are faced with decision alternatives of whether to produce the product. The states of the exchange rates are: dollar weaker, dollar stays the same, and dollar stronger. The probabilities of these states occurring are .35, .25, and .40, respectively. Some negative payoffs will result from not producing the product because of sunk development and market research costs and because of lost market opportunity. If the product is not produced, the payoffs are -\$700 when the dollar gets weaker, -\$200 when the dollar remains about the same, and \$150 when the dollar gets stronger. If the product is produced, the payoffs are \$1,800 when the dollar gets weaker, \$400 when the exchange rates stay about the same, and -\$1,600 when the dollar gets stronger.

Use this information to construct a decision tree and a decision table for this decision-making situation. Use the probabilities to compute the expected monetary values of the decision alternatives. On the basis of this information, which decision choice should the company make? Compute the expected monetary value of perfect information and the value of perfect information.

19.23 a. A small retailer began as a mom-and-pop operation selling crafts and consignment items. During the past two years, the store's volume grew significantly. The owners are trying to decide whether to purchase an automated checkout system. Their present manual system is slow. They are concerned about lost business due to inability to ring up sales quickly. The automated system would also offer some accounting and inventory advantages. The problem is that the automated system carries a large fixed cost, and the owners feel that sales volume would have to grow to justify the cost.

The following decision table contains the decision alternatives for this situation, the possible states of future sales, prior probabilities of those states occurring, and the payoffs. Use this information to compute the expected monetary payoffs for the alternatives.

		State of Sales		
		Reduction (.15)	Constant (.35)	Increase (.50)
Decision Alternative	Automate	-\$40,000	-\$15,000	\$60,000
	Don't Automate	\$5,000	\$10,000	-\$30,000

b. For a fee, the owners can purchase a sales forecast for the near future. The forecast is not always perfect. The probabilities of these forecasts being correct for particular states of sales are shown here. Use these probabilities to revise the prior state probabilities. Compute the expected monetary value on the basis of sample information. Determine the value of the sample information.

		State of Sales		
		Reduction	Constant	Increase
Forecast	Reduction	.60	.10	.05
	Constant	.30	.80	.25
	Increase	.10	.10	.70

19.24 a. A city is considering airport expansion. In particular, the mayor and city council are trying to decide whether to sell bonds to construct a new terminal. The problem is that at present demand for gates is not strong enough to warrant construction of a new terminal. However, a major airline is investigating several cities to determine which will become its new hub. If this city is selected, the new terminal will easily pay for itself. The decision to build the terminal must be made by the city before the airline will say whether the city has been chosen as its hub. Shown here is a decision table for this dilemma. Use this information to compute expected monetary values for the alternatives and reach a conclusion.

		State of Nature	
		City Chosen (.20)	City Not Chosen (.80)
Decision Alternative	Build Terminal	\$12,000	-\$8,000
	Don't Build Terminal	-\$1,000	\$2,000

b. An airline industry expert indicates that she will sell the city decision makers her best "guess" as to whether the city will be chosen as hub for the airline. The probabilities of her being right or wrong are given.

Use these probabilities to revise the prior probabilities of the city being chosen as the hub and then calculate the expected monetary value by using the sample information. Determine the value of the sample information for this problem.

		State of Nature	
		City Chosen	City Not Chosen
Forecast	City Chosen	.45	.40
	City Not Chosen	.55	.60

ANALYZING THE DATABASES

1. A carpet and rug manufacturer in the manufacturing database is faced with the decision of making expenditures in the form of machinery in anticipation of strong growth or not making the expenditures and losing market opportunity. Experts claim that there is about a 40% probability of having strong growth in the industry and a 60% probability of not having strong growth in the industry. If there is strong growth, the company will realize a payoff of \$5,500,000 if the capital expenditure is made. Under strong growth, if the company does not make the capital expenditure, it will still realize a \$750,000 payoff in new business, which it can handle with existing equipment. If there is not strong growth in the industry and the company has made the capital expenditure, the payoff will be $-\$2,000,000$. If there is not strong growth and the capital expenditure has not been made, the company will receive a payoff of \$500,000. Analyze this situation by using the decision analysis techniques presented in this chapter.
2. Suppose you are the CEO of a hospital in the hospital database. You are considering expansion of the physical facility. What are some decision alternatives to consider? What are some states of nature that can occur in this decision-making environment? How would you go about calculating the payoffs for such a decision?
3. Suppose you are the CEO of a company such as Procter & Gamble in the financial database. What are some decisions that you might make in which you would consider decision alternatives? Name three arenas in which you would be making substantial strategic decisions (e.g., marketing, finance, production, and human resources). Delineate at least three decision alternatives in each of these arenas. Examine and discuss at least two states of nature that could occur under these decision alternatives in each arena.

CASE

FLETCHER-TERRY: ON THE CUTTING EDGE

The Fletcher-Terry Company of Farmington, Connecticut, is a worldwide leader in the development of glass-cutting tools and accessories for professional glaziers, glass manufacturers, glass artisans, and professional framers. The company can trace its roots back to 1868 when a young engineer, Samuel Monce, developed and patented a hardened steel tool that could effectively replace expensive diamonds as a cutting device. Using this invention as his centerpiece, Monce formed the Monce Company, which went on to become a leader in glass-cutting devices for several decades. Meanwhile, in 1894, Monce's nephew Fred Fletcher got a patent on a replaceable-wheel cutter. Ten years later he went into business with his father-in-law, Franklin Terry, forming the Fletcher-Terry Company. In 1935, the Fletcher-Terry Company bought the Monce Company, thereby combining the assets and knowledge of the two companies.

For the next four decades, Fletcher-Terry had much success making its traditional product lines of hand-held glass cutters and cutting wheels for the glass, glazing, and hardware markets. However, by the 1980s, Fletcher-Terry was facing a crisis. Its two largest customers, distributors of cutting devices, decided to introduce their own private-label cutters made overseas. By the end of 1982, Fletcher-Terry's sales of hand-held glass cutters were down 45%.

Fletcher-Terry responded by investing heavily in technology with the hope that automation would cut costs; however, the technology never worked. The company then decided to expand its line of offerings by creating private lines through imports, but the dollar weakened and any price advantage was lost. Eventually, Fletcher-Terry had to write off this line with a substantial loss.

Company managers realized that if they did not change the way they did business, the company would not survive. They began a significant strategic planning process in which they set objectives and redefined the mission of the company. Among the new objectives were to increase market share where the company was already strong, penetrate new markets with new products, provide technological expertise for product development, promote greater employee involvement and growth, and achieve a sales growth rate twice that of the gross domestic product.

To accomplish these objectives, the company invested in plant and process improvements that reduced costs and improved quality. Markets were researched for both old and new products and marketing efforts were launched to reestablish the company's products as being "the first choice of professionals." A participatory management system was implemented that encouraged risk taking and creativity among employees.