Properties of Stock Options

Notation

- c: European call option price
- p: European put option price
- S_0: Stock price today
- K: Strike price
- T: Life of option
- σ: Volatility of stock price
- C: American call option price
- P: American put option price
- S_T: Stock price at option maturity
- D: PV of dividends paid during life of option
- r Risk-free rate for maturity T with cont. comp.

Effect of Variables on Option Pricing *(Table 11.1, page 232)*

Variable	c	p	C	P
S_0	+	−	+	−
K	−	+	−	+
T	?	?	+	+
σ	+	+	+	+
r	+	−	+	−
D	−	+	−	+

American vs European Options

An American option is worth at least as much as the corresponding European option

$$C \geq c$$

$$P \geq p$$

Calls: An Arbitrage Opportunity?

- Suppose that

$c = 3$	$S_0 = 20$
$T = 1$	$r = 10\%$
$K = 18$	$D = 0$

- Is there an arbitrage opportunity?

Lower Bound for European Call Option Prices; No Dividends (Equation 11.4, page 237)

$$c \geq \max(S_0 - Ke^{-rT}, 0)$$

Puts: An Arbitrage Opportunity?

- Suppose that

$p = 1$ $\quad S_0 = 37$

$T = 0.5$ $\quad r = 5\%$

$K = 40$ $\quad D = 0$

- Is there an arbitrage opportunity?

Lower Bound for European Put Prices; No Dividends

(Equation 11.5, page 238)

$$p \geq \max(Ke^{-rT} - S_0, 0)$$

Put-Call Parity: No Dividends

- Consider the following 2 portfolios:
  - Portfolio A: European call on a stock + zero-coupon bond that pays K at time T
  - Portfolio C: European put on the stock + the stock

Values of Portfolios

		$S_T > K$	$S_T < K$
Portfolio A	Call option	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K
Portfolio C	Put Option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

The Put-Call Parity Result
(Equation 11.6, page 239)

- Both are worth max(S_T, K) at the maturity of the options
- They must therefore be worth the same today. This means that

$$c + Ke^{-rT} = p + S_0$$

Arbitrage Opportunities

- Suppose that

$c = 3$ $S_0 = 31$

$T = 0.25$ $r = 10\%$

$K = 30$ $D = 0$

- What are the arbitrage possibilities when

$$p = 2.25 \ ?$$

$$p = 1 \ ?$$

Early Exercise

- Usually there is some chance that an American option will be exercised early
- An exception is an American call on a non-dividend paying stock
- This should never be exercised early

An Extreme Situation

- For an American call option:

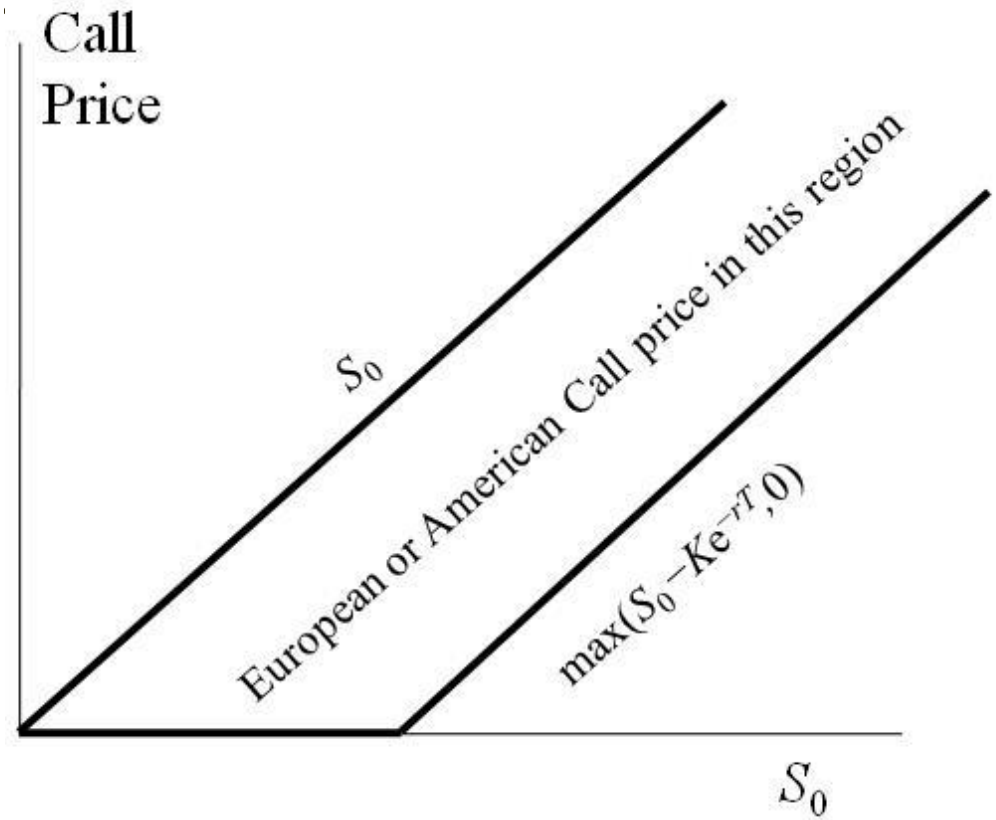
  $$S_0 = 100;\ T = 0.25;\ K = 60;\ D = 0$$

