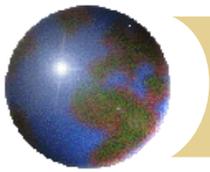


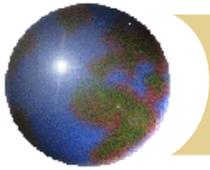
Options on Stock Indices and Currencies



Index Options (page 365-367)

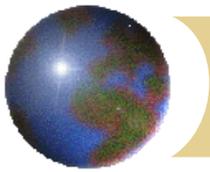
- ⊕ The most popular underlying indices in the U.S. are
 - ⊞ The S&P 100 Index (OEX and XEO)
 - ⊞ The S&P 500 Index (SPX)
 - ⊞ The Dow Jones Index times 0.01 (DJX)
 - ⊞ The Nasdaq 100 Index (NDX)

- ⊕ Exchange-traded contracts are on 100 times index; they are settled in cash; OEX is American; the XEO and all others are European



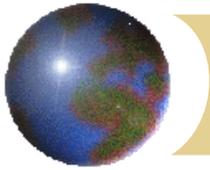
Index Option Example

- ✚ Consider a call option on an index with a strike price of 880
- ✚ Suppose 1 contract is exercised when the index level is 900
- ✚ What is the payoff?



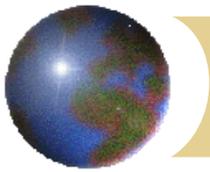
Using Index Options for Portfolio Insurance

- ✦ Suppose the value of the index is S_0 and the strike price is K
- ✦ If a portfolio has a β of 1.0, the portfolio insurance is obtained by buying 1 put option contract on the index for each $100S_0$ dollars held
- ✦ If the β is not 1.0, the portfolio manager buys β put options for each $100S_0$ dollars held
- ✦ In both cases, K is chosen to give the appropriate insurance level



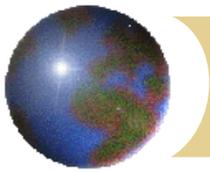
Example 1

- ✚ Portfolio has a beta of 1.0
- ✚ It is currently worth \$500,000
- ✚ The index currently stands at 1000
- ✚ What trade is necessary to provide insurance against the portfolio value falling below \$450,000?



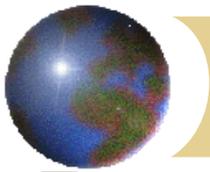
Example 2

- ✚ Portfolio has a beta of 2.0
- ✚ It is currently worth \$500,000 and index stands at 1000
- ✚ The risk-free rate is 12% per annum
- ✚ The dividend yield on both the portfolio and the index is 4%
- ✚ How many put option contracts should be purchased for portfolio insurance?



Calculating Relation Between Index Level and Portfolio Value in 3 months

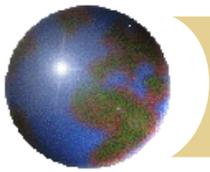
- ✚ If index rises to 1040, it provides a 40/1000 or 4% return in 3 months
- ✚ Total return (incl. dividends) = 5%
- ✚ Excess return over risk-free rate = 2%
- ✚ Excess return for portfolio = 4%
- ✚ Increase in Portfolio Value = $4+3-1=6\%$
- ✚ Portfolio value=\$530,000



Determining the Strike Price (Table 17.2, page 367)

Value of Index in 3 months	Expected Portfolio Value in 3 months (\$)
1,080	570,000
1,040	530,000
1,000	490,000
960	450,000
920	410,000

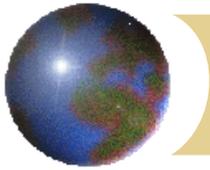
An option with a strike price of 960 will provide protection against a 10% decline in the portfolio value



European Options on Assets Providing a Known Yield

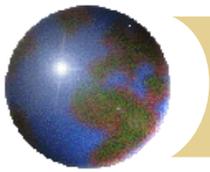
We get the same probability distribution for the asset price at time T in each of the following cases:

1. The asset starts at price S_0 and provides a yield $= q$
2. The asset starts at price $S_0 e^{-qT}$ and provides no income



European Options on Assets Providing Known Yield continued

We can value European options by reducing the asset price to $S_0 e^{-qT}$ and then behaving as though there is no income



Extension of Chapter 11 Results

(Equations 17.1 to 17.3)

Lower Bound for calls:

$$c \geq \max(S_0 e^{-qT} - Ke^{-rT}, 0)$$

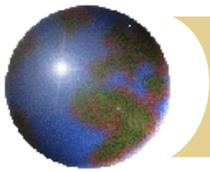
Lower Bound for puts

$$p \geq \max(Ke^{-rT} - S_0 e^{-qT}, 0)$$

Put Call Parity

$$c + Ke^{-rT} = p + S_0 e^{-qT}$$

$$c + Ke^{-rT} = p + F_0 e^{-rT}$$



Extension of Chapter 15 Results

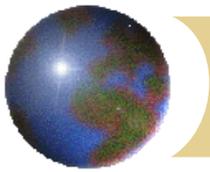
(Equations 17.4 and 17.5)

$$c = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2)$$

$$p = Ke^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

where $d_1 = \frac{\ln(S_0 / K) + (r - q + \sigma^2 / 2)T}{\sigma\sqrt{T}}$

$$d_2 = \frac{\ln(S_0 / K) + (r - q - \sigma^2 / 2)T}{\sigma\sqrt{T}}$$



Alternative Formulas (page 375)

$$c = e^{-rT} [F_0 N(d_1) - KN(d_2)]$$

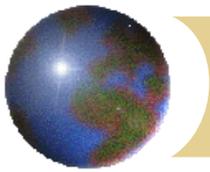
$$p = e^{-rT} [KN(-d_2) - F_0 N(-d_1)]$$

$$d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

where

$$F_0 = S_0 e^{(r-q)T}$$



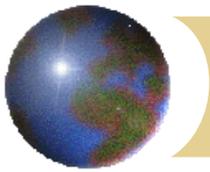
Valuing European Index Options

We can use these formulas for an option on an asset paying a dividend yield

Set S_0 = current index level

Set F_0 = futures or forward index price for a contract maturing at the same time as the option

Set q = average dividend yield expected during the life of the option

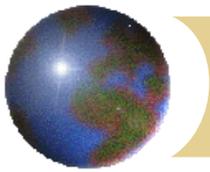


Implied Forward Prices and Dividend Yields

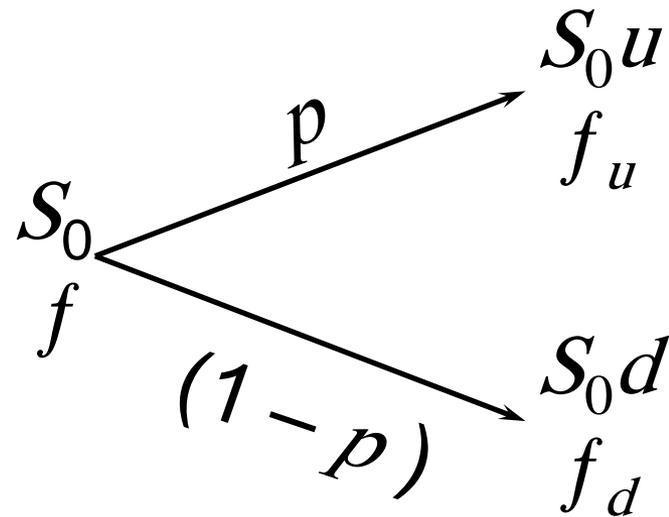
- From European calls and puts with the same strike price and time to maturity

$$F_0 = K + (c - p)e^{rT} \qquad q = -\frac{1}{T} \ln \frac{c - p + Ke^{-rT}}{S_0}$$

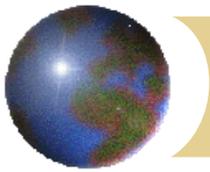
- These formulas allow term structures of forward prices and dividend yields to be estimated
- OTC European options are typically valued using the forward prices (Estimates of q are not then required)
- American options require the dividend yield term structure



The Binomial Model



$$f = e^{-rT} [pf_u + (1-p)f_d]$$



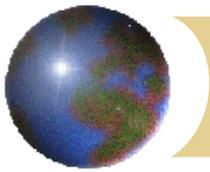
The Binomial Model *continued*

- ✚ In a risk-neutral world the asset price grows at $r-q$ rather than at r when there is a dividend yield at rate q
- ✚ The probability, p , of an up movement must therefore satisfy

$$pS_0u + (1-p)S_0d = S_0e^{(r-q)T}$$

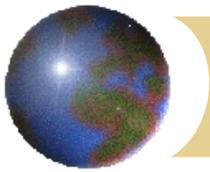
so that

$$p = \frac{e^{(r-q)T} - d}{u - d}$$



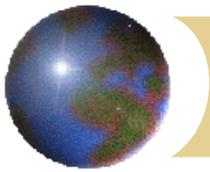
Currency Options

- ✚ Currency options trade on NASDAQ OMX
- ✚ There also exists a very active over-the-counter (OTC) market
- ✚ Currency options are used by corporations to buy insurance when they have an FX exposure