  Should you exercise immediately?
- What should you do if
  - You want to hold the stock for the next 3 months?
  - You do not feel that the stock is worth holding for the next 3 months?

Reasons For Not Exercising a Call Early (No Dividends)

- No income is sacrificed
- You delay paying the strike price
- Holding the call provides insurance against stock price falling below strike price

Bounds for European or American Call Op



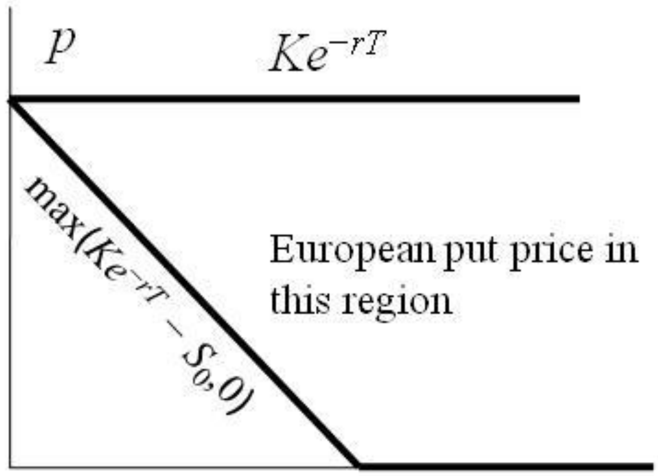
Should Puts Be Exercised Early ?

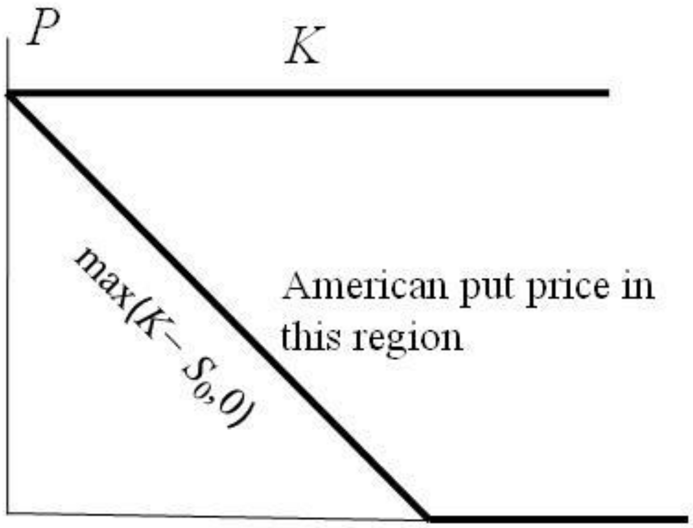
Are there any advantages to exercising an American put when

$S_0 = 60;\ T = 0.25;\ r = 10\%$

$K = 100;\ D = 0$

Bounds for European and American Put Options (No Dividends)





The Impact of Dividends on Lower Bounds to Option Prices

(Equations 11.8 and 11.9, page 246-247)

$$c \geq S_0 - D - Ke^{-rT}$$

$$p \geq D + Ke^{-rT} - S_0$$

Extensions of Put-Call Parity

- American options; $D = 0$

  $$S_0 - K < C - P < S_0 - Ke^{-rT}$$

  Equation 11.7 p. 240

- European options; $D > 0$

  $$c + D + Ke^{-rT} = p + S_0$$

  Equation 11.10 p. 247

- American options; $D > 0$

  $$S_0 - D - K < C - P < S_0 - Ke^{-rT}$$

  Equation 11.11 p. 247