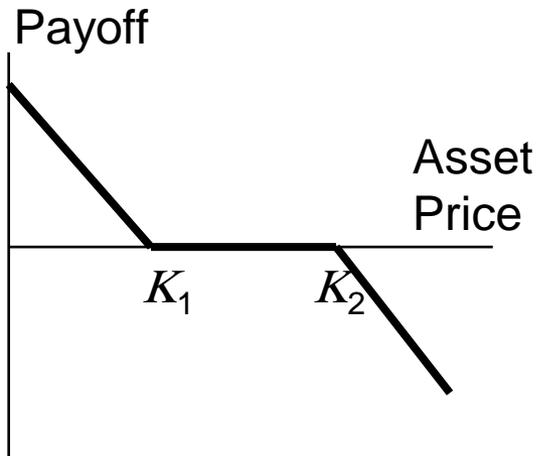
Range Forward Contracts

- Have the effect of ensuring that the exchange rate paid or received will lie within a certain range
- When currency is to be paid it involves selling a put with strike K_1 and buying a call with strike K_2 (with $K_2 > K_1$)
- When currency is to be received it involves buying a put with strike K_1 and selling a call with strike K_2
- Normally the price of the put equals the price of the call

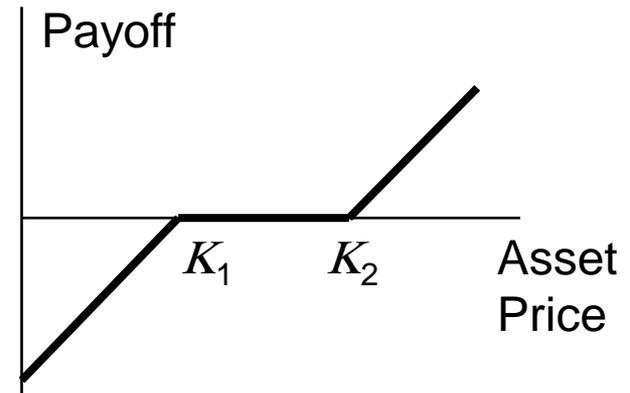


Range Forward Contract continued

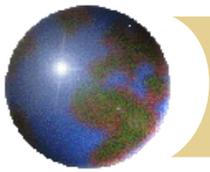
Figure 17.1, page 368



Short
Position

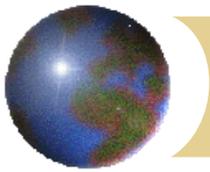


Long
Position



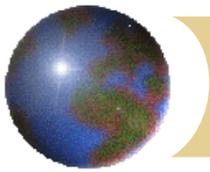
The Foreign Interest Rate

- ✦ We denote the foreign interest rate by r_f
- ✦ When a U.S. company buys one unit of the foreign currency it has an investment of S_0 dollars
- ✦ The return from investing at the foreign rate is $r_f S_0$ dollars
- ✦ This shows that the foreign currency provides a yield at rate r_f



Valuing European Currency Options

- ✿ A foreign currency is an asset that provides a yield equal to r_f
- ✿ We can use the formula for an option on a stock paying a dividend yield :
 - ✿ S_0 = current exchange rate
 - ✿ $q = r_f$



Formulas for European Currency

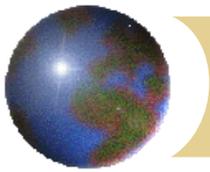
Options (Equations 17.11 and 17.12, page 375)

$$c = S_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-r_f T} N(-d_1)$$

$$\text{where } d_1 = \frac{\ln(S_0 / K) + (r - r_f + \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - r_f - \sigma^2 / 2)T}{\sigma \sqrt{T}}$$



Alternative Formulas (Equations 17.13 and 17.14)

Using $F_0 = S_0 e^{(r-r_f)T}$

$$c = e^{-rT} [F_0 N(d_1) - KN(d_2)]$$

$$p = e^{-rT} [KN(-d_2) - F_0 N(-d_1)]$$

$$d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